Concepts of Math: Recitations 14 and 15

October 19, 2015

Combinatorics Formulas

1. Given sets A and B, $|A| = s \geq 1$, $|B| = t \geq 1$.

   (a) How many functions from A to B are there?

   (b) How many injective functions from A to B are there?

   (c) How many functions from A to B are bijections?

   (d) How many functions from A to B have their range consisting of exactly one element of B (constant functions)?

2. (a) A man has 10 distinct candies and he puts them in two distinct bags such that each bag contains 5 candies. In how many ways can he do it?

   (b) A man has 10 identical candies and he puts them in two distinct bags such that each bag contains 5 candies. In how many ways can he do it?

   (c) A person has 10 distinct candies and he puts them in two identical bags such that no bag is empty. In how many ways can he do it?

   (d) A person has 10 identical candies and he puts them in two identical bags such that no bag is empty. In how many ways can he do it? The answer is 5. We know no formula for this one, just count all the possibilities.

3. There are 15 questions on a multiple-choice exam and five possible answers to each question.

   (a) In how many ways can the exam be answered?

   (b) In how many ways can the exam be answered with exactly eight answers correct?

4. There are 20 varieties of chocolates available and Linda wants to buy eight chocolates.

   (a) How many choices does she have?

   (b) How many choices does she have if her friend insists that at least one chocolate should have a cherry center?

(c) How many choices does she have if there remain only two chocolates with caramel center? (At least eight chocolates of all other varieties are available.)

5. In how many ways can 12 people form four groups of three if

   (a) the groups have names?
   (b) the groups are unnamed?

6. In how many ways can 18 different books be given to Tara, Danny, Shannon, and Mike, so that one person has six books, one has two books, and the other two people have five books each?

7. In class we proved that the number of non-negative integer solutions $(x_1, x_2, \ldots, x_k)$ to

$$x_1 + x_2 + \ldots + x_k = n$$

is

$$\binom{n + k - 1}{k}.$$

Count the positive integer solutions $(x_1, x_2, \ldots, x_k)$ to

$$x_1 + x_2 + \ldots + x_k = n.$$

Your answer should be a single binomial coefficient.

8. By counting a set it two ways, give a combinatorial proof of

$$n^2 = 2\binom{n}{2} + n.$$

9. By counting a set it two ways, give a combinatorial proof of

$$n^3 = 6\binom{n}{3} + 6\binom{n}{2} + n.$$

10. By counting a set it two ways, give a combinatorial proof of

$$\binom{2n}{n} = 2\binom{2n - 1}{n - 1}.$$

11. Using the Manhattan walk interpretation of Pascal's triangle, give a combinatorial proof to

$$\binom{n}{k}\binom{k}{m} = \binom{n}{m}\binom{n - m}{k - m}$$

12. Three couples, the Smiths, Jonses, and Murphys, are going to form a line.

   (a) In how many such lines will Mr. and Mrs. Jones be next to each other?

   (b) In how many such lines will Mr. and Mrs. Jones be next to each other and Mr. and Mrs. Murphy be next to each other?

   (c) In how many such lines will at least one couple be next to each other? Do not use inclusion-exclusion (we have not done that yet).

   (d) Find the number of arrangements of "CINCINNATI".