Concepts of Math: Recitation 22

November 11, 2015

Linear Diophantine Equations

In order to solve a Diophantine equation $am + bn = c$, we first need to find a particular solution (m, n) using the Euclidean Algorithm. The general solution has the for

$$\left(m + \frac{bk}{\gcd(a,b)}, n - \frac{ak}{\gcd(a,b)}\right)$$

for $k \in \mathbb{Z}$. Find the general solution of $170x + 28y = 518$. Use the Euclidean Algorithm to find a particular solution, do not guess!

Binary Relations

1. In class I defined reflexive and symmetric relations. Define a transitive relation as follows: a relation $\mathcal{R}$ on A is transitive if the following implication holds:

$$(a,b) \in \mathcal{R} \text{ and } (b,c) \in \mathcal{R} \Rightarrow (a,c) \in \mathcal{R}.$$

   Show that $\mathcal{R} = \{(x,y) \in \mathbb{R}^2 \mid x \leq y\}$ is a transitive relation on $\mathbb{R}$. Show that $\mathcal{R} = \{(x,y) \in \mathbb{N}^2 \mid y \text{ divides } x\}$ is a transitive relation on $\mathbb{N}$.

2. Determine whether the following relations on the set of people are reflexive/symmetric/transitive.

   (a) is a father of

   (b) is a friend of

   (c) is a descendant of

   (d) have the same parents

   (e) is an uncle of

3. Let S be a set with at least two elements in it. Determine whether the following relations on $\mathcal{P}(S)$ are reflexive/symmetric/transitive.

   (a) $\mathcal{R} = \{(X,Y) \mid X, Y \in \mathcal{P}(S),\ X \subseteq Y\}$

(b) $\mathcal{R} = \{(X, Y) \mid X, Y \in \mathcal{P}(S),\ X \subsetneq Y\}$

(c) $\mathcal{R} = \{(X, Y) \mid X, Y \in \mathcal{P}(S),\ X \cap Y = \emptyset\}$

4. Define an equivalence relation as a binary relation on a set A that is reflexive, symmetric and transitive.

5. Define $\mathcal{R}$ on $\mathbb{R}^2$ as following: $((x, y), (u.v)) \in \mathcal{R}$ if $x^2 + y^2 = u^2 + v^2$. Prove that $\mathcal{R}$ is an equivalence relation on $\mathbb{R}^2$.