Concepts of Math: Recitation 20

November 4, 2015

The Principle of Inclusion-Exclusion

If you have not finished the problems from last time, please finish them.

Divisibility

1. Use contradiction to prove that the set of prime numbers is not finite.
2. Let n be a positive integer. Construct a set of n consecutive positive integers that are not prime.
3. Find the largest integer k such that 5^k divides 250!. First you will need to express the exponent of a prime p in the factorization of $k!$ as a finite sum.
4. Please tell the students that the following two equations should be memorized (here $n > 1$ is a natural number).

$$a^n - b^n = (a-b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \ldots + b^{n-1})$$
$$a^{2n+1} + b^{2n+1} = (a+b)(a^{2n} - a^{2n-1}b + a^{2n-2}b^2 - \ldots + b^{2n})$$

   Prove these equations. Use these equations to show that $2^6 + 1$ and $2^{100} + 1$ are not primes.
5. Prove that if $2^n - 1$ is prime, then n is prime. Mention that primes of form $2^n - 1$ are called Mersenne primes. Give a few examples of Mersenne primes.