Concepts of Math: Recitation 3

September 4, 2015

DNF and CNF

1. Find the DNF of $p \to (q \wedge r)$ in two ways. First do it by using logical equivalences and property $x \Leftrightarrow ((x \wedge y) \vee (x \wedge (\neg y)))$. Then do it using the truth table. If confused, see page 26 in the Goodaire and Parmenter file I posted in DROPBOX, under OTHER.

2. Find the CNF of $p \to (q \wedge r)$ by using logical equivalences and property $x \Leftrightarrow ((x \vee y) \wedge (x \vee (\neg y)))$.

3. Here is one easy way to find the CNF of a statement $P(p_1, p_2, \ldots, p_n)$. First use the truth table to write the DNF of $\neg P(p_1, p_2, \ldots, p_n)$, then negate this DNF using De Morgan's Rules. The result is the CNF of $P(p_1, p_2, \ldots, p_n)$. Use this method to find the CNF of $p \to (q \wedge r)$.

Quantifiers

1. (a) What is the truth value of $\forall x : P(x)$, where $P(x)$ is the statement "$x^2 < 10$" and the universe consists of the positive integers not exceeding 4?

   (b) What is the truth value of $\exists x : P(x)$, where $P(x)$ is the statement "$x^2 > 10$" and the universe consists of the positive integers not exceeding 4?

   (c) What is the negation of $\forall x : x^2 > x$?

   (d) What is the negation of $\exists x : x^2 = 2)$?

   (e) Let $Q(x)$ denote "$x + y = 0$." What are the truth values of the quantifications $\exists y \forall x : Q(x, y)$ and $\forall x \exists y : Q(x, y)$?

2. Write the negation of the following statements:

   (a) Every positive even number $n \geq 6$ can be written as a sum of two prime numbers.

   (b) For every x and y, such that $x < y$, we have that $f(x) < f(y)$.