Concepts of Math: Recitation 11

October 5, 2015

Bijective Functions

1. Let A and B be two finite sets. Let $f : A \to B$ be a function. Show that if f is surjective, then $|A| \geq |B|$. Show that if f is injective, then $|A| \leq |B|$. Conclude that, if f is bijective, then $|A| = |B|$.

2. Show that f from $[2, \infty)$ to $[-3, \infty)$ defined by $f(x) = x^2 - 4x + 1$ is a bijection.

3. Show that f from $\mathbb{R}^2$ to $\mathbb{R}^2$ defined by $f(x, y) = (x + y, x + y)$ is NOT a bijection.

4. Show that f from $\mathbb{R}^2$ to $\mathbb{R}^2$ defined by $f(x, y) = (x + y, x - y)$ is a bijection.

5. For $n \in \mathbb{N}$ we define $[n] = \{1, 2, 3, \ldots, n\}$. By convention $[0] = \emptyset$. Consider the function $f : \mathbb{N} \to \mathbb{N}$ defined by $f(x) = 2x - 1$. For $n \in \mathbb{N}$ find the set $f([n])$.

6. Let $\mathbf{A}$ be the set of all subsets of $[n]$ with an even number of elements and $\mathbf{B}$ be the set of all subsets of $[n]$ with an odd number of elements. Find a bijection from $\mathbf{A}$ to $\mathbf{B}$. Note that when n is odd, $f(S) = S^c$ works. When n is even, some creativity is necessary. Maybe they can play with it for a while to get an answer to verify.

7. If there is time left, answer questions about the homework.