Reinforcement Learning
(Model-free RL)

- R&N Chapter 21

- Demos and Data Contributions from
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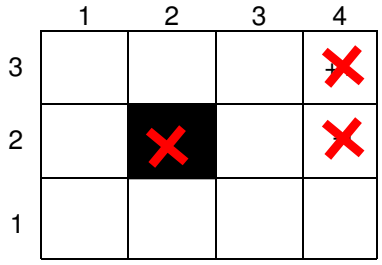
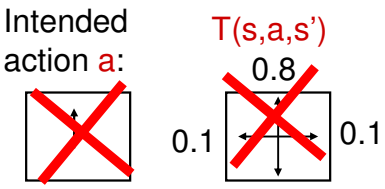
Reinforcement Learning





- Same (fully observable) MDP as before except:
  - We don't know the model of the environment
  - We don't know T(.,.,.)
  - We don't know R(.)
- Task is still the same:
  - Find an optimal policy

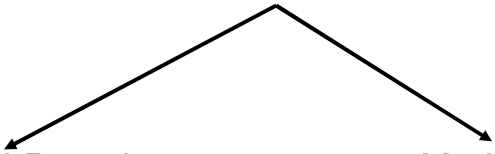
General Problem

- All we can do is try to execute actions and record the resulting rewards
  - World: You are in state **102**, you have a choice of 4 actions
  - Robot: I'll take action 2
  - World: You get a reward of 1 and you are now in state 63, you have a choice of 3 actions
  - Robot: I'll take action 3
  - World: You get a reward of -10 and you are now in state 12, you have a choice of 4 actions
  - ……..

*Notice we have **state-observability**!*

Classes of Techniques

Reinforcement Learning



Model-Based

- Try to learn an explicit model of T(.,.,.) and R(.)

Model-Free

- Recover an optimal policy without ever estimating a model

Model-Free

- We are not interested in T(.,.,.), we are only interested in the resulting values and policies
- Can we compute something without an explicit model of T(.,.,.) ?

- First, let's fix a policy and compute the resulting values

Temporal Differencing

- Upon action $a = \pi(s)$, the values satisfy:

$$U(s) = R(s) + \gamma \sum_{s'} T(s,a,s') U(s')$$

For any s' successor of s, $U(s)$ is "in between":

The new value considering only the actual s' reached:

$$R(s) + \gamma U(s')$$

and the old value

$$U(s)$$

The γ is the discount of future reward.

Temporal Differencing

- Upon moving from s to s' by using action a, the new estimate of U(s) is approximated by:

$$U(s) = (1-\alpha)\, U(s) + \alpha\, (R(s) + \gamma\, U(s'))$$

- *Temporal Differencing*: When moving from any state s to a state s', update:

$$U(s) \leftarrow U(s) + \alpha\, (R(s) + \gamma\, U(s') - U(s))$$

Temporal Differencing

Discrepancy between current value and new guess at a value after moving to s'

Current value

$$U(s) \leftarrow U(s) + \alpha\, (R(s) + \gamma\, U(s') - U(s))$$

The transition probabilities do not appear anywhere!!!

Temporal Differencing

Learning rate

$$U(s) \leftarrow U(s) + \alpha\,(R(s) + \gamma\,U(s') - U(s))$$

How to choose $0 < \alpha < 1$?

- Too small: Converges slowly; tends to always trust the current estimate of U
- Too large: Changes very quickly; tends to always replace the current estimate by the new guess

Temporal Differencing

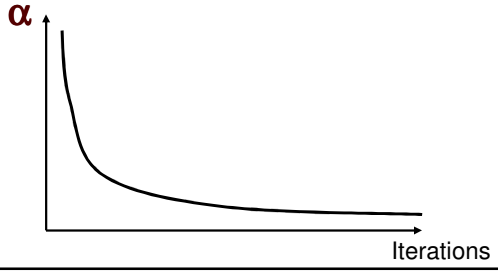
How to choose $0 < \alpha < 1$?

- Start with large α

→ Not confident in our current estimate so we can change it a lot

- Decrease α as we explore more

→ We are more and more confident in our estimate so we don't want to change it a lot



Summary

- Learning exploring environment and recording received rewards
- Model-Based techniques
  - Evaluate transition probabilities and apply previous MDP techniques to find values and policies
  - More efficient: Single value update at each state
  - Selection of "interesting" states to update: Prioritized sweeping
- Exploration strategies
- Model-Free Techniques (so far)
- Temporal update to estimate values without ever estimating the transition model
- Parameter: Learning rate must decay over iterations

Temporal Differencing

Discrepancy between current value and new guess at a value after moving to *s'*

Current value

$$U(s) \leftarrow U(s) + \alpha\,(R(s) + \gamma\,U(s') - U(s))$$

The transition probabilities do not appear anywhere!!!

But how to find the optimal policy?

Q-Learning

- $U(s)$ = Utility of state s = expected sum of future discounted rewards

- $Q(s,a)$ = Value of taking action a at state s = expected sum of future discounted rewards after taking action a at state s

Q-Learning

- $U(s)$ = Ut… sum of future dis…

(s,a) = "state-action" pair. Maintain table of $Q(s,a)$ instead of $U(s)$

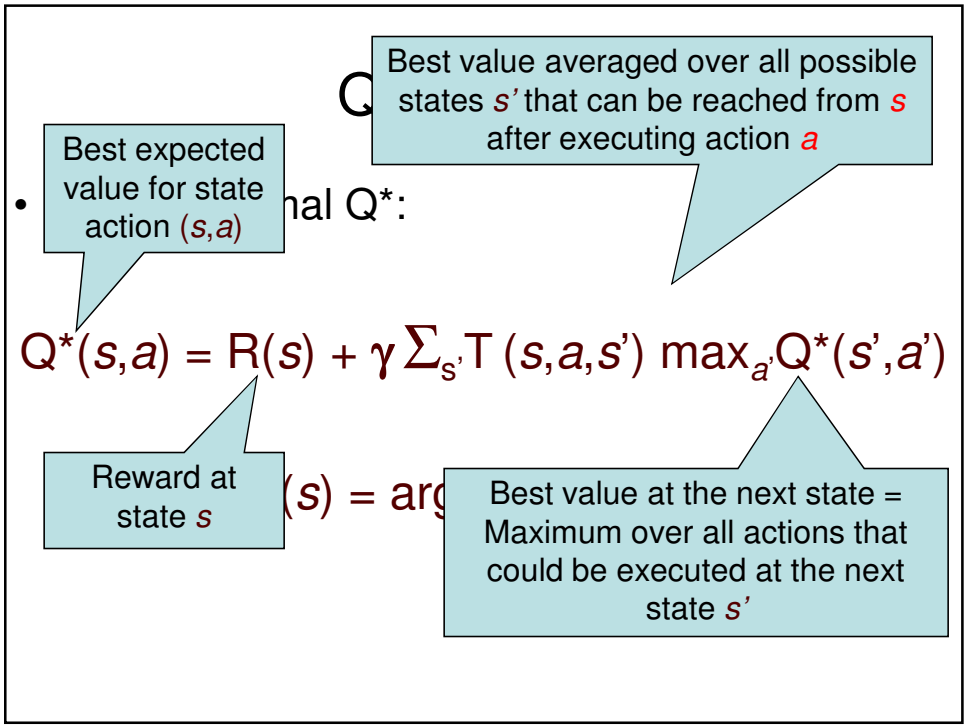
- $Q(s,a)$ = Value of taking action a at state s = expected sum of future discounted rewards after taking action a at state s

Q-Learning

- For the optimal Q*:

$$Q^*(s,a) = R(s) + \gamma \sum_{s'} T(s,a,s') \max_{a'} Q^*(s',a')$$

$$\pi^*(s) = \text{argmax}_a \, Q^*(s,a)$$



Q-Learning: Updating Q without a Model

Use temporal differencing; after moving from state *s* to state *s'* using action *a*:

$$Q(s,a) \leftarrow Q(s,a) + \alpha(R(s) + \gamma \max_{a'} Q(s',a') - Q(s,a))$$

Q-Learning: Updating Q without a Model

After moving from state *s* to state *s'* using action *a*:

Old estimate of Q(s,a)

Difference between old estimate and new guess after taking action *a*

$$Q(s,a) \leftarrow Q(s,a) + \alpha(R(s) + \gamma \max_{a'} Q(s',a') - Q(s,a))$$

New estimate of Q(s,a)

Learning rate $0 < \alpha < 1$

Q-Learning: Estimating the policy

Q-Update: After moving from state *s* to state *s'* using action *a:*

$$Q(s,a) \leftarrow Q(s,a) + \alpha(R(s) + \gamma \max_{a'} Q(s',a') - Q(s,a))$$

Policy estimation:

$$\pi(s) = \text{argmax}_a\ Q(s,a)$$

Q-Learning: Estimating the policy

Key Point: We do not use T(.,.,.) anywhere → We can compute optimal values and policies without ever computing a model of the MDP!

$$Q(s,a) \leftarrow Q(s,a) + \alpha(R(s) + \gamma \max_{a'} Q(s',a') - Q(s,a))$$

Policy estimation:

$$\pi(s) = \text{argmax}_a\ Q(s,a)$$

Q-Learning: Convergence

- Q-learning guaranteed to converge to an optimal policy (Watkins)
- Very general procedure (because completely model-free)
- May be slow (because completely model-free)

(Start = S_1, Action = a_1, Reward = 10, End = S_2)

	S_1	S_2
a_1		
a_2		

(Start = S_2, Action = a_2, Reward = -10, End = S_1)

	S_1	S_2
a_1		
a_2		

(Start = S_1, Action = a_2, Reward = 10, End = S_1)

	S_1	S_2
a_1		
a_2		

(Start = S_1, Action = a_1, Reward = 10, End = S_1)

	S_1	S_2
a_1		
a_2		

(Start = S_1, Action = a_1, Reward = 10, End = S_2)

	S_1	S_2
a_1	5.0	0.0
a_2	0.0	0.0

(Start = S_2, Action = a_2, Reward = -10, End = S_1)

	S_1	S_2
a_1	5.0	0.0
a_2	0.0	-3.75

(Start = S_1, Action = a_2, Reward = 10, End = S_1)

	S_1	S_2
a_1	5.0	0.0
a_2	6.25	-3.75

(Start = S_1, Action = a_1, Reward = 10, End = S_1)

	S_1	S_2
a_1	9.0625	0.0
a_2	6.25	-3.75

$\pi^*(S_1) = a_1$

$\pi^*(S_2) = a_1$

Q-Learning: Exploration Strategies

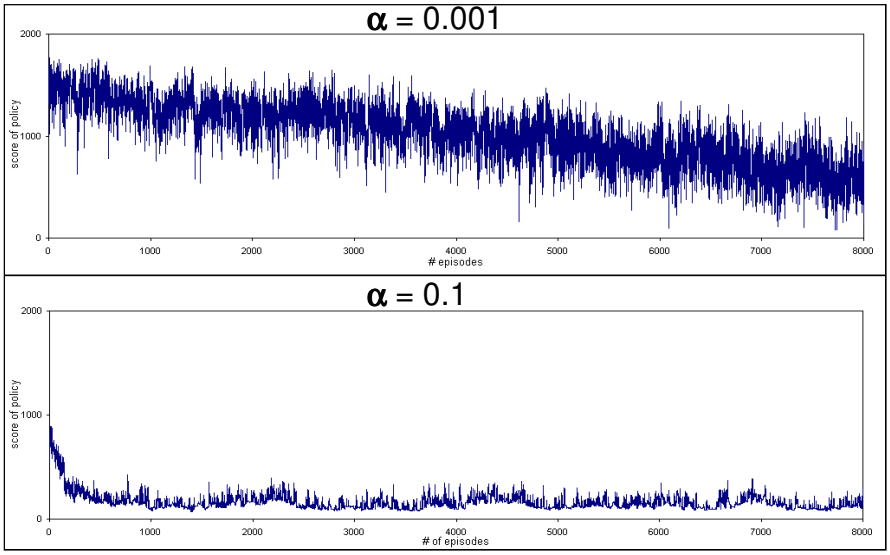
- How to choose the next action **while we're learning**?
  - Random
  - Greedy: Always choose the estimated best action $\pi(s)$
  - ε-Greedy: Choose the estimated best with probability $1-\varepsilon$
  - Boltzmann: Choose the estimated best with probability proportional to $e^{Q(s,a)/T}$

Evaluation

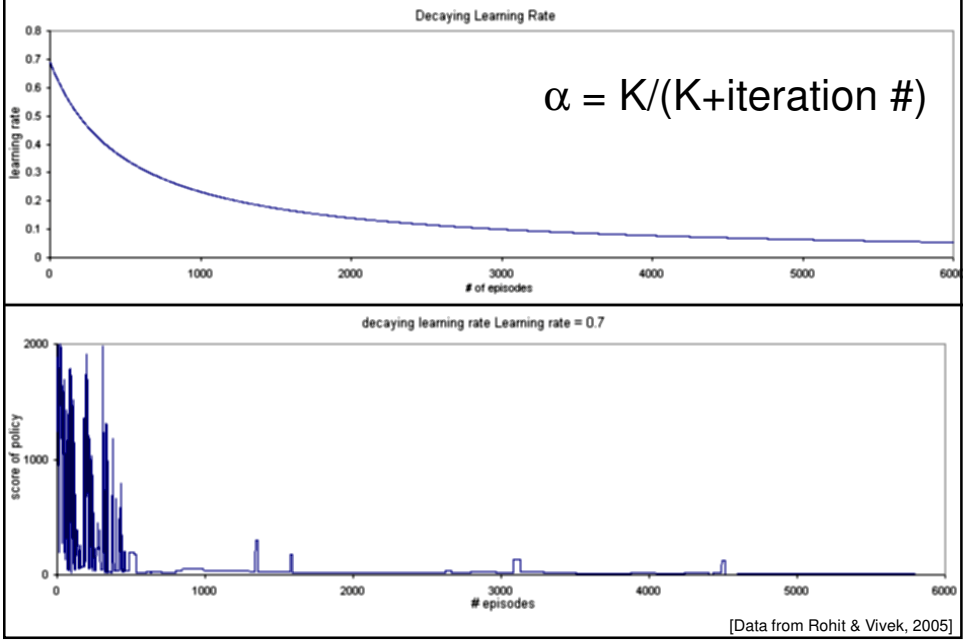
- How to measure how well the learning procedure is doing?
- $U(s)$ = Value estimated at s at the current learning iteration
- $U^*(s)$ = Optimal value if we knew everything about the environment

$$\text{Error} = |U - U^*|$$

Constant Learning Rate

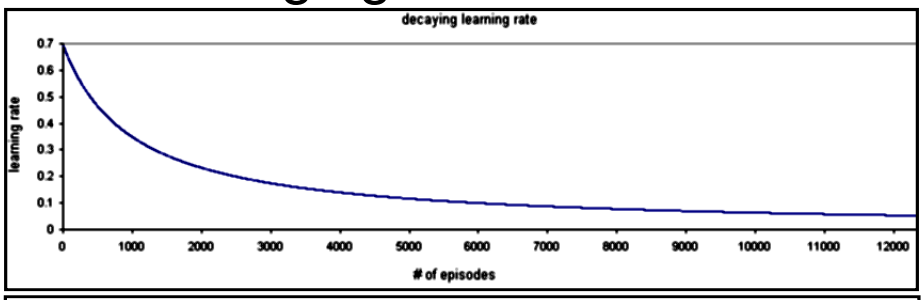


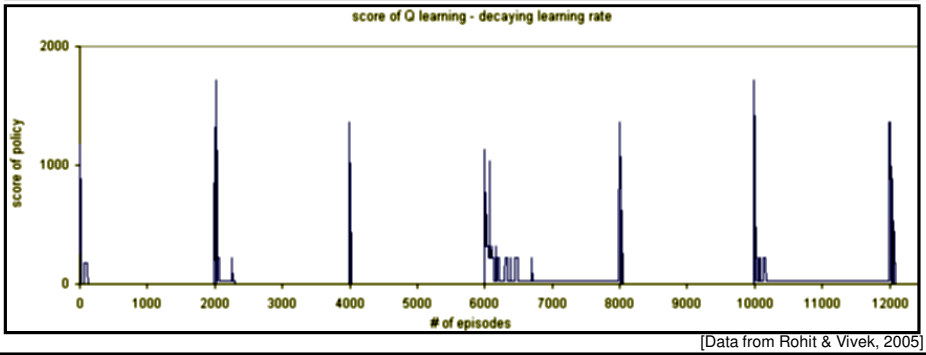
Decaying Learning Rate



[Data from Rohit & Vivek, 2005]

Changing Environments





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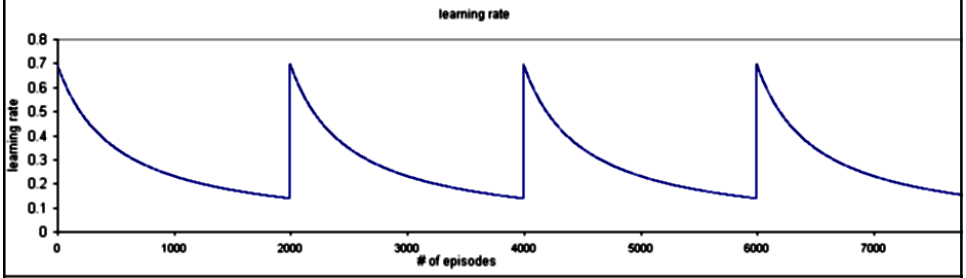
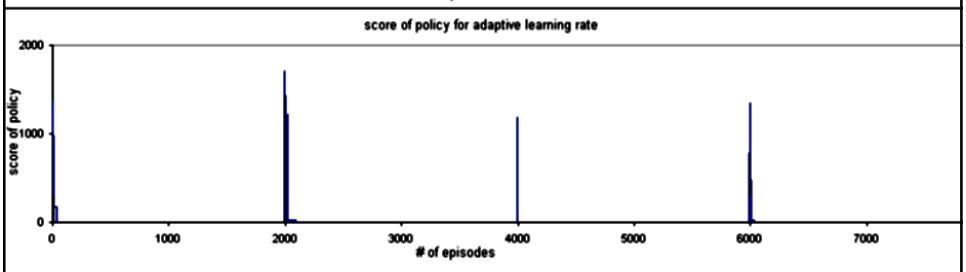
Adaptive Learning Rate





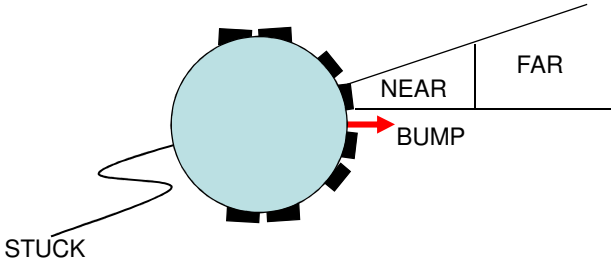
[Data from Rohit & Vivek, 2005]

Example: Pushing Robot

- Task: Learn how to push boxes around.
- States: Sensor readings
- Actions: Move forward, turn

Example from Mahadevan and Connell, "Automatic Programming of Behavior-based Robots using Reinforcement Learning, Proceedings AAAI 1991

Example: Pushing Robot



- State = 1 bit for each NEAR and FAR gates x 8 sensors + 1 bit for BUMP + 1 bit for STUCK = 18 bits
- Actions = move forward or turn +/- 22^{o} or turn +/- 45^{o} = 5 actions

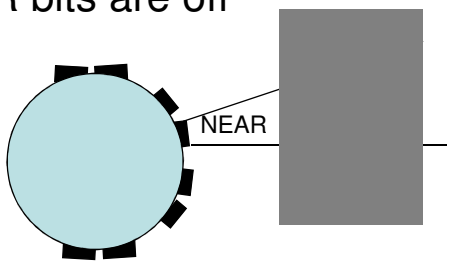
Example from Mahadevan and Connell, "Automatic Programming of Behavior-based Robots using Reinforcement Learning, Proceedings AAAI 1991

Learn How to Find the Boxes

- Box is found when the NEAR bits are on for all the front sonars.
- Reward:

R(*s*) = +3 if NEAR bits are on

R(*s*) = -1 if NEAR bits are off

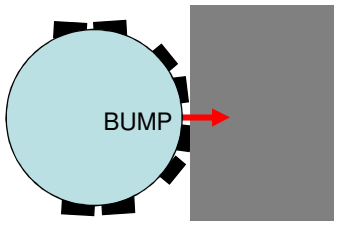


Learn How to Push the Box

- Try to maintain contact with the box while moving forward
- Reward:

R(*s*) = +1 if BUMP while moving forward

R(*s*) = -3 if robot loses contact

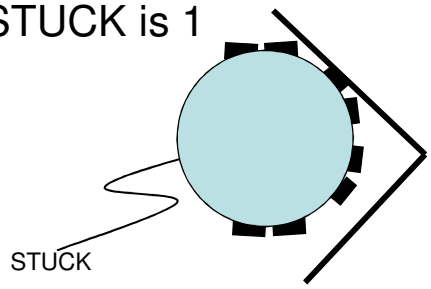


Learn how to Get Unwedged

- Robot may get wedged against walls, in which the STUCK bit is raised.
- Reward:

R(*s*) = +1 if STUCK is 0

R(*s*) = -3 if STUCK is 1



Q-Learning

- Initialize Q(*s*,*a*) to 0 for all state-action pairs
- Repeat:
  - Observe the current state *s*
    - 90% of the time, choose the action *a* that maximimizes Q(*s*,*a*)
    - Else choose a random action *a*
  - Update Q(*s*,*a*)

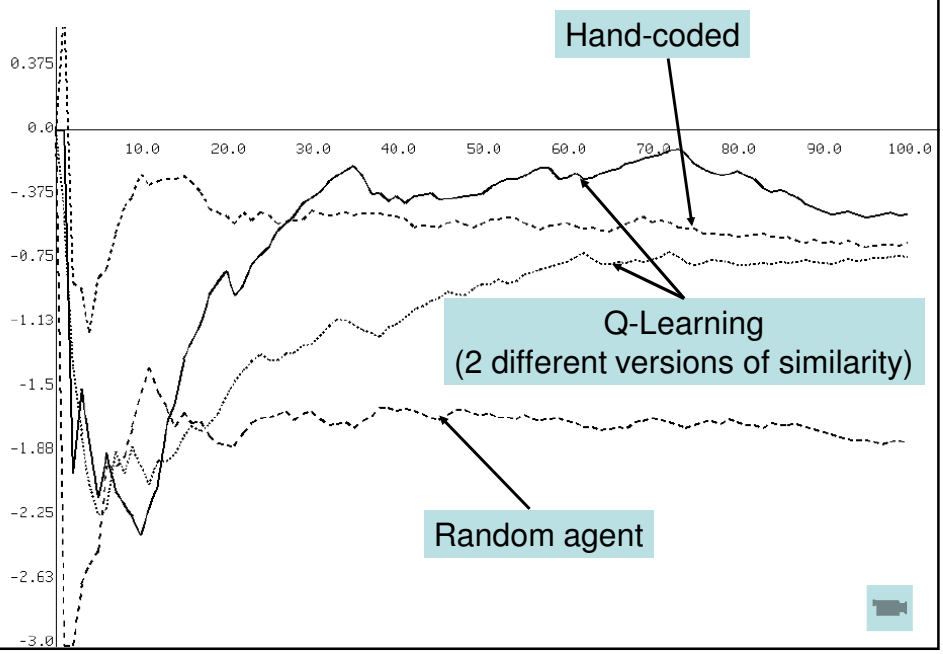
Q-Learning

- Initialize Q(*s*,*a*) to 0 for all state-action pairs
- Repeat:
  - Observe the current state *s*
    - 90% of the time, choose the action *a* that maximimizes Q(*s*,*a*)
    - Else choose a random action *a*
  - Update Q(*s*,*a*)

Improvement:
Update also all the states *s'* that are "similar" to *s*.

In this case: Similarity between *s* and *s'* is measured by the Hamming distance between the bit strings

Performance

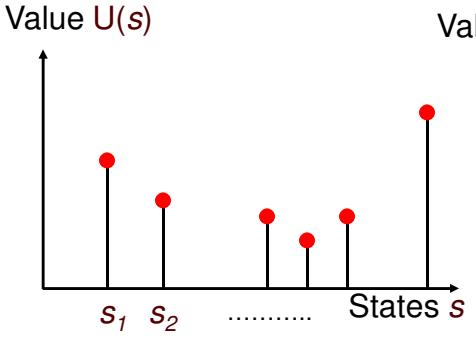


Generalization

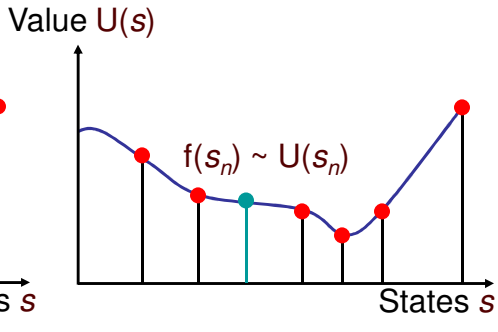
- In real problems: Too many states (or state-action pairs) to store in a table
- Example: Backgammon → 10^{20} states!

- Need to:
  - Store U for a subset of states $\{s_1,..,s_K\}$
  - Generalize to compute U(s) for any other states s

Generalization



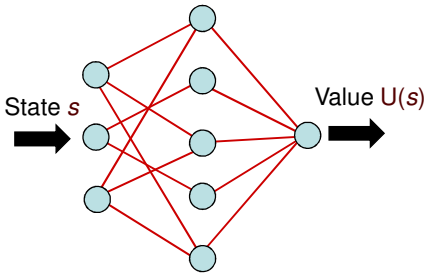
We have sample values of U for some of the states s_1, s_2



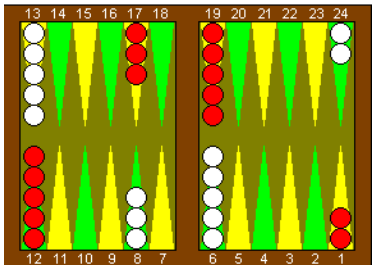
We interpolate a function f(.), such that for any query state s_n, f(s_n) approximates U(s_n)

Generalization

- Possible function approximators:
  - Neural networks
  - Memory-based methods
- …… and many others solutions to representing U over large state spaces:
  - Decision trees
  - Clustering
  - Hierarchical representations



Example: Backgammon



- States: Number of red and white checkers at each location

→ Order 10^{20} states!!!!

→ Branching factor prevents direct search

- Actions: Set of legal moves from any state

Example from: G. Tesauro. Temporal Difference Learning and TD-Gammon. Communications of the ACM, 1995

Example: Backgammon

- Represent mapping from states to expected outcomes by multilayer neural net
- Run a large number of "training games"
  - For each state *s* in a training game:
  - Update using temporal differencing
  - At every step of the game → Choose best move according to current estimate of U
- Initially: Random moves
- After learning: Converges to good selection of moves

Performance

- Can learn starting with no knowledge at all!
- Example: 200,000 training games with 40 hidden units.
- Enhancements use better encoding and additional hand-designed features

- Example:
  - 1,500,000 training games
  - 80 hidden units
  - -1 pt/40 games (against world-class opponent)

Example: Control and Robotics

- Devil-stick juggling (Schaal and Atkeson): Non-linear control at 200ms per decision. Program learns to keep juggling after ~40 trials. A human requires 10 times more practice.
- Helicopter control (Andrew Ng): Control of a helicopter for specific flight patterns. Learning policies from simulator. Learns policies for control pattern that are difficult even for human experts (e.g., inverted flight).
  http://heli.stanford.edu/

Summary

- Certainty equivalent learning for estimating future rewards
- Exploration strategies
- One-backup update, prioritized sweeping
- Model free (Temporal Differencing = TD) for estimating future rewards
- Q-Learning for model-free estimation of future rewards and optimal policy
- Exploration strategies and selection of actions

(Some) References

- S. Sutton and A.G. Barto. Reinforcement Learning: An Introduction. MIT Press.
- L. Kaelbling, M. Littman and A. Moore. Reinforcement Learning: A Survey. Journal of Artificial Intelligence Research. Volume 4, 1996.
- G. Tesauro. TD-Gammon, a self-teaching backgammon program, achieves master-level play. Neural Computation 6(2), 1995.
- http://ai.stanford.edu/~ang/
- http://www-all.cs.umass.edu/rlr/