Concepts of Math: Recitation 13

October 12, 2015

Introduction to Combinatorics

In class I proved that the number of arrangements, also known as permutations, (ordered lists of length k, without repeats, from a set of size n) is

$$\frac{n!}{(n-k)!},$$

a direct result of the rule of product. The number of selections, also known as combinations, (subsets of size k from a set of size n) is

$$\binom{n}{k} = \frac{n!}{k!(n-k)!},$$

and was explained by noting that each selection occurs as an arrangement exactly $k!$ times.

1. Consider the set R of all 6-digit numbers where each digit is non-zero.
   (a) How many numbers are there in the set R?
   (b) How many numbers in R have distinct digits?
   (c) How many numbers in R have 1 as their first digit?
   (d) How many numbers in R have distinct digits as well as 2 as their first digit and 4 as their last digit?
2. In how many ways can one choose 8 people from 18 people and seat them
   (a) in a row from left to right?
   (b) in a circle?
   (c) in a square with 2 on each side?
   (d) in two rows of 4 facing each other?
3. The following problem is important. If you have not done it last time, please do it. Prove that if sets A and B are countable, then $A \times B$ is countable.

4. Count the bijections from A to B, given that $|A| = |B| = n$.

5. We wish to choose 9 cards from a usual deck of 52 playing cards.

   (a) In how many ways can be achieve this?

   (b) In how many ways can we achieve this if we are required to choose all cards from the same suit?

   (c) In how many ways can we achieve this if we are required to choose exactly 3 aces and 3 kings?

   (d) In how many ways can we achieve this if we are required to choose cards of different values (assuming that the 13 cards in each suit are of different values)?