CMU 15-451 lecture 11/29/07

An Algorithms-based Intro to Machine Learning

- Models and basic issues
- An interesting algorithm for "combining expert advice"

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[Based on a talk given at the 2003 National Academy of Sciences "Frontiers of Science" symposium]

Machine learning can be used to...

- recognize speech,
- identify patterns in data,
- steer a car,
- play games,
- adapt programs to users,
- categorize documents, ...

From a scientific perspective: can we develop models to understand learning as a computational problem, and what types of guarantees might we hope to achieve?

A typical setting

- Imagine you want a computer program to help you decide which email messages are spam and which are important.
- Might represent each message by n features. (e.g., return address, keywords, spelling, etc.)
- Take sample S of data, labeled according to whether they were/weren't spam.
- Goal of algorithm is to use data seen so far produce good prediction rule (a "hypothesis") *h(x)* for future data.

The concept learning setting

E.g.,

sales	size	Mr.	bad spelling	known-sender	spam?

The concept learning setting

E.g.,

sales	size	Mr.	bad spelling	known-sender	spam?
Y	N	Y	Y	N	Y
N	N	N	Y	Y	N
N	Y	N	N	N	Y
Y	N	N	N	Y	N
N	N	Y	N	Y	N
Y	N	N	Y	N	Y
N	N	Y	N	N	N
N	Y	N	Y	N	Y

Given data, some reasonable rules might be:

- Predict SPAM if unknown AND (size OR sales)
- Predict SPAM if sales + size – known > 0.
- ...

Big questions

(A) How might we automatically generate rules that do well on observed data? [algorithm design]

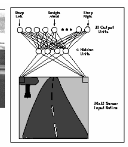
(B) What kind of confidence do we have that they will do well in the future? [confidence bound / sample complexity]

for a given learning alg, how much data do we need...

Power of basic paradigm

Many problems solved by converting to basic "concept learning from structured data" setting.

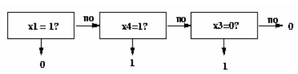
- E.g., document classification
  - convert to bag-of-words
  - Linear separators do well
- E.g., driving a car
  - convert image into features.
  - Use neural net with several outputs.



Natural formalization (PAC)

- We are given sample S = {(x,y)}.
  - Assume x's come from some fixed probability distribution D over instance space.
  - View labels y as being produced by some target function f.
- Alg does optimization over S to produce some hypothesis (prediction rule) h.
- Goal is for h to do well on new examples also from D. I.e., $Pr_D[h(x) \neq f(x)] < \varepsilon$.

Example of analysis: Decision Lists



Say we suspect there might be a good prediction rule of this form.

1. Design an efficient algorithm **A** that will find a consistent DL if one exists.
2. Show that if |S| is of reasonable size, then Pr[exists consistent DL h with err(h) > ε] < δ.
3. This means that **A** is a good algorithm to use if f is, in fact, a DL.

   (a bit of a toy example since usually never a perfect DL)

How can we find a consistent DL?

x_1	x_2	x_3	x_4	x_5	label
1	0	0	1	1	+
0	1	1	0	0	–
1	1	1	0	0	+
0	0	0	1	0	–
1	1	0	1	1	+
1	0	0	0	1	–

if (x_1=0) then -, else
if (x_2=1) then +, else
if (x_4=1) then +, else -

Decision List algorithm

- Start with empty list.
- Find if-then rule consistent with data.
  (and satisfied by at least one example)
- Put rule at bottom of list so far, and cross off examples covered. Repeat until no examples remain.

If this fails, then:
- No DL consistent with remaining data.
- So, no DL consistent with original data.

OK, fine. Now why should we expect it to do well on future data?

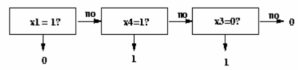
Confidence/sample-complexity

- Consider some DL h with err(h)>ε, that we're worried might fool us.
- Chance that h survives m examples is at most $(1-\varepsilon)^m$.
- Let |H| = number of DLs over n Boolean features. $|H| < n!4^n$. (for each feature there are 4 possible rules, and no feature will appear more than once)

So, Pr[some DL h with err(h)>ε is consistent] < $|H|(1-\varepsilon)^m$.

- This is <0.01 for m > $(1/\varepsilon)[\ln(|H|) + \ln(100)]$
  or about $(1/\varepsilon)[n \ln n + \ln(100)]$

Example of analysis: Decision Lists



Say we suspect there might be a good prediction rule of this form.

1. Design an efficient algorithm **A** that will find a consistent DL if one exists. DONE
2. Show that if |S| is of reasonable size, then Pr[exists consistent DL h with err(h) > ε] < δ. DONE
3. So, if f is in fact a DL, then whp **A**'s hypothesis will be approximately correct. "PAC model"

Confidence/sample-complexity

- What's great is there was nothing special about DLs in our argument.
- All we said was: "if there are not *too* many rules to choose from, then it's unlikely one will have fooled us just by chance."
- And in particular, the number of examples needs to only be proportional to log(|H|).

(notice big difference between 100 and e^{100}.)

Occam's razor

William of Occam (~1320 AD):

"entities should not be multiplied unnecessarily" (in Latin)

Which we interpret as: "in general, prefer simpler explanations".

Why? Is this a good policy? What if we have different notions of what's simpler?

Occam's razor (contd)

A computer-science-ish way of looking at it:

- Say "simple" = "short description".
- At most 2^s explanations can be < s bits long.
- So, if the number of examples satisfies:

Think of as 10x #bits to write down h.

$$m > (1/\varepsilon)[s \ln(2) + \ln(100)]$$

Then it's unlikely a bad simple explanation will fool you just by chance.

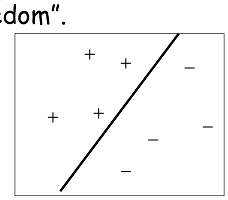
Occam's razor (contd)^2

Nice interpretation:

- Even if we have different notions of what's simpler (e.g., different representation languages), we can both use Occam's razor.
- Of course, there's no guarantee there will be a short explanation for the data. That depends on your representation.

Further work

- Replace log(|H|) with "effective number of degrees of freedom".



  - There are infinitely many linear separators, but not that many really different ones.
- Other more refined analyses.

Online learning

- What if we don't want to make assumption that data is coming from some fixed distribution? Or any assumptions on data?
- Can no longer talk about past performance predicting future results.
- Can we hope to say anything interesting at all??

Idea: regret bounds.

Ø Show that our algorithm does nearly as well as best predictor in some large class.

Using "expert" advice

Say we want to predict the stock market.

- We solicit n "experts" for their advice. (Will the market go up or down?)
- We then want to use their advice somehow to make our prediction. E.g.,

Expt 1	Expt 2	Expt 3	neighbor's dog	truth
down	up	up	up	up
down	up	up	down	down
…	…	…	…	…

Basic question: Is there a strategy that allows us to do nearly as well as best of these in hindsight?

["expert" = someone with an opinion. Not necessarily someone who knows anything.]

Simpler question

- We have n "experts".
- One of these is perfect (never makes a mistake). We just don't know which one.
- Can we find a strategy that makes no more than lg(n) mistakes?

Answer: sure. Just take majority vote over all experts that have been correct so far.

Ø Each mistake cuts # available by factor of 2.

Ø Note: this means ok for n to be very large.

What if no expert is perfect?

Intuition: Making a mistake doesn't completely disqualify an expert. So, instead of crossing off, just lower its weight.

Weighted Majority Alg:

- Start with all experts having weight 1.
- Predict based on weighted majority vote.
- Penalize mistakes by cutting weight in half.

					prediction	correct
weights	1	1	1	1		
predictions	Y	Y	Y	N	Y	Y
weights	1	1	1	.5		
predictions	Y	N	N	Y	N	Y
weights	1	.5	.5	.5		

Analysis: do nearly as well as best expert in hindsight

- M = # mistakes we've made so far.
- m = # mistakes best expert has made so far.
- W = total weight (starts at n).
- After each mistake, W drops by at least 25%. So, after M mistakes, W is at most $n(3/4)^M$.
- Weight of best expert is $(1/2)^m$. So,

$$(1/2)^m \leq n(3/4)^M$$
$$(4/3)^M \leq n2^m$$
$$M \leq 2.4(m + \lg n)$$

With improved settings/tweaks, can get $M < 1.07m + 8 \ln n$.

Randomized Weighted Majority

2.4(m + lg n) not so good if the best expert makes a mistake 20% of the time. Can we do better? Yes.

- Instead of taking majority vote, use weights as probabilities. (e.g., if 70% on up, 30% on down, then pick 70:30) Idea: smooth out the worst case.
- Also, generalize $\frac{1}{2}$ to 1- ε.

Solves to: $M \leq \frac{-m \ln(1-\varepsilon) + \ln(n)}{\varepsilon} \approx (1 + \varepsilon/2)m + \frac{1}{\varepsilon}\ln(n)$

M = expected #mistakes

$M \leq 1.39m + 2\ln n \quad \leftarrow \varepsilon = 1/2$

$M \leq 1.15m + 4\ln n \quad \leftarrow \varepsilon = 1/4$

$M \leq 1.07m + 8\ln n \quad \leftarrow \varepsilon = 1/8$

Analysis

- Say at time t we have fraction F_t of weight on experts that made mistake.
- So, we have probability F_t of making a mistake, and we remove an εF_t fraction of the total weight.
  - $W_{final} = n(1-\varepsilon F_1)(1 - \varepsilon F_2)\ldots$
  - $\ln(W_{final}) = \ln(n) + \sum_t [\ln(1 - \varepsilon F_t)] \leq \ln(n) - \varepsilon \sum_t F_t$ (using $\ln(1-x) < -x$)
    $= \ln(n) - \varepsilon M$. ($\sum F_t = E[\#\text{ mistakes}]$)
- If best expert makes m mistakes, then $\ln(W_{final}) > \ln((1-\varepsilon)^m)$.
- Now solve: $\ln(n) - \varepsilon M > m \ln(1-\varepsilon)$.

$$M \leq \frac{-m\ln(1-\varepsilon)+\ln(n)}{\varepsilon} \approx (1+\varepsilon/2)m + \frac{1}{\varepsilon}\log(n)$$

What can we use this for?

- Can use to combine multiple algorithms to do nearly as well as best in hindsight.
  - E.g., online control policies.
- Extension: "sleeping experts". E.g., one for each possible keyword. Try to do nearly as well as best "coalition".
- More extensions: "bandit problem", movement costs.

Other models

Some scenarios allow more options for algorithm.

- "Active learning": have large unlabeled sample and alg may choose among these.
  - E.g., web pages, image databases.
- Or, allow algorithm to construct its own examples. "Membership queries"
  - E.g., features represent variable-settings in some experiment, label represents outcome.
  - Gives algorithm more power.

Conclusions/lessons

- Simple theoretical models can give insight into basic issues. E.g., Occam's razor.
- Even if models aren't perfect, can often lead to good algorithms.
- Often diverse problems best solved by fitting into basic paradigm(s).
- A lot of ongoing research into better algorithms, models that capture specific issues, incorporating Machine Learning into broader classes of applications.

Additional notes

- Some courses at CMU on machine learning:
  - 10-601 Machine Learning
  - 10-701/15-781 Machine Learning
  - 15-859(B) Machine Learning Theory.
- There is also a web site for the area as a whole at www.learningtheory.org, with pointers to survey articles, course notes, tutorials, and textbooks.