Problem Set 3

16825 Learning for 3D Vision (Spring 2024)
https://piazza.com/cmu/spring2024/16825

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Instructor: Shubham Tulsiani
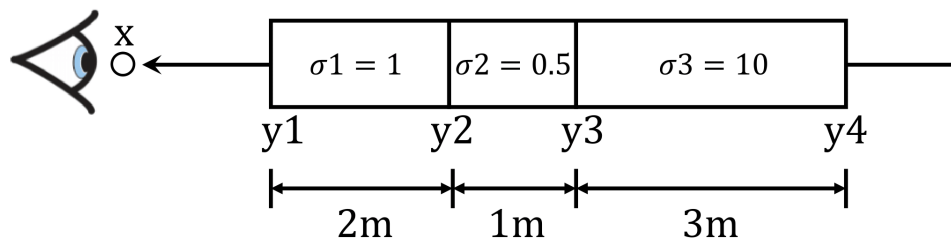TAs: Anurag Ghosh, Ayush Jain, Bharath Raj, Ruihan Gao, Shun Iwase

1. [**10 pts**]

Figure 1: A ray through a non-homogeneous medium. The medium is composed of 3 segments ($y1y2$, $y2y3$, $y3y4$). Each segment has a different absorption coefficient, shown as $\sigma_1, \sigma_2, \sigma_3$ in the figure. The length of each segment is also annotated in the figure (1m means 1 meter).

As shown in Figure 1, we observe a ray going through a non-homogeneous medium. Please compute the following transmittance:

- $T(y1, y2)$
- $T(y2, y4)$
- $T(x, y4)$
- $T(x, y3)$

一条射线从位置 a 走到 b，穿过吸收系数为 σ(s) 的介质，透射率：

$$T(a,b) = \exp\left(-\int_a^b \sigma(s)\,ds\right)$$

若介质**分段常数**（每一段吸收系数恒定），就变成**每段指数的乘积**：

$$T = \prod_i \exp\left(-\sigma_i\,\Delta\ell_i\right) = \exp\left(-\sum_i \sigma_i\,\Delta\ell_i\right)$$

你题里的情形：介质分成三段 $[y_1, y_2], [y_2, y_3], [y_3, y_4]$，对应 $\sigma_1, \sigma_2, \sigma_3$。记每段长度为 $\ell_{12}, \ell_{23}, \ell_{34}$。

- $T(y_1, y_2)$

  只穿过第一段： $T(y_1, y_2) = \exp(-\sigma_1\,\ell_{12})$.

- $T(y_2, y_4)$

  穿过第二 + 第三段： $T(y_2, y_4) = \exp\left(-(\sigma_2\,\ell_{23} + \sigma_3\,\ell_{34})\right)$.

- $T(x, y_4)$

  要看 x 在哪一段：

  - 若 $x \in [y_1, y_2]$，先走完第一段的剩余长度 $(\ell_{12}^{\mathrm{rem}} = |y_2 - x|)$，再过第二、第三段：

    $$T(x, y_4) = \exp\left(-(\sigma_1\,|y_2 - x| + \sigma_2\,\ell_{23} + \sigma_3\,\ell_{34})\right).$$

  - 若 $x \in [y_2, y_3]$：

    $$T(x, y_4) = \exp\left(-(\sigma_2\,|y_3 - x| + \sigma_3\,\ell_{34})\right).$$

  - 若 $x \in [y_3, y_4]$： $T(x, y_4) = \exp(-\sigma_3\,|y_4 - x|)$.

  - 若 x 在 y_1 之前或 y_4 之后，同理加上/减掉对应段的长度和系数。

- $T(x, y_3)$

  同理按 x 所在段分情况：

  - $x \in [y_1, y_2]$： $T(x, y_3) = \exp\left(-(\sigma_1\,|y_2 - x| + \sigma_2\,\ell_{23})\right)$.

  - $x \in [y_2, y_3]$： $T(x, y_3) = \exp(-\sigma_2\,|y_3 - x|)$.