Concepts of Math: Recitation 7

September 22, 2015

Weak and Strong Induction

1. Let S be a set with n elements, $n \geq 0$. Notation: $|S| = n$. Use (weak) induction to prove that $|\mathcal{P}(S)| = 2^n$. Stress the importance of this result.

2. A triominoe is a 2×2 board with a corner removed. Use (weak) induction to show that a $2^n \times 2^n$ board with a corner removed can be tiled with triominoes for any positive integer n.

3. Let $\{a_i\}_{i=0}^{\infty}$ be the sequence satisfying $a_1 = 1$, $a_2 = 13$ and $a_n = a_{n-1} + 6a_{n-2}$ for $n \geq 3$. Use (strong) induction to prove that

$$a_n = (-2)^n + 3^n, \text{ for } n \in \mathbb{N}.$$

4. Show that the sum of the angles of a convex n-gon is $(n-2)180$ degrees. The induction step is to split a convex $(n+1)$-gon into a triangle and a convex n-gon. This is where you use the convexity of the figure.