Outline

1. Review: Inference using a Trained Network: Forward Propagation

2. Review: Training a Neural Network: Backpropagation

3. Optimizing SGD Parameters for Faster Convergence

4. Universality and Depth

5. Deep Neural Networks (DNNs)
Review: Inference using a Trained Network: Forward Propagation
How do you perform inference using a trained neural network?

- Expressing outputs $z$ in terms of inputs $x$ is called **forward-propagation**.
  - Express inputs $u_j$ to the hidden layer in terms of $x$
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Nodes in the hidden layer

Apply $g_j$ to $u_j$
Review: Training a Neural Network: Backpropagatiopn
Learning Parameters

How to learn the parameters?

- Choose the right loss function
  - Regression: Least-square loss (in class)
    \[
    \min \sum_n (f(x_n) - t_n)^2
    \]
  - Classification: cross-entropy loss (in the homework)
    \[
    \min -\sum_n \sum_k t_{nk} \log f_k(x_n) + (1 - t_{nk}) \log (1 - f_k(x_n))
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  - Hard optimization problem because \( f \) (the output of the neural network) is a complicated function of \( x_n \)
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    \[ \min - \sum_n \sum_k t_{nk} \log f_k(x_n) + (1 - t_{nk}) \log (1 - f_k(x_n)) \]

• Hard optimization problem because \( f \) (the output of the neural network) is a complicated function of \( x_n \)
  • Solution: Use Stochastic gradient descent (SGD)
  • Many optimization tricks are applied to speed-up SGD convergence
**Stochastic gradient descent**

- Randomly pick a data point \((x_n, t_n)\)
- Compute the gradient using only this data point, for example,

\[
\Delta = \frac{\partial [f(x_n) - t_n]^2}{\partial w}
\]

- Update the parameters: \(w \leftarrow w - \eta \Delta\)
- Iterate the process until some (pre-specified) stopping criteria
Back-propagate the error. Given parameters $w, b$:

- **Step 1**: Forward-propagate to find $z_k$ in terms of the input (the “feed-forward signals”).
- **Step 2**: Calculate output error $E$ by comparing the predicted output $z_k$ to its true value $t_k$.
- **Step 3**: Back-propagate $E$ by weighting it by the gradients of the associated activation functions and the weights in previous layers.
- **Step 4**: Calculate the gradients $\frac{\partial E}{\partial w}$ and $\frac{\partial E}{\partial b}$ for the parameters $w, b$ at each layer based on the backpropagated error signal and the feedforward signals from the inputs.
- **Step 5**: Update the parameters using the calculated gradients $w \leftarrow w - \eta \frac{\partial E}{\partial w}, b \leftarrow b - \eta \frac{\partial E}{\partial b}$ where $\eta$ is the step size.
Illustrative example

- \( w_{ij} \): weights connecting node \( i \) in layer \((\ell - 1)\) to node \( j \) in layer \( \ell \).
- \( b_j, b_k \): bias for nodes \( j \) and \( k \).
- \( u_j, u_k \): inputs to nodes \( j \) and \( k \) (where \( u_j = b_j + \sum_i x_i w_{ij} \)).
- \( g_j, g_k \): activation function for node \( j \) (applied to \( u_j \)) and node \( k \).
- \( y_j = g_j(u_j), z_k = g_k(u_k) \): output/activation of nodes \( j \) and \( k \).
- \( t_k \): target value for node \( k \) in the output layer.
Illustrative example (steps 1 and 2)

- Step 1: **Forward-propagate** for each output $z_k$.

$$z_k = g_k(u_k) = g_k(b_k + \sum_j y_j w_{jk}) = g_k(b_k + \sum_j g_j(b_j + \sum_i x_i w_{ij}) w_{jk})$$

Nodes in the hidden layer

Apply $g_j$ to $u_j$
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- Step 2: Find the error. Let's assume that the error function is the sum of the squared differences between the target values $t_k$ and the network output $z_k$: $E = \frac{1}{2} \sum_{k \in K} (z_k - t_k)^2$. 
Step 3: Backpropagate the error. Let’s start at the output layer with weight $w_{jk}$, recalling that $E = \frac{1}{2} \sum_{k \in K} (z_k - t_k)^2$, $u_k = b_k + \sum_j w_{jk} y_j$:

$$\frac{\partial E}{\partial w_{jk}} = \frac{\partial E}{\partial z_k} \frac{\partial z_k}{\partial u_k} \frac{\partial u_k}{\partial w_{jk}} = (z_k - t_k) \frac{\partial z_k}{\partial u_k} \frac{\partial u_k}{\partial w_{jk}}$$
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$$= (z_k - t_k)g'_k(u_k) \frac{\partial}{\partial w_{jk}} u_k$$

where $\delta_k = (z_k - t_k)g'_k(u_k)$ is called the error in $u_k$. 

Nodes in the output layer

Apply $g_k$ to $u_k$
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$$= (z_k - t_k) g'_k(u_k) \frac{\partial}{\partial w_{jk}} u_k = (z_k - t_k) g'_k(u_k) y_j = \delta_k y_j$$

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Apply $g_k$ to $u_k$
Illustrative example (step 3, hidden layer)

Step 3 (cont’d): Now let’s consider $w_{ij}$ in the hidden layer, recalling $u_j = b_i + \sum_i x_i w_{ij}$, $u_k = b_k + \sum_j g_j(u_j) w_{jk}$, $z_k = g_k(u_k)$:

$$\frac{\partial E}{\partial w_{ij}} = \sum_{k \in K} \frac{\partial E}{\partial u_k} \frac{\partial u_k}{\partial y_j} \frac{\partial y_j}{\partial u_j} \frac{\partial u_j}{\partial w_{ij}} = \sum_{k \in K} \delta_k w_{jk} \frac{\partial y_j}{\partial u_j} \frac{\partial u_j}{\partial w_{ij}}$$
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$$= \sum_{k \in K} \delta_k w_{jk} g_j'(u_j) x_i = \delta_j x_i$$

where we substituted $\delta_j = g_j'(u_j) \sum_{k \in K} (z_k - t_k) g_k'(u_k) w_{jk}$, the error in $u_j$. 

Apply $g_j$ to $u_j$

Nodes in the hidden layer
Illustrative example (steps 3 and 4)

Step 3 (cont’d): We similarly find that \( \frac{\partial E}{\partial b_k} = \delta_k \), \( \frac{\partial E}{\partial b_j} = \delta_j \).

Step 4: Calculate the gradients. We have found that

\[
\frac{\partial E}{\partial w_{ij}} = \delta_j x_i \quad \text{and} \quad \frac{\partial E}{\partial w_{jk}} = \delta_k y_j.
\]

where

\[
\delta_k = (z_k - t_k)g'_k(u_k), \quad \delta_j = g'_j(u_j) \sum_{k \in K} (z_k - t_k)g'_k(u_k)w_{jk}.
\]

Now since we know the \( z_k, y_j, x_i, u_k \) and \( u_j \) for a given set of parameter values \( w, b \), we can use these expressions to calculate the gradients at each iteration and update them.
Illustrative example (steps 4 and 5)

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where \(\delta_k = (z_k - t_k)g'_k(u_k)\), \(\delta_j = g'_j(u_j) \sum_{k \in K}(z_k - t_k)g'_k(u_k)w_{jk}\).

- Step 5: Update the weights and biases with learning rate \(\eta\). For example

\[
w_{jk} \leftarrow w_{jk} - \eta \frac{\partial E}{\partial w_{jk}} \quad \text{and} \quad w_{ij} \leftarrow w_{ij} - \eta \frac{\partial E}{\partial w_{ij}}
\]
High-level Procedure: Can be Used with More Hidden Layers

Final Layer

- Error in each of its outputs is $z_k - t_k$.
- Error in input $u_k$ to the final layer is $\delta_k = g'_k(u_k)(z_k - t_k)$

Hidden Layer

- Error in output $y_j$ is $\sum_{k \in K} \delta_k w_{jk}$.
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The gradient w.r.t. $w_{ij}$ is $x_i \delta_j$. 
Vectorized Implementation

Much faster than implementing a loop over all neurons in each layer

Forward-Propagation

- Represent the weights between layers $l-1$ and $l$ as a matrix $W^{(l)}$ and biases as a row vector $b^{(l)}$
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Forward-Propagation

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- Outputs of layer \( l - 1 \) are in a row vector \( \mathbf{y}^{(l-1)} \). Then we have
  \[
  \mathbf{u}^{(l)} = \mathbf{y}^{(l-1)} \mathbf{W}^{(l)} + \mathbf{b}^{(l)}.
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Forward-Propagation

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- Outputs of layer $l - 1$ are in a row vector $y^{(l-1)}$. Then we have $u^{(l)} = y^{(l-1)}W^{(l)} + b^{(l)}$.
- Outputs of layer $l$ are in the row vector $y^{(l)} = g(u^{(l)})$. 

Nodes in the hidden layer

Apply $g_j$ to $u_j$
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![Diagram of a neural network with nodes and weights](image)

Back-Propagation

- For each layer $l$ find $\Delta^{(l)}$, the vector of errors in $u^{(l)}$ in terms of the final error

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- Update weights $W^{(l)}$ using $\Delta^{(l)}$
- Recursively find $\Delta^{(l-1)}$ in terms $\Delta^{(l)}$
Optimizing SGD Parameters for Faster Convergence
Mini-batch SGD

- Recall the empirical risk loss function that we considered for the backpropagation discussion

\[
E = \sum_{n=1}^{N} \frac{1}{2} (f(x_n) - t_n)^2
\]
Mini-batch SGD

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- For large training datasets (large \( N \)), then computing gradients with respect to each datapoint is expensive. For example, for the last year, the batch gradients are

\[ \frac{\partial E}{\partial w_{jk}} = \sum_{n=1}^{N} (z_k - t_k) \]
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\[ \frac{\partial E}{\partial w_{jk}} = \sum_{n=1}^{N} (z_k - t_k) \]

- Therefore we use stochastic gradient descent (SGD), where we choose a random data point \( x_n \) and use \( E = \frac{1}{2} (f(x_n) - t_n)^2 \) instead of the entire sum
• Mini-batch SGD is in between these two extremes
Mini-batch SGD

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- In each iteration, we choose a set $S$ of $m$ samples from the $N$ training samples and use

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for backpropagation
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• Large $m$ reduces gradient noise and gives better error convergence, but increases computing cost per iteration
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- Small \( m \) saves per-iteration computing cost, but increases noise in the gradients and yields worse error convergence
- Large \( m \) reduces gradient noise and typically gives better error convergence, but increases computing cost per iteration
How to Choose Mini-batch size $m$

- Small training datasets – use batch gradient descent $m = N$
- Large training datasets – typical $m$ are 64, 128, 256 ... whatever fits in the CPU/GPU memory
- Mini-batch size is another hyperparameter that you have to tune

Vectorized Implementation of Mini-batch SGD

Much faster than implementing a loop over all neurons in each layer and all samples in a mini-batch

Forward-Propagation

- Represent the weights between layers $l-1$ and $l$ as a matrix $W^{(l)}$ and biases as a vector $b^{(l)}$ (dimensions do NOT depend on $m$)
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Forward-Propagation

- Represent the weights between layers $l - 1$ and $l$ as a matrix $\mathbf{W}^{(l)}$ and biases as a vector $\mathbf{b}^{(l)}$ (dimensions do NOT depend on $m$)
- Outputs of layer $l - 1$ are arranged in an $m \times n_{l-1}$ size matrix $\mathbf{Y}^{(l-1)}$, where each row is the layer $l - 1$ outputs for one sample.
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Forward-Propagation (contd)

- Then we have $m \times n_l$ matrix $U^{(l)} = Y^{(l-1)}W^{(l)} + [1, 1, \ldots, 1]^T b^{(l)}$ of layer $l$ inputs, where $[1, 1, \ldots, 1]^T$ is a column of $m$ ones
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- Outputs of layer $l$ is an $m \times n_{(l)}$ matrix $Y^{(l)} = g(U^{(l)})$. 

Nodes in the hidden layer
Apply $g_j$ to $u_j$
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Back-Propagation

- For each layer $l$ find the $m \times n(l)$ size matrix $\Delta^{(l)}$ of errors in $U^{(l)}$ in terms of the final error $E$
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![Diagram of a neural network with nodes and weights](image)

**Back-Propagation**

- For each layer $l$ find the $m \times n_{(l)}$ size matrix $\Delta^{(l)}$ of errors in $U^{(l)}$ in terms of the final error $E$
- Update weights $W^{(l)}$ using $\Delta^{(l)}$
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Back-Propagation

- For each layer \( l \) find the \( m \times n_{(l)} \) size matrix \( \Delta^{(l)} \) of errors in \( U^{(l)} \) in terms of the final error \( E \)
- Update weights \( W^{(l)} \) using \( \Delta^{(l)} \)
- Recursively find \( \Delta^{(l-1)} \) in terms \( \Delta^{(l)} \)
Learning Rate

- **SGD Update Rule**
  \[ w^{(t+1)} = w^{(t)} - \eta \frac{\partial E}{\partial w^{(t)}} = w^{(t)} - \eta \nabla E(w^{(t)}) \]

- **Large \( \eta \)**: Faster convergence, but higher error floor (the flat portion of each curve)
- **Small \( \eta \)**: Slow convergence, but lower error floor (the blue curve will eventually go below the red curve)
- To get the best of both worlds, decay \( \eta \) over time
A common way to decay $\eta$

- Start with some learning rate, say $\eta = 0.1$

Image Source: http://www.bdhammel.com/learning-rates/
How to Decay the Learning Rate?

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- Start with some learning rate, say $\eta = 0.1$
- Monitor the training loss and wait till it flattens
- Reduce $\eta$ by a fixed factor, say 5. New $\eta = 0.02$. 

How to Decay the Learning Rate?

A common way to decay $\eta$

- Start with some learning rate, say $\eta = 0.1$
- Monitor the training loss and wait till it flattens
- Reduce $\eta$ by a fixed factor, say 5. New $\eta = 0.02$.
- Reduce again by the same factor when curve flattens

Image Source: http://www.bdhammel.com/learning-rates/
An alternate approach – AdaGrad [Duchi et al 2011]

- Divide the learning rate $\eta$ by the square root of the sum of squares of gradients until that time.

$$w(t+1)_i = w(t)_i - \eta \sqrt{g(t)_i^2 + \epsilon} \nabla E(w(t)_i)$$

- In a modified version AdaDelta, you take the sum of square gradients over a fixed size sliding window instead of all times from 1 to $t$. 
An alternate approach – AdaGrad [Duchi et al 2011]

- Divide the learning rate $\eta$ by the square root of the sum of squares of gradients until that time.
- This scaling factor is different for each parameter depending upon the corresponding gradients.

$$
\mathbf{w}_i^{(t+1)} = \mathbf{w}^{(t)} - \frac{\eta}{\sqrt{g_i^{(t)}} + \epsilon} \nabla E(\mathbf{w}_i^{(t)})
$$

where $g_i^{(t)} = \sum_{k=1}^{t} (\nabla E(\mathbf{w}_i^{(k)}))^2$. 
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$$w_i^{(t+1)} = w_i^{(t)} - \eta \frac{\nabla E(w_i^{(t)})}{\sqrt{g_i^{(t)}} + \epsilon}$$

where $g_i^{(t)} = \sum_{k=1}^{t} (\nabla E(w_i^{(k)}))^2$.

- In a modified version AdaDelta, you take the sum of square gradients over a fixed size sliding window instead of all times from 1 to $t$. 

How to Decay the Learning Rate?
• Remember the update to \( w \) in the previous iteration, that is, 
\[
 w(t) - w(t-1)
\] 
• \( \alpha \) is called the momentum, and it is typically set to around 0.9 in neural network training.

If current speed is fast, then we move even faster in the next iteration.
Momentum – Accelerating SGD Convergence

- Remember the update to $w$ in the previous iteration, that is, $w^{(t)} - w^{(t-1)}$
- Add it to the next iteration’s update, that is,

$$w^{(t+1)} = w^{(t)} - \eta \nabla E(w^{(t)}) + \alpha (w^{(t)} - w^{(t-1)})$$

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Universality and Depth
Architecture Design

- First layer: \( h^{(1)} = g^{(1)}(W^{(1)^T}x + b^{(1)}) \)
- Second layer: \( h^{(2)} = g^{(2)}(W^{(2)^T}h^{(1)} + b^{(2)}) \)
- How do we decide depth, width?
- In theory how many layers suffice?
Universality

- **Theoretical result** [Cybenko, 1989]: 2-layer net with linear output with some squashing non-linearity in hidden units can approximate any continuous function over compact domain to arbitrary accuracy (given enough hidden units!)
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- But not guaranteed that our training algorithm will be able to learn that function

- Gives no guidance on how large the network will be (exponential size in worst case)
Advantages of Depth

Figure 1: Goodfellow et al., 2014

- Increasing the depth of a neural network generally improves test accuracy
Advantages of Depth

• Control experiments show that other increases to model size don’t yield the same effect.

• These are a lot of parameters...

Figure 2: Goodfellow et al., 2014
Preventing Overfitting

- **Approach 1** Get more data
Preventing Overfitting

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  - Always best if possible!
Preventing Overfitting

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  - If no natural ones, use data augmentation

- **Approach 2** Regularization
  - Add regularizer term to the objective function
  - Need to incorporate this in back-propagation

- **Approach 3** Choose network structure with the right capacity:
  - Enough to fit the true regularities.
  - Not enough to also fit spurious regularities (if they are weaker).
  - Requires parameter tuning, hard to guess the right size.

- **Approach 4** Average many different models
  - Models with different forms to encourage diversity
  - Train on different subsets of data
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Consider a fully connected neural net with $H$ nodes in hidden layers.
Dropout

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• Each time we present a training example (for each iteration of SGD), we randomly omit each hidden unit with probability 0.5.

• So we are randomly sampling from $2^H$ different architectures.

• All architectures share weights.
Dropout as preventing co-adaptation

- If a hidden unit knows which other hidden units are present, it can **co-adapt to them** on the training data.
Dropout as preventing co-adaptation

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- But complex co-adaptations are likely to go wrong on new test data.

![Diagram](image-url)

(a) Standard Neural Net
(b) After applying dropout.

Present with probability $p$
(a) At training time

Always present
(b) At test time
Dropout as preventing co-adaptation

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- Dropout as orthogonalization
Dropout as form of model averaging

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- We sample from $2^H$ models. So only a few of the models ever get trained, and they only get one training example.
- The sharing of the weights means that every model is very strongly regularized.
  - It’s a much better regularizer than L2 or L1 penalties that pull the weights towards zero.
  - Note that it’s hard to generalize dropout to other types of ML models, unlike L2 or L1 penalties.
What do we do at test time?

- We could sample many different architectures and take the geometric mean of their output distributions.

- It's better to use all of the hidden units, but to halve their outgoing weights.

- This exactly computes the geometric mean of the predictions of all $2^n$ models (why?).

- This is not exactly the same as averaging all the separate dropped-out models, but it's a pretty good approximation, and it's fast.

- Alternatively, run the stochastic model (i.e., the different architectures) several times on the same input.

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Some dropout tips

- Dropout lowers your **capacity**
  - Increase network size by $n/p$ where $n$ is number of hidden units in original, $p$ is probability of dropout

- Dropout slows down error convergence
- Increase learning rate by 10 to 100
- Or increase momentum (e.g. from 0.9 to 0.99)
- These can cause large weight growths, use weight regularization
- May require more iterations to converge
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Deep Neural Networks (DNNs)
Basic idea behind DNNs

Architecturally, a big neural network (with a lot of variants)

- **in depth**: 4-5 layers are commonly (Google LeNet uses more than 20)
- **in width**: each layer might have a few thousand hidden units
- **the number of parameters**: hundreds of millions, even billions
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Algorithmically, many new things, including:

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Computing

- Requires **fast computations** and coping with a lot of data
- Ex: fast Graphics Processing Unit (GPUs) are almost indispensable
• Deep supervised neural networks are generally too difficult to train
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• One notable exception: **Convolutional neural networks (CNN)**
Deep Convolutional Networks

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- Convolutional nets were inspired by the visual system’s structure
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One notable exception: Convolutional neural networks (CNN).

Convolutional nets were inspired by the visual system’s structure.

They typically have more than five layers, a number of layers which makes fully-connected neural networks almost impossible to train properly when initialized randomly.
Deep Convolutional Networks

- Compared to standard feedforward neural networks with similarly-sized layer
  - CNNs have much fewer connections and parameters
  - and so they are easier to train
  - while their theoretically-best performance is likely to be only slightly worse.
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  - CNNs have much fewer connections and parameters
  - and so they are easier to train
  - while their theoretically-best performance is likely to be only slightly worse.

- Usually applied to image datasets (where convolutions have a long history).

LeNet 5

Convolution

- Continuous functions:

\[(f \ast g)(t) = \int_{-\infty}^{\infty} f(\tau)g(t - \tau)d\tau\]

\[= \int_{-\infty}^{\infty} f(t - \tau)g(\tau)d\tau\]
Convolution

- **Continuous functions**:
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  \]

- **Discrete functions**:
  \[
  (f \ast g)[n] = \sum_{m=-\infty}^{\infty} f[m]g[n - m]
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  \]
Convolution

- If discrete $g$ has support on $-M,...,M$:

\[
(f * g)[n] = \sum_{m=-M}^{M} f[n - m]g[m]
\]

Where $g[m]$ is the kernel
Convolution

- If discrete g has support on \(-M,...,M\):

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- Product of polynomials

\[ [1, 2] \ast [3, 2, 5] = (x + 2) \ast (3x^2 + 2x + 5) = 3x^3 + 8x^2 + 9x + 10 \]
\[ [1 \times 3 + 2 \times 0, 1 \times 2 + 2 \times 3, 1 \times 5 + 2 \times 2, 1 \times 0 + 2 \times 5] = [3, 8, 9, 10] \]

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Where $[1, 2]$ is the kernel of convolution
2-Dimensional Convolution

\[ f[x, y] \ast g[x, y] = \sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} f[n_1, n_2] \cdot g[x - n_1, y - n_2] \]

https://graphics.stanford.edu/courses/cs178/applets/convolution.html

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Convolve subsets of an image with a small filter.

- Each pixel in the output image is a weighted sum of the filter and a subset of the input.
- Learn the values in the filter (these are your parameters, or weights).
LeNet 5, LeCun 1998

- **Input**: 32 × 32 pixel image. Largest character is 20 × 20 (All important info should be in the center of the receptive field of the highest level feature detectors)
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- Black and White pixel values are normalized:
  Eg. White=−0.1, Black=−1.175 (Mean of pixels = 0, Standard deviation of pixels = 1)
C1: Convolutional layer with 6 feature maps of size 28×28 $C^k_1 (k = 1..6)$
Each unit of C1 has 5x5 receptive field in the input layer.
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Each unit of C1 has 5x5 receptive field in the input layer.

- Topological structure
- Sparse connections
- Shared weights

$$ (5 \times 5 + 1) \times 6 = 156 \text{ parameters to learn} $$

Connections: $28 \times 28 \times (5 \times 5 + 1) \times 6 = 122304$
If it was fully connected, we had $(32\times 32 + 1) \times (28 \times 28) \times 6 \text{ parameters} $
LeNet 5, Layer S2

S2: Sub-sampling layer with 6 feature maps of size $14 \times 14$
$2 \times 2$ non-overlapping receptive fields in C1

$$S2^k_{ij} = \tanh(w_1^k \sum_{s,t=0}^{1} C1^k_{2i-s,2j-t} + w_2^k)$$

Layer S2: $6*2=12$ trainable parameters
Connections: $14 \times 14 \times (2 \times 2 + 1) \times 6 = 5880$
LeNet 5, Layer S2

S2: Sub-sampling layer with 6 feature maps of size $14 \times 14$

$2 \times 2$ non-overlapping receptive fields in C1

These days, we typically use

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2 × 2 Max-Pool

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- C5: Convolutional layer with 120 feature maps of size 1x1
- Each unit in C5 is connected to all 16 5x5 receptive fields in S4
LeNet 5, Layer C5

- C5: **Convolutional layer with 120 feature maps of size 1x1**
- Each unit in C5 is connected to all 16 5x5 receptive fields in S4

Layer C5: $120 \times (16 \times 25 + 1) = 48120$ trainable parameters and connections (Fully Connected)
• Layer F6: **84 fully connected nodes.** $84 \times (120 + 1) = 10164$ trainable parameters and connections.
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• Output layer: 10 RBF (One for each digit)

$$y_i = \sum_{j=1}^{84} (x_j - w_{ij})^2$$

Where $i = 1, 2, \ldots, 10$
LeNet 5, Layer F6

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Where $i = 1, 2, .., 10$

84 = 7x12, stylized image

**Weight update:** Backpropagation
Today’s networks can go much deeper than LeNet!
• Used to model **temporal data** (e.g., speech recognition).
Recurrence Neural Networks

- Used to model \textit{temporal data} (e.g., speech recognition).
- Results can flow backwards (we use hidden node outputs from previous times as inputs to the current node).
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- Creates an “internal state” of the hidden node input/outputs.

- Several variants, e.g., long short-term memory (LSTM) networks.
You should know:

- How to use dropout to prevent overfitting.
- Differences between a convolutional and feedforward neural network.