

Concepts of Math: Recitation 15

October 21, 2015

Combinatorics, Probability and the Pigeonhole Principle

Important: the pigeonhole principle will not be in Exam 2.

1. What is the coefficient of x^5y^7 in the expansion of $(2x - y)^{12}$? What is the coefficient of x^5y^{28} in the expansion of $(2x - 3y^2)^{19}$?
2. We wish to elect 10 members to a committee from 30 candidates, and you and two friends are among the candidates.
 - (a) What is the probability that you will be successful?
 - (b) Your best friend is one of the candidates. What is the probability that both of you are successful?
 - (c) What is the probability that you and exactly one of your two friends are elected?
 - (d) What is the probability that you and at least one of your two friends are elected?
 - (e) What is the probability that both your friends are elected but you are not?
3. There are 20 varieties of chocolates available and Linda wants to buy eight chocolates.
 - (a) What is the probability that at least one chocolate has a cherry center and at least one has caramel center?
 - (b) What is the probability that exactly one chocolate has a cherry center and exactly two chocolates have caramel center?
4. Let S be a subset of $[3n] = \{1, 2, \dots, 3n\}$ having size $2n+1$. Prove that S must contain three consecutive numbers. Show that this is the best possible by exhibiting a set of size $2n$ for which the conclusion is false.
5. Prove that every set of five points in the square of area 1 has two points separated by distance at most $\sqrt{2}/2$. Prove that this is best possible by exhibiting five points with no pair less than $\sqrt{2}/2$ apart.
6. A six by six checkerboard with 36 squares can be covered by exactly 18 dominoes. Prove that each such tiling can be cut by a horizontal or vertical line across the board without cutting any dominoes.