15-381
Artificial **Intelligence**
Lecture 7: Probabilistic Reasoning

PERCEPTION

ACTUAL SOUND

1. The ?eel is on the shoe
2. The ?eel is on the car
3. The ?eel is on the table
4. The ?eel is on the orange

Slide credit: David Mumford

PERCEPTION

ACTUAL SOUND

1. The ?eel is on the shoe
2. The ?eel is on the car
3. The ?eel is on the table
4. The ?eel is on the orange

PERCEIVED WORDS

1. The heel is on the shoe
2. The wheel is on the car
3. The meal is on the table
4. The peel is on the orange

(Warren & Warren, 1970)

Slide credit: David Mumford

VISUAL PERCEPTION

“His outline is lost in clutter, shadows and wrinkles; except for one ear, his face is invisible. No known algorithm will find him.”

Slide credit: David Mumford

REASONING UNDER UNCERTAINTY

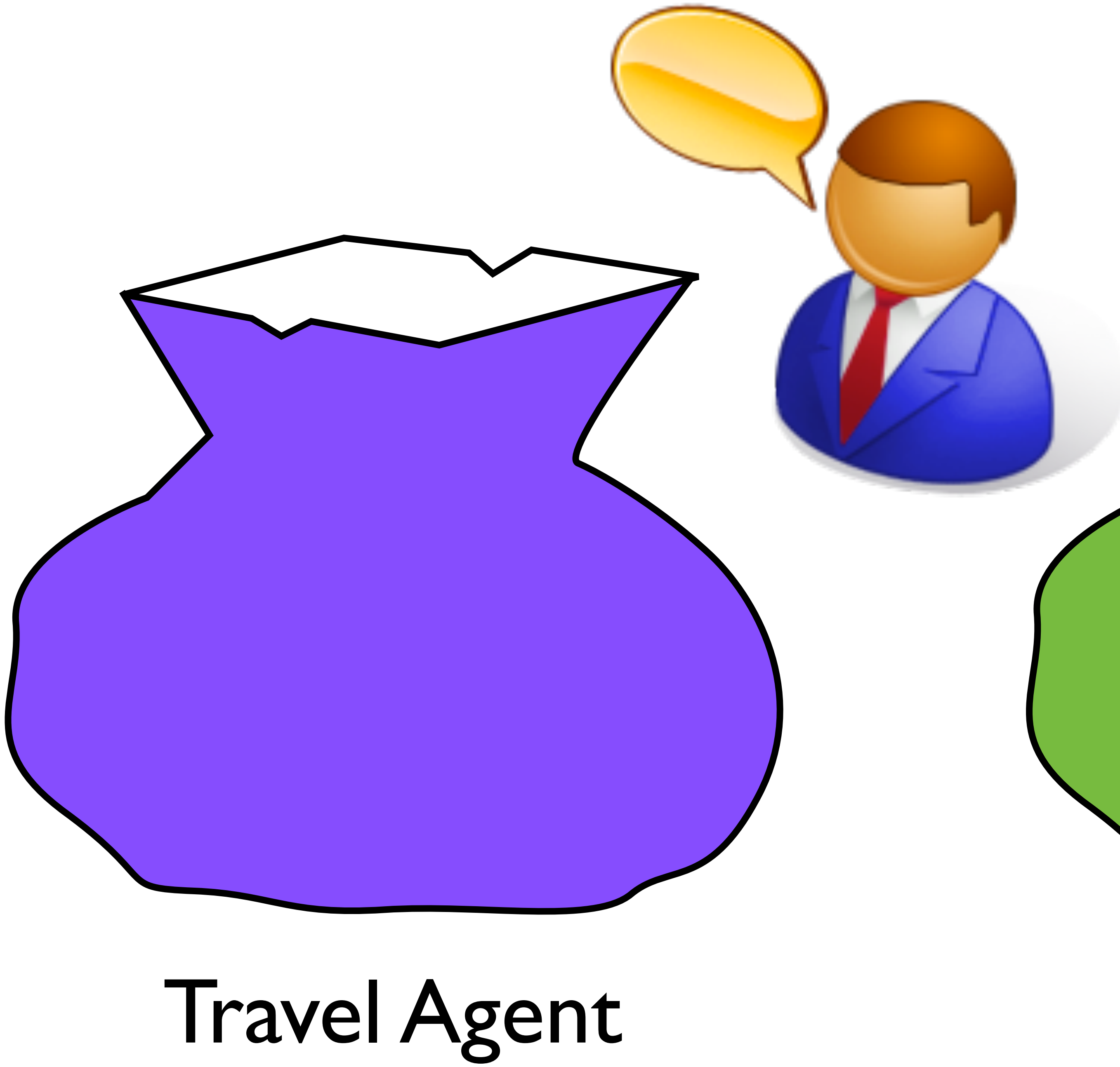
- MEASUREMENT ERROR
- INFORMATION INCOMPLETENESS
- MODEL ERROR

BAG OF WORDS

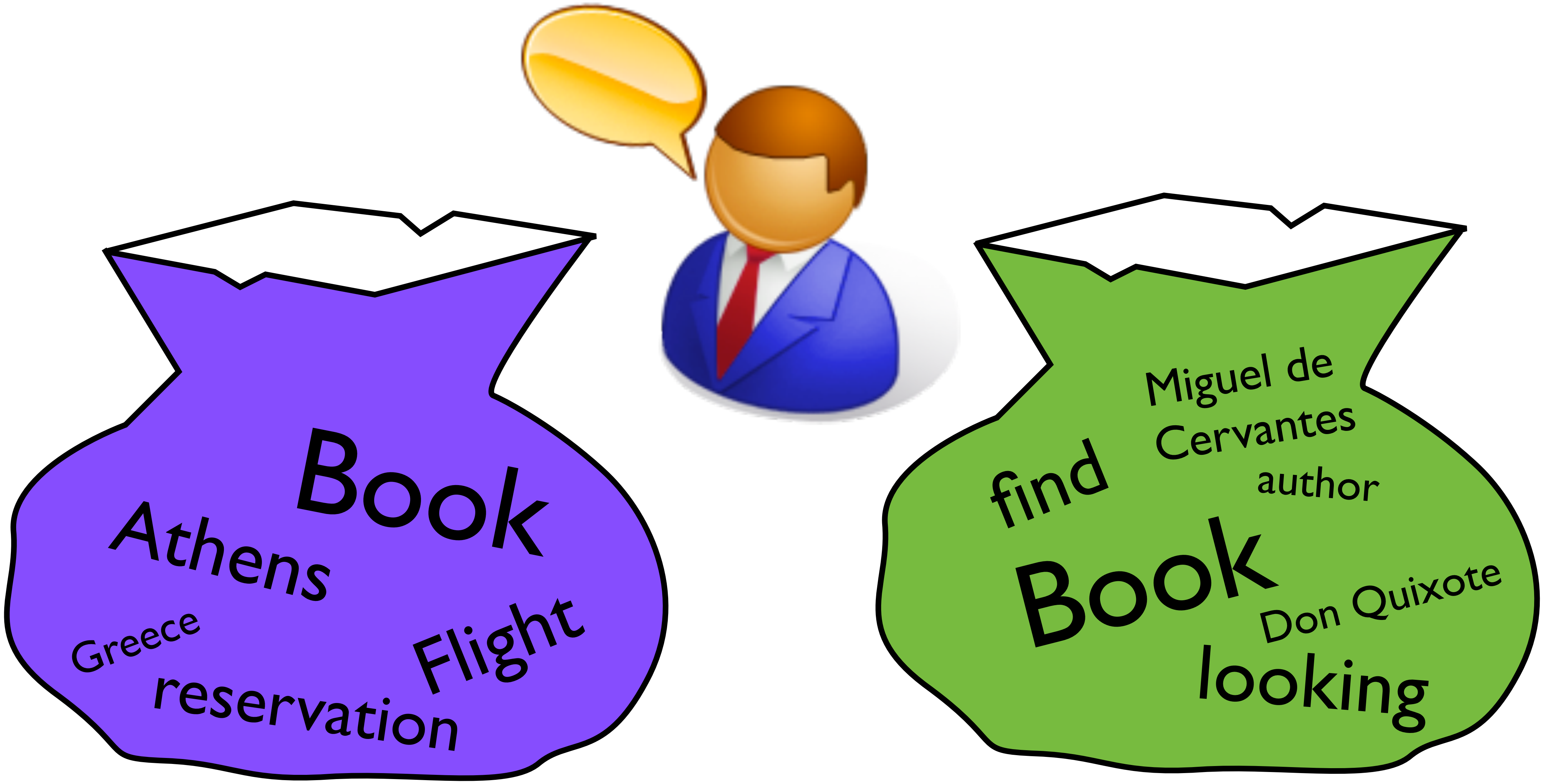
Travel Agent

Book Store

BAG OF WORDS



BAG OF WORDS

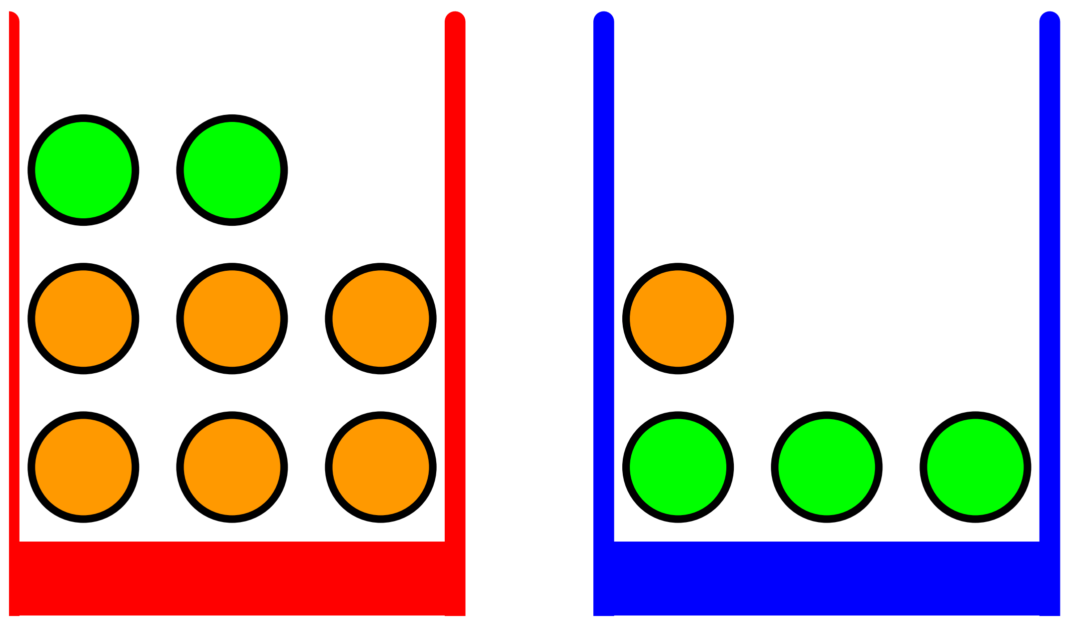


Travel Agent

Book Store

RANDOM VARIABLE

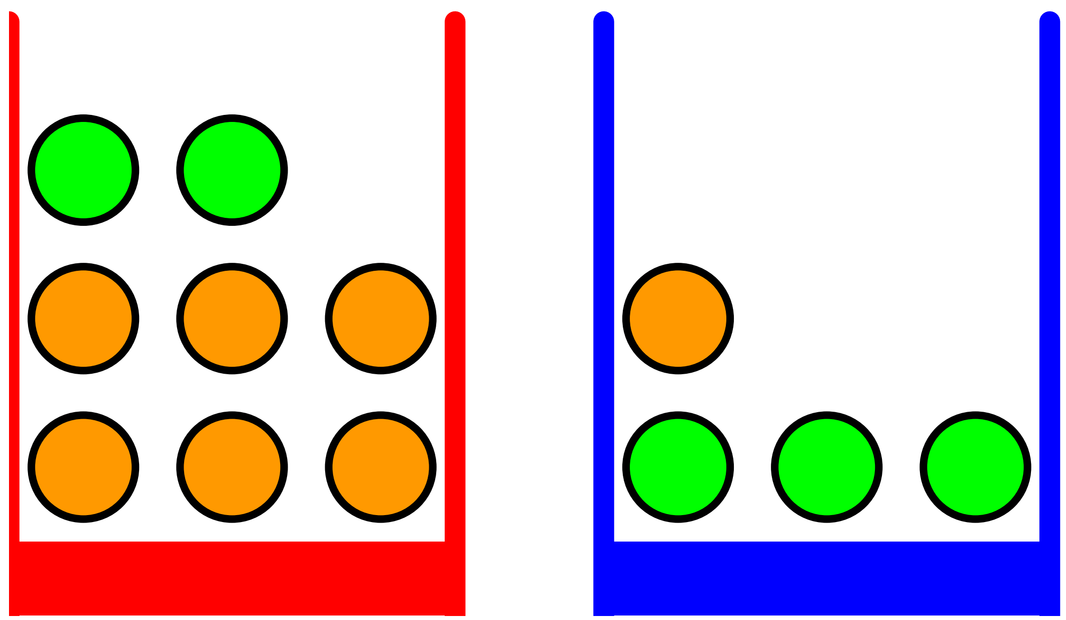
DISCRETE



Figures from Pattern Recognition and Machine Learning (Bishop)

RANDOM VARIABLE

DISCRETE



Red Box (*r*)

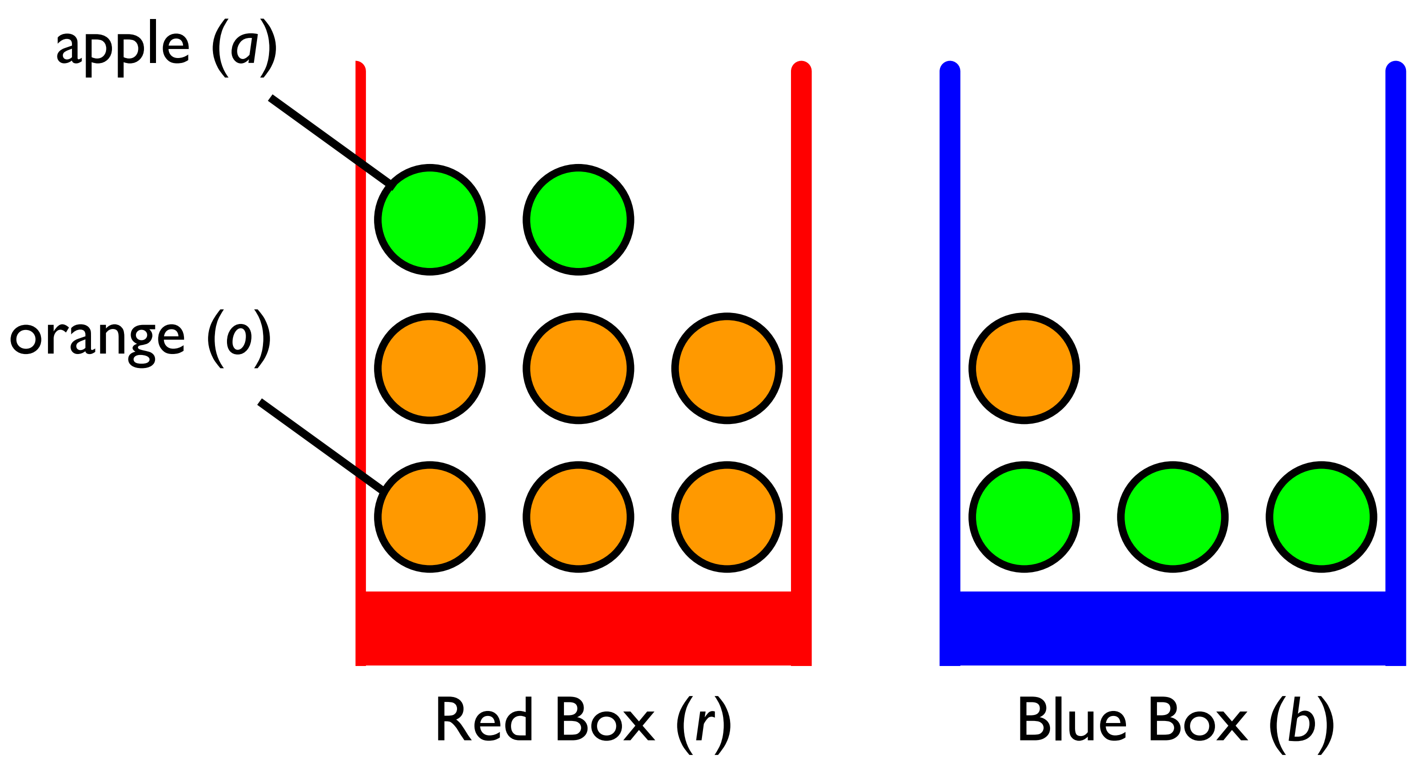
Blue Box (*b*)

$$\text{Box} : B \in \{r, b\}$$

Figures from Pattern Recognition and Machine Learning (Bishop)

RANDOM VARIABLE

DISCRETE



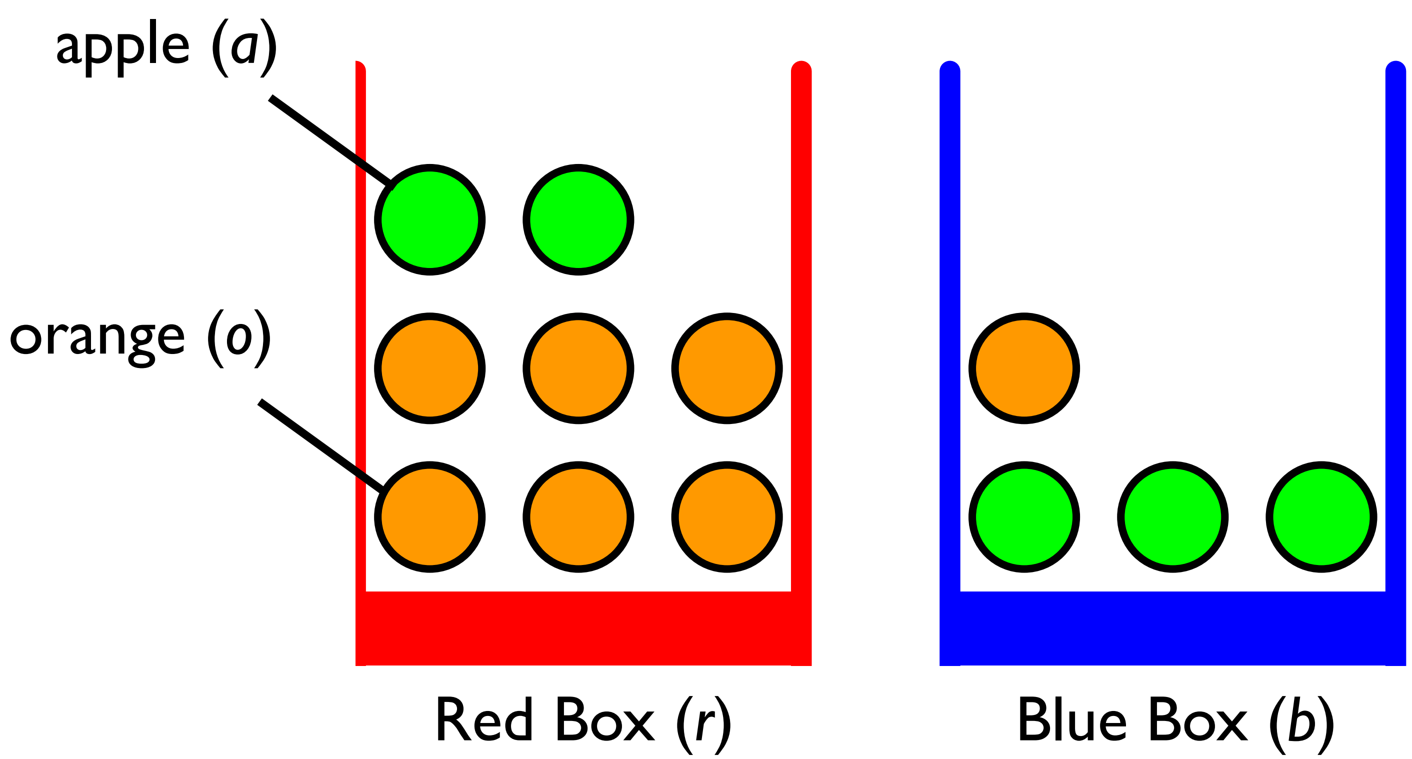
$$\text{Box} : B \in \{r, b\}$$

$$\text{Fruit} : F \in \{a, o\}$$

Figures from Pattern Recognition and Machine Learning (Bishop)

RANDOM VARIABLE

DISCRETE



$$\text{Box} : B \in \{r, b\}$$

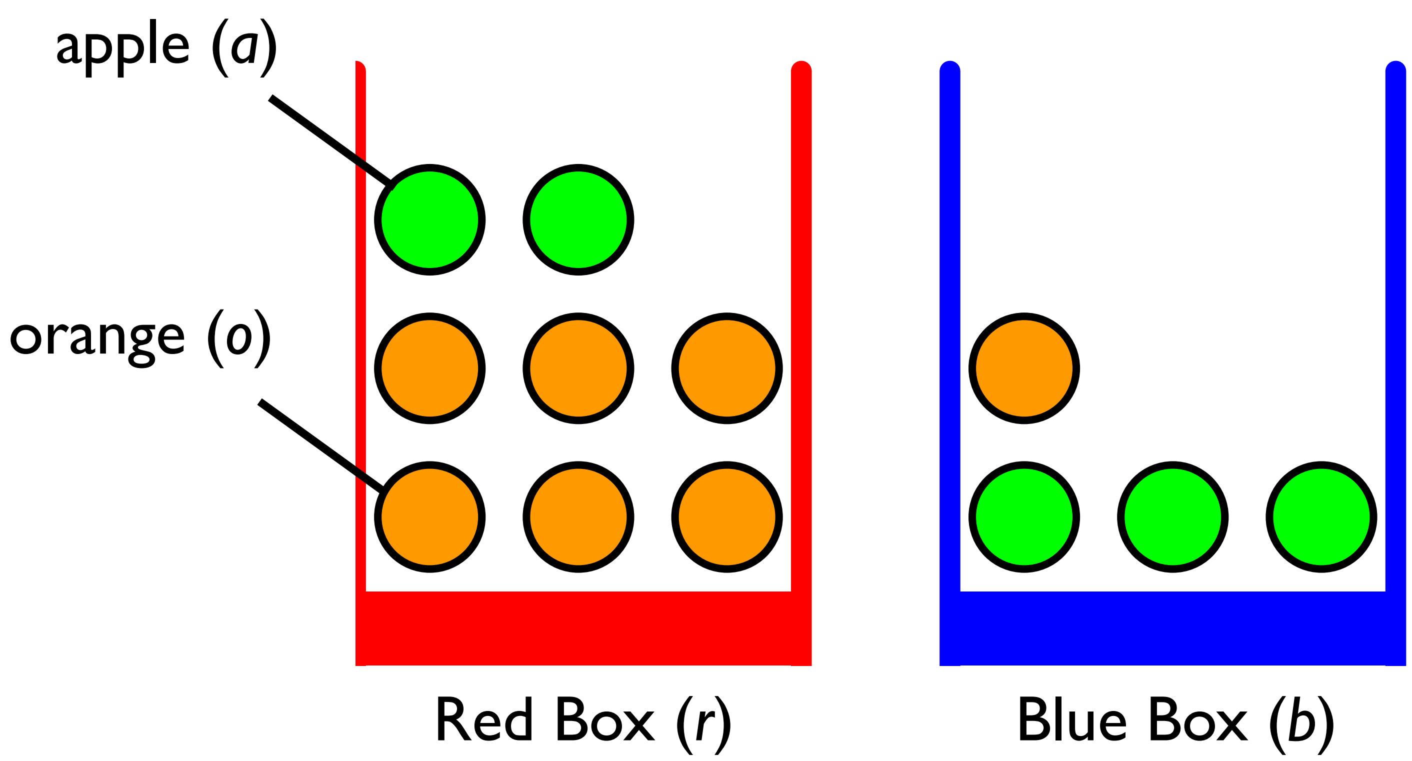
$$\text{Fruit} : F \in \{a, o\}$$

$$p(B = r) = 0.4$$
$$p(B = b) = 0.6$$

Figures from Pattern Recognition and Machine Learning (Bishop)

RANDOM VARIABLE

DISCRETE



$$\text{Box} : B \in \{r, b\}$$

$$\text{Fruit} : F \in \{a, o\}$$

$$p(B = r) = 0.4$$
$$p(B = b) = 0.6$$

$$p(B = b) + p(B = r) = 1$$

Figures from Pattern Recognition and Machine Learning (Bishop)

RANDOM VARIABLE

DISCRETE

WHAT IS THE PROBABILITY THAT THE SELECTION PROCEDURE WILL PICK AN APPLE?

RANDOM VARIABLE

DISCRETE

WHAT IS THE PROBABILITY THAT THE SELECTION PROCEDURE WILL PICK AN APPLE?

GIVEN THAT AN ORANGE WAS SELECTED WHAT IS THE PROBABILITY THAT THE BOX WE CHOSE WAS BLUE?

RANDOM VARIABLE

DISCRETE

WHAT IS THE PROBABILITY THAT THE SELECTION PROCEDURE WILL PICK AN APPLE?

GIVEN THAT AN ORANGE WAS SELECTED WHAT IS THE PROBABILITY THAT THE BOX WE CHOSE WAS BLUE?

SUM RULE AND **PRODUCT RULE**

RANDOM VARIABLE

- DISCRETE R.V.
  - BOOLEAN R.V.
- CONTINUOUS R.V.

RANDOM VARIABLE

BINARY/BOOLEAN

$$\text{Box} : B \in \{r, b\} \qquad \text{Fruit} : F \in \{a, o\}$$

RANDOM VARIABLE

BINARY/BOOLEAN

$$\text{Box} : B \in \{r, b\} \qquad \text{Fruit} : F \in \{a, o\}$$

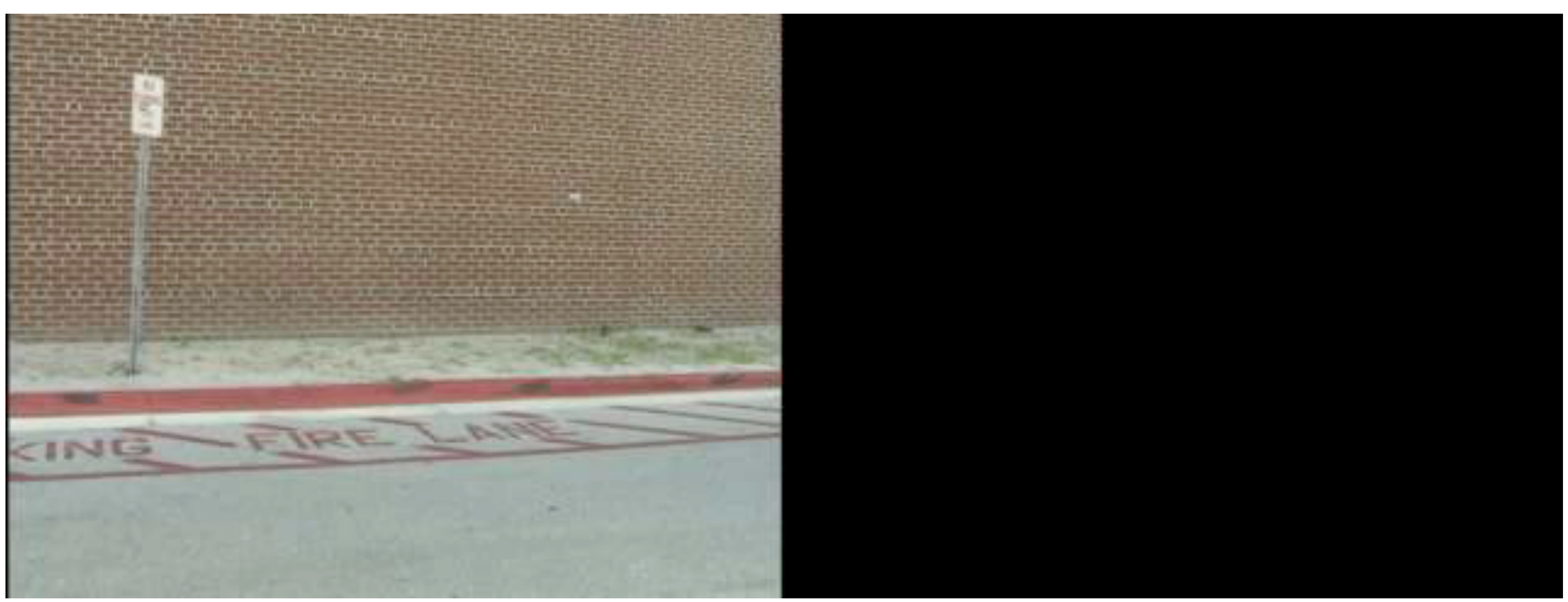
ON/OFF

RANDOM VARIABLE

BINARY/BOOLEAN

$$\text{Box} : B \in \{r, b\} \qquad \text{Fruit} : F \in \{a, o\}$$

ON/OFF



RANDOM VARIABLE

DISCRETE

RANDOM VARIABLE

DISCRETE

MUCHO

RANDOM VARIABLE

DISCRETE

MUCHO MACHO

RANDOM VARIABLE

DISCRETE

MUCHO MACHO MUCHO MACHO

RANDOM VARIABLE

DISCRETE

MUCHO MACHO MUCHO MACHO

SEDAN

SUV

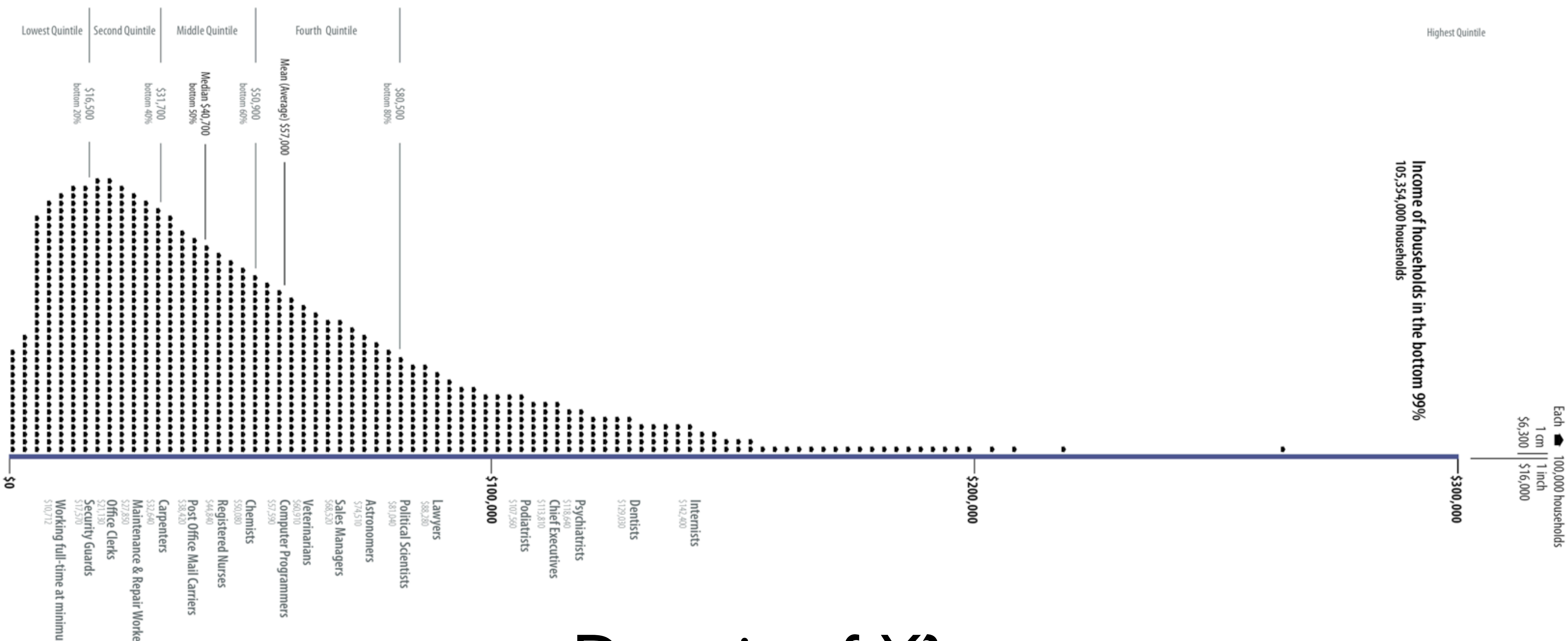
COUPE

TRUCK

RANDOM VARIABLE

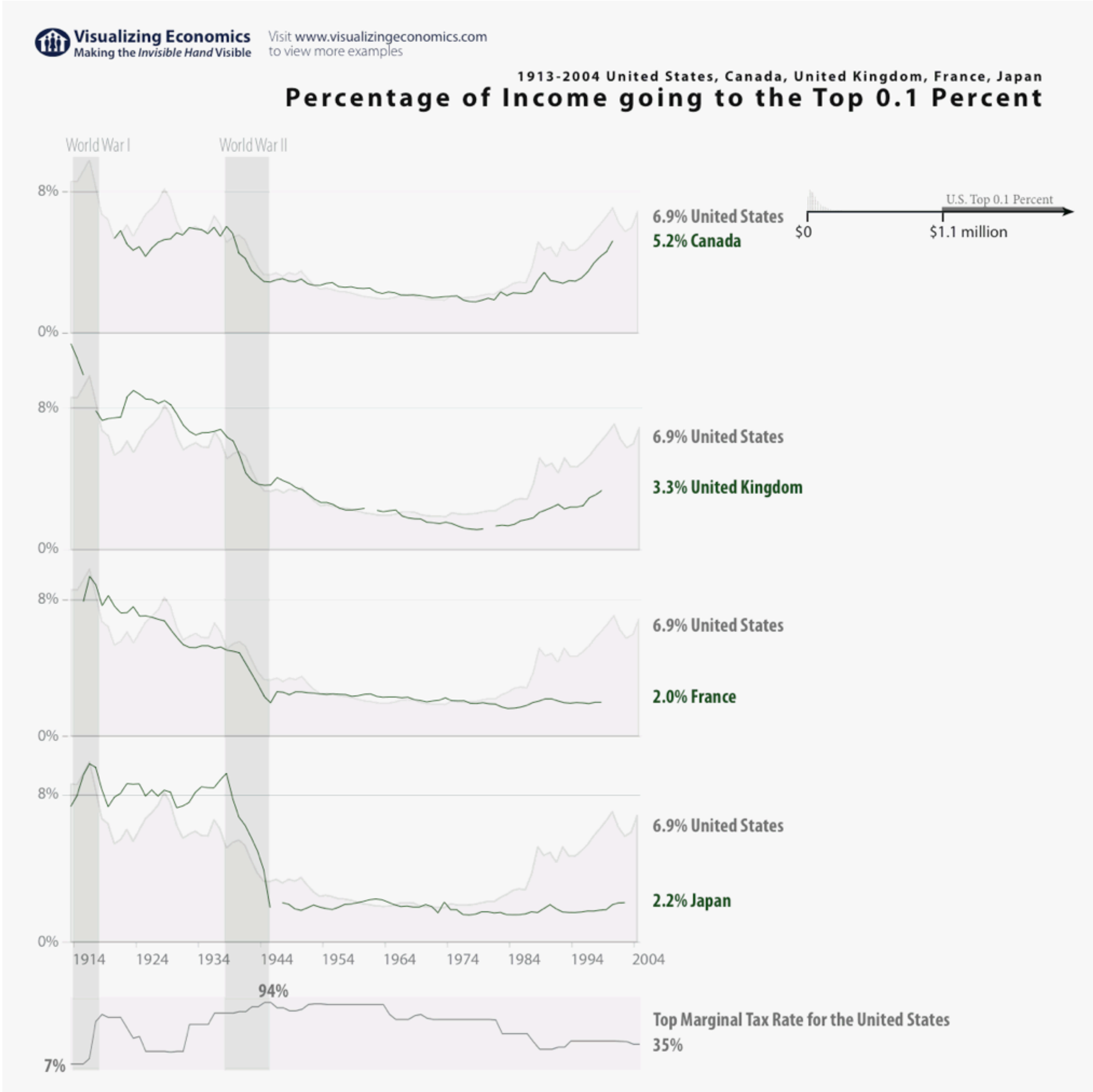
CONTINUOUS

X: Income of Households



Domain of *X*?

Figure Credit: Catherine Mulbrandon


Visualizing Economics
Making the Invisible Hand Visible
Visit www.visualizingeconomics.com
to view more examples
1913-2004 United States, Canada, United Kingdom, France, Japan
Percentage of Income going to the Top 0.1 Percent
World War I
World War II
8%
0%
6.9% United States
5.2% Canada
U.S. Top 0.1 Percent
$0
$1.1 million
3.3% United Kingdom
2.0% France
2.2% Japan
1914
1924
1934
1944
1954
1964
1974
1984
1994
2004
94%
Top Marginal Tax Rate for the United States
35%
7%

X: ?

Domain of X?

RANDOM VARIABLE

CONTINUOUS

HEIGHT IN 15-381?

CAR RECOGNITION

Yan Li, Leon Gu, Takeo Kanade, "A robust shape model for multi-view car alignment," CVPR 2009.

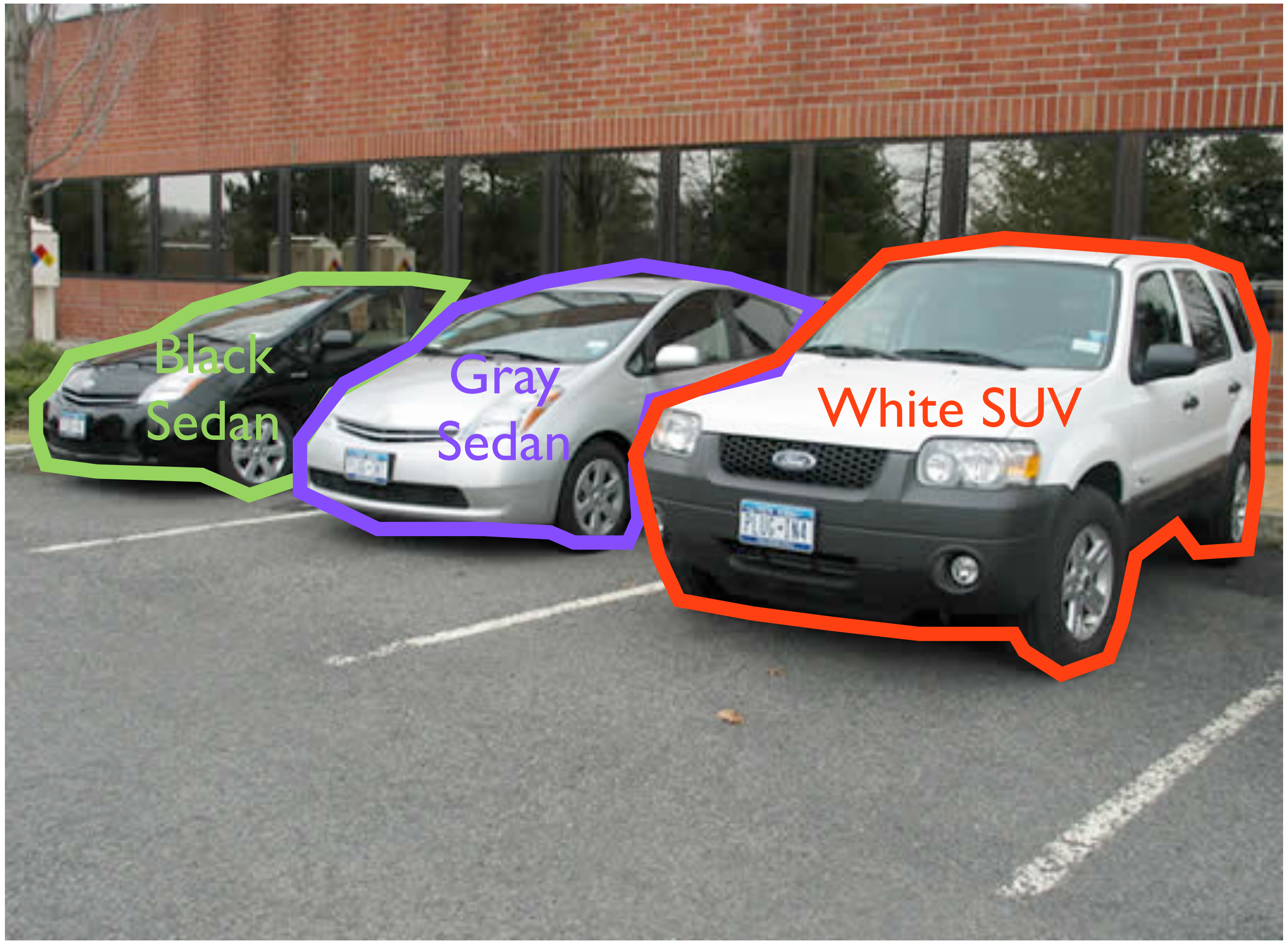
CAR RECOGNITION

Yan Li, Leon Gu, Takeo Kanade, "A robust shape model for multi-view car alignment," CVPR 2009.

CAR RECOGNITION

Yan Li, Leon Gu, Takeo Kanade, "A robust shape model for multi-view car alignment," CVPR 2009.

CAR RECOGNITION



Yan Li, Leon Gu, Takeo Kanade, "A robust shape model for multi-view car alignment," CVPR 2009.

What is the probability of observing a blue coupe?

JOINT PROBABILITY

DISCRETE

JOINT PROBABILITY

DISCRETE

$$X \in \{\mathrm{Sedan}, \mathrm{SUV}, \mathrm{Coupe}, \mathrm{Truck}\}$$

JOINT PROBABILITY

DISCRETE

$$X \in \{\text{Sedan}, \text{SUV}, \text{Coupe}, \text{Truck}\}$$

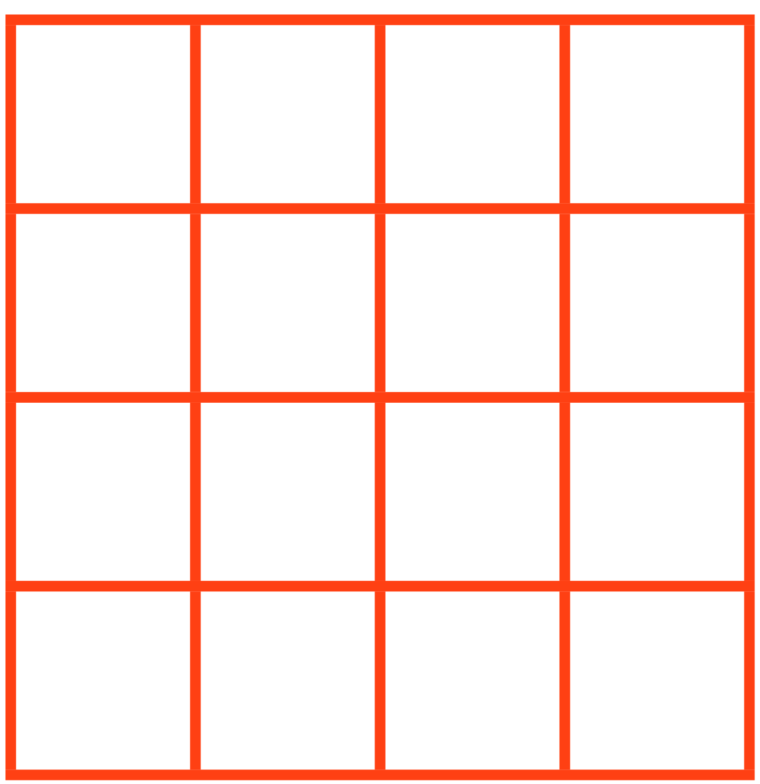
$$Y \in \{\text{Red}, \text{White}, \text{Blue}, \text{Beige}\}$$

JOINT PROBABILITY

DISCRETE

$$X \in \{\text{Sedan}, \text{SUV}, \text{Coupe}, \text{Truck}\}$$

$$Y \in \{\text{Red}, \text{White}, \text{Blue}, \text{Beige}\}$$



JOINT PROBABILITY

DISCRETE

$$X \in \{\text{Sedan}, \text{SUV}, \text{Coupe}, \text{Truck}\}$$

$$Y \in \{\text{Red}, \text{White}, \text{Blue}, \text{Beige}\}$$

Red				
White				
Blue				
Beige				

JOINT PROBABILITY

DISCRETE

$$X \in \{\text{Sedan}, \text{SUV}, \text{Coupe}, \text{Truck}\}$$

$$Y \in \{\text{Red}, \text{White}, \text{Blue}, \text{Beige}\}$$

	Sedan	SUV	Coupe	Truck
Red				
White				
Blue				
Beige				

JOINT PROBABILITY

DISCRETE

$$X \in \{\mathrm{Sedan}, \mathrm{SUV}, \mathrm{Coupe}, \mathrm{Truck}\}$$

$$Y \in \{\mathrm{Red}, \mathrm{White}, \mathrm{Blue}, \mathrm{Beige}\}$$

	Sedan	SUV	Coupe	Truck
Red				
White				
Blue				
Beige				

Joint Probability

$$p(X = x_i, Y = y_j)$$

JOINT PROBABILITY

DISCRETE

$$X \in \{\text{Sedan}, \text{SUV}, \text{Coupe}, \text{Truck}\}$$

$$Y \in \{\text{Red}, \text{White}, \text{Blue}, \text{Beige}\}$$

x_i

	Sedan	SUV	Coupe	Truck
Red				
White				
y_j Blue				
Beige				

Joint Probability

$$p(X = x_i, Y = y_j)$$

JOINT PROBABILITY

DISCRETE

$$X \in \{\text{Sedan}, \text{SUV}, \text{Coupe}, \text{Truck}\}$$

$$Y \in \{\text{Red}, \text{White}, \text{Blue}, \text{Beige}\}$$

	Sedan	SUV	x_i Coupe	Truck
Red				
White				
y_j Blue			n_{ij}	
Beige				

Joint Probability

$$p(X = x_i, Y = y_j)$$

JOINT PROBABILITY

DISCRETE

$$X \in \{\text{Sedan}, \text{SUV}, \text{Coupe}, \text{Truck}\}$$

$$Y \in \{\text{Red}, \text{White}, \text{Blue}, \text{Beige}\}$$

	Sedan	SUV	x_i Coupe	Truck
Red				
White				
y_j Blue			n_{ij}	
Beige				

Joint Probability

$$p(X = x_i, Y = y_j) = \frac{n_{ij}}{\sum_{ij} n_{ij}}$$

JOINT PROBABILITY

DISCRETE

$$X \in \{\text{Sedan}, \text{SUV}, \text{Coupe}, \text{Truck}\}$$

$$Y \in \{\text{Red}, \text{White}, \text{Blue}, \text{Beige}\}$$

	Sedan	SUV	x_i Coupe	Truck
Red				
White				
y_j Blue			n_{ij}	
Beige				

Joint Probability

$$p(X = x_i, Y = y_j) = \frac{n_{ij}}{\sum_{ij} n_{ij}}$$

$$p(X = x_i, Y = y_j) = \frac{n_{ij}}{N}$$

JOINT PROBABILITY

DISCRETE

$$X \in \{\text{Sedan}, \text{SUV}, \text{Coupe}, \text{Truck}\}$$

$$Y \in \{\text{Red}, \text{White}, \text{Blue}, \text{Beige}\}$$

	Sedan	SUV	x_i Coupe	Truck
Red				
White				
y_j Blue			n_{ij}	
Beige				

Joint Probability

$$p(X = x_i, Y = y_j) = \frac{n_{ij}}{\sum_{ij} n_{ij}}$$

$$p(X = x_i, Y = y_j) = \frac{n_{ij}}{N}$$

$$p(x_i, y_j) = \frac{n_{ij}}{N}$$

JOINT PROBABILITY

DISCRETE

$$X \in \{\text{Sedan}, \text{SUV}, \text{Coupe}, \text{Truck}\}$$

$$Y \in \{\text{Red}, \text{White}, \text{Blue}, \text{Beige}\}$$

	Sedan	SUV	x_i Coupe	Truck
Red				
White				
y_j Blue			n_{ij}	
Beige				

Joint Probability

$$p(X = x_i, Y = y_j) = \frac{n_{ij}}{\sum_{ij} n_{ij}}$$

$$p(X = x_i, Y = y_j) = \frac{n_{ij}}{N}$$

$$p(x_i, y_j) = \frac{n_{ij}}{N}$$

$$N \to \infty$$

What is the probability of a coupe of *any* color?

SUM RULE

DISCRETE

$$X \in \{\text{Sedan}, \text{SUV}, \text{Coupe}, \text{Truck}\}$$

$$Y \in \{\text{Red}, \text{White}, \text{Blue}, \text{Beige}\}$$

	Sedan	SUV	x_i Coupe	Truck
Red				
White				
y_j Blue			n_{ij}	
Beige				

SUM RULE

DISCRETE

$$X \in \{\text{Sedan}, \text{SUV}, \text{Coupe}, \text{Truck}\}$$

$$Y \in \{\text{Red}, \text{White}, \text{Blue}, \text{Beige}\}$$

			x_i	
	Sedan	SUV	Coupe	Truck
Red				
White				
y_j Blue			n_{ij}	
Beige				

$$p(X = x_i) = \frac{\sum_j n_{ij}}{N}$$

SUM RULE

DISCRETE

$$X \in \{\text{Sedan}, \text{SUV}, \text{Coupe}, \text{Truck}\}$$

$$Y \in \{\text{Red}, \text{White}, \text{Blue}, \text{Beige}\}$$

	Sedan	SUV	Coupe (x_i)	Truck
Red				
White				
y_j Blue			n_{ij}	
Beige				

Marginal Probability

$$p(X = x_i) = \frac{\sum_j n_{ij}}{N}$$

SUM RULE

DISCRETE

$$X \in \{\text{Sedan}, \text{SUV}, \text{Coupe}, \text{Truck}\}$$

$$Y \in \{\text{Red}, \text{White}, \text{Blue}, \text{Beige}\}$$

	Sedan	SUV	Coupe (x_i)	Truck
Red				
White				
y_j Blue			n_{ij}	
Beige				
			c_i	

Marginal Probability

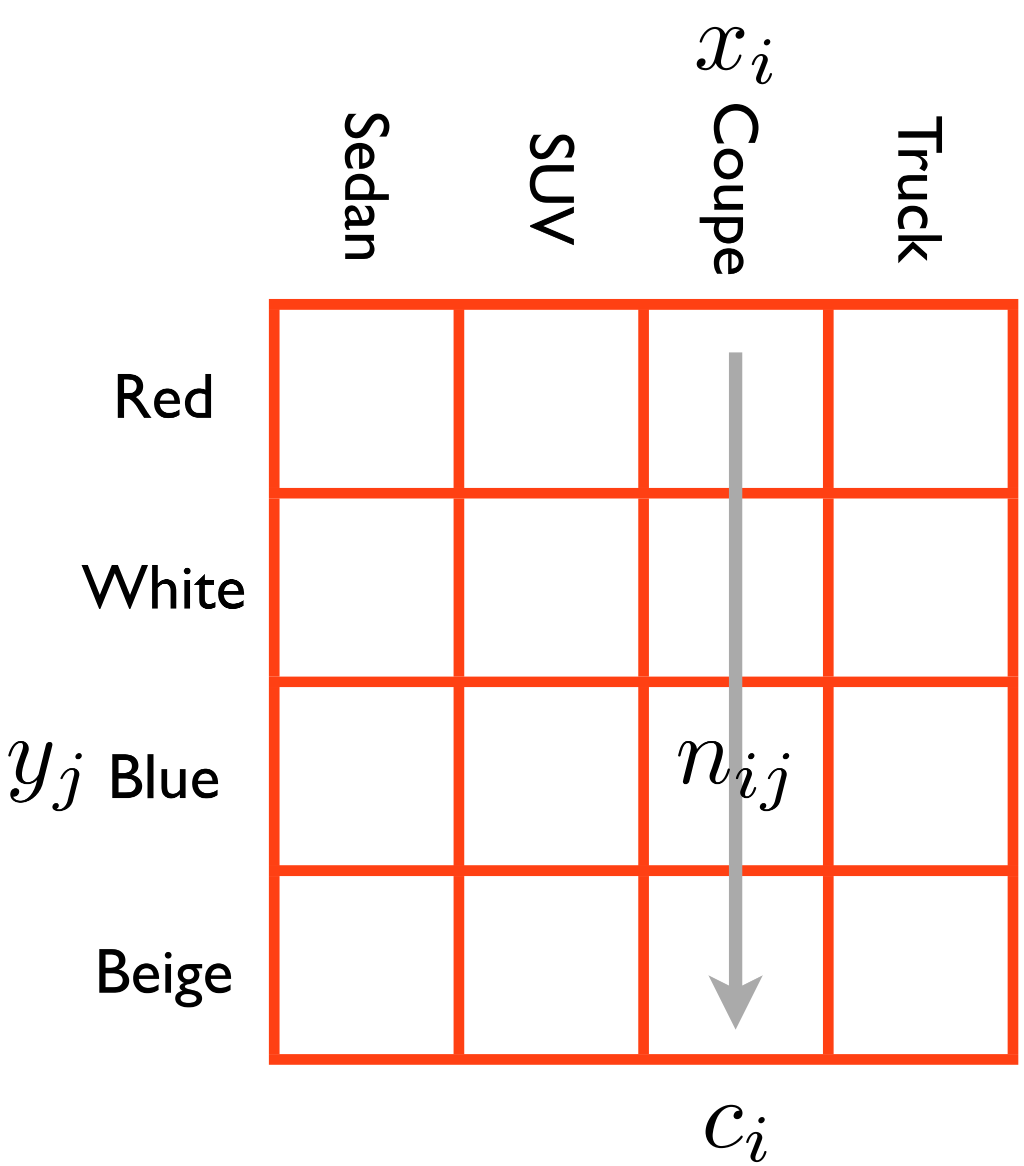
$$p(X = x_i) = \frac{\sum_j n_{ij}}{N}$$

SUM RULE

DISCRETE

$$X \in \{\text{Sedan}, \text{SUV}, \text{Coupe}, \text{Truck}\}$$

$$Y \in \{\text{Red}, \text{White}, \text{Blue}, \text{Beige}\}$$



Marginal Probability

$$p(X = x_i) = \frac{\sum_j n_{ij}}{N}$$

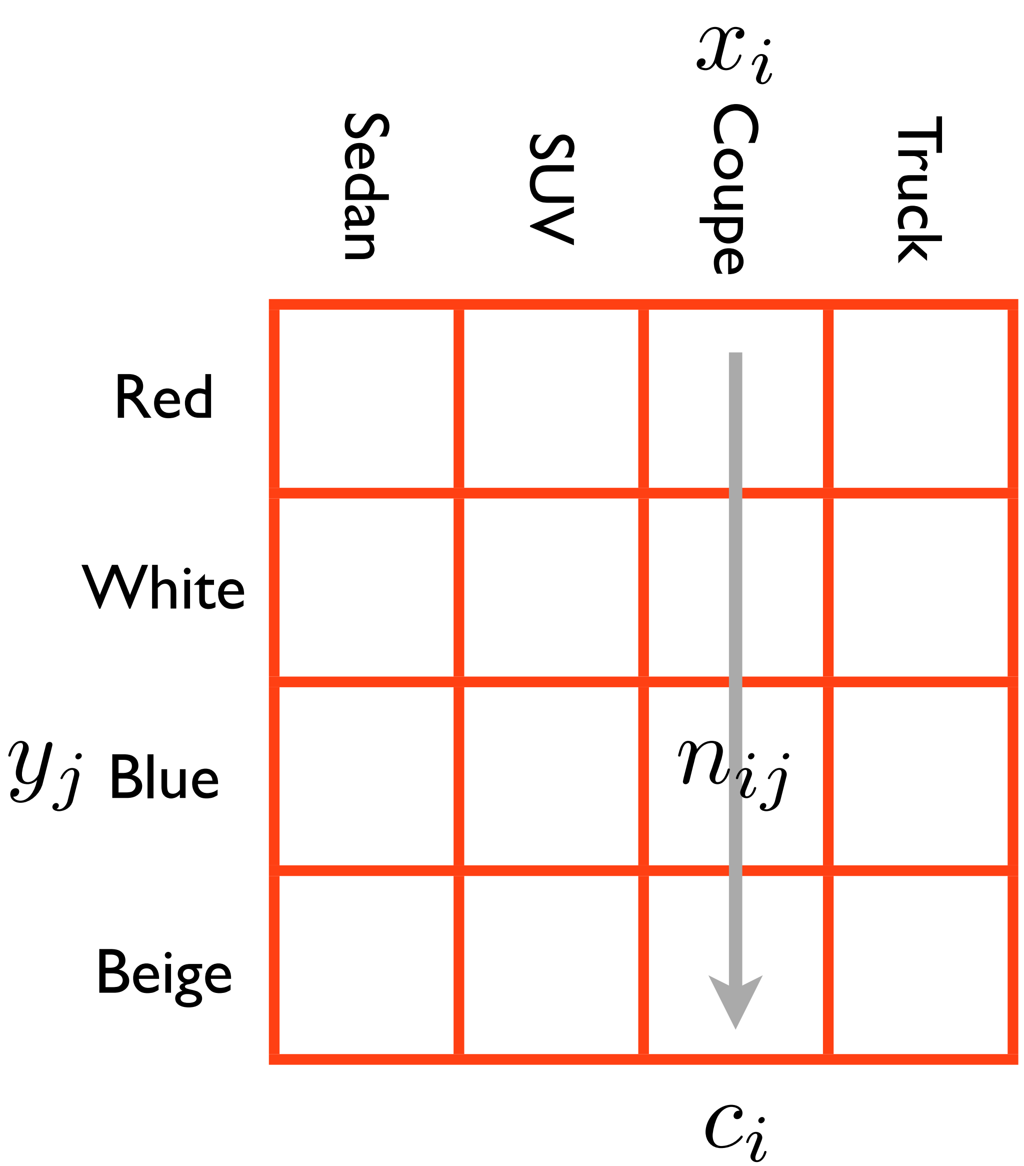
$$c_i = \sum_j n_{ij}$$

SUM RULE

DISCRETE

$$X \in \{\text{Sedan}, \text{SUV}, \text{Coupe}, \text{Truck}\}$$

$$Y \in \{\text{Red}, \text{White}, \text{Blue}, \text{Beige}\}$$



Marginal Probability

$$p(X = x_i) = \frac{\sum_j n_{ij}}{N}$$

$$c_i = \sum_j n_{ij}$$

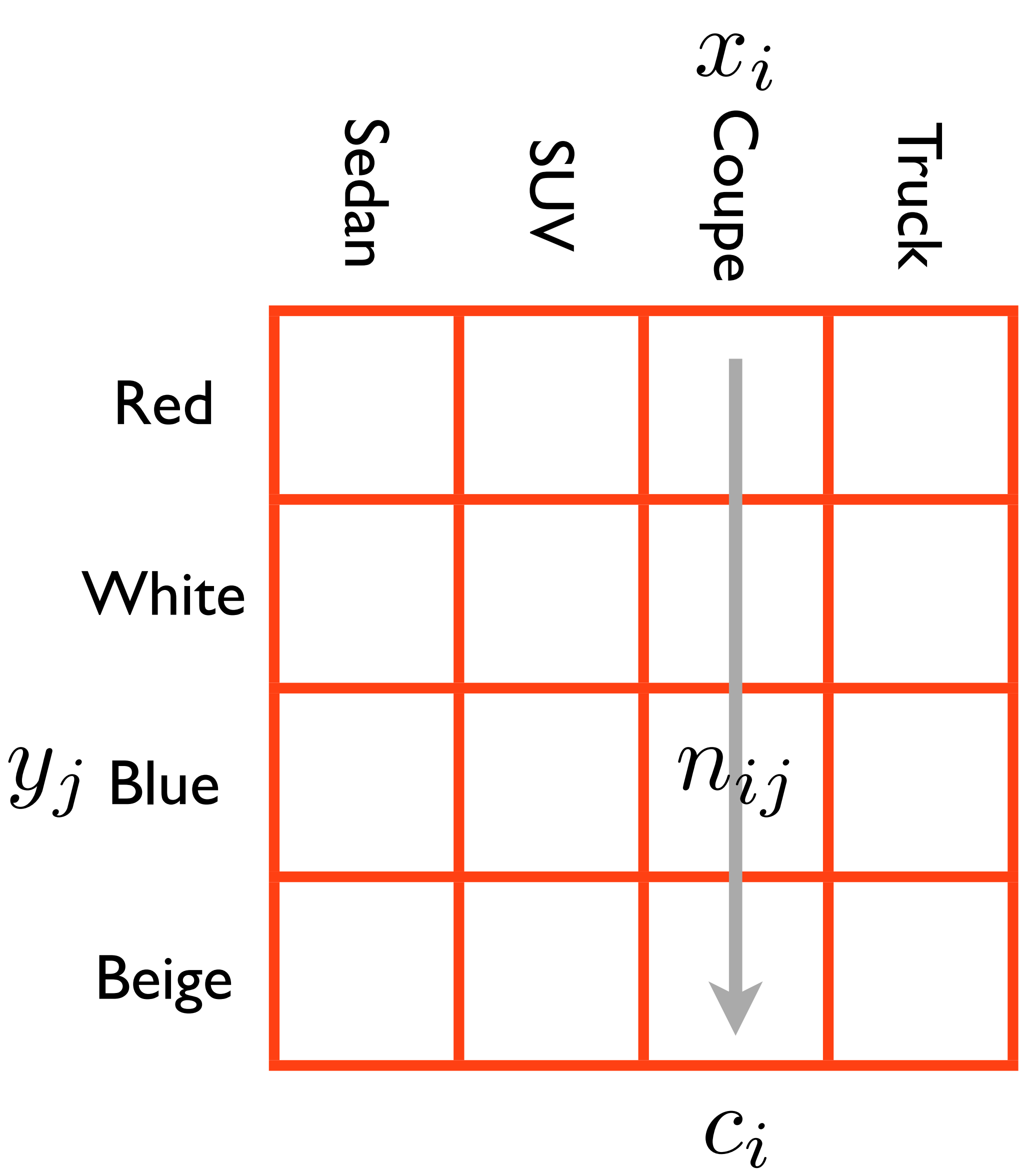
$$p(X = x_i) = \frac{c_i}{N}$$

SUM RULE

DISCRETE

$$X \in \{\text{Sedan}, \text{SUV}, \text{Coupe}, \text{Truck}\}$$

$$Y \in \{\text{Red}, \text{White}, \text{Blue}, \text{Beige}\}$$



Marginal Probability

$$p(X = x_i) = \frac{\sum_j n_{ij}}{N}$$

$$c_i = \sum_j n_{ij}$$

$$p(X = x_i) = \frac{c_i}{N}$$

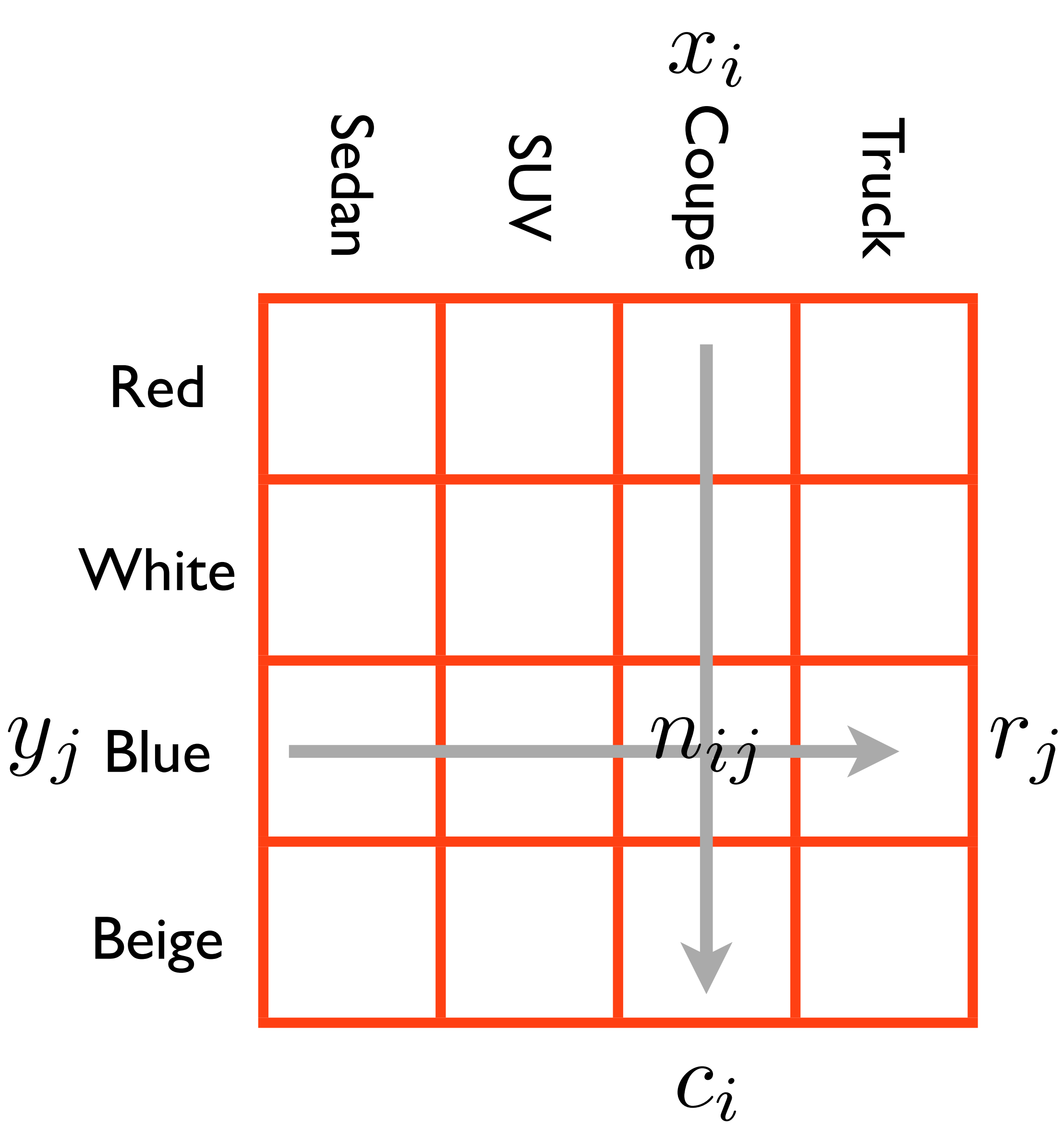
$$\boxed{p(X = x_i) = \sum_j p(X = x_i, Y = y_j)}$$

SUM RULE

DISCRETE

$$X \in \{\text{Sedan}, \text{SUV}, \text{Coupe}, \text{Truck}\}$$

$$Y \in \{\text{Red}, \text{White}, \text{Blue}, \text{Beige}\}$$



Marginal Probability

$$p(X = x_i) = \frac{\sum_j n_{ij}}{N}$$

$$c_i = \sum_j n_{ij}$$

$$p(X = x_i) = \frac{c_i}{N}$$

$$\boxed{p(X = x_i) = \sum_j p(X = x_i, Y = y_j)}$$

SUM RULE

DISCRETE

SUM RULE

DISCRETE

$$p(X = x_i) = \sum_j p(X = x_i, Y = y_j)$$

SUM RULE

DISCRETE

$$p(X = x_i) = \sum_j p(X = x_i, Y = y_j)$$

$$p(x_i) = \sum_j p(x_i, y_j)$$

SUM RULE

DISCRETE

$$p(X = x_i) = \sum_j p(X = x_i, Y = y_j)$$

$$p(x_i) = \sum_j p(x_i, y_j)$$

$$p(y_j) = \sum_i p(x_i, y_j)$$

SUM RULE

DISCRETE

$$p(X = x_i) = \sum_j p(X = x_i, Y = y_j)$$

$$p(x_i) = \sum_j p(x_i, y_j)$$

$$p(y_j) = \sum_i p(x_i, y_j)$$

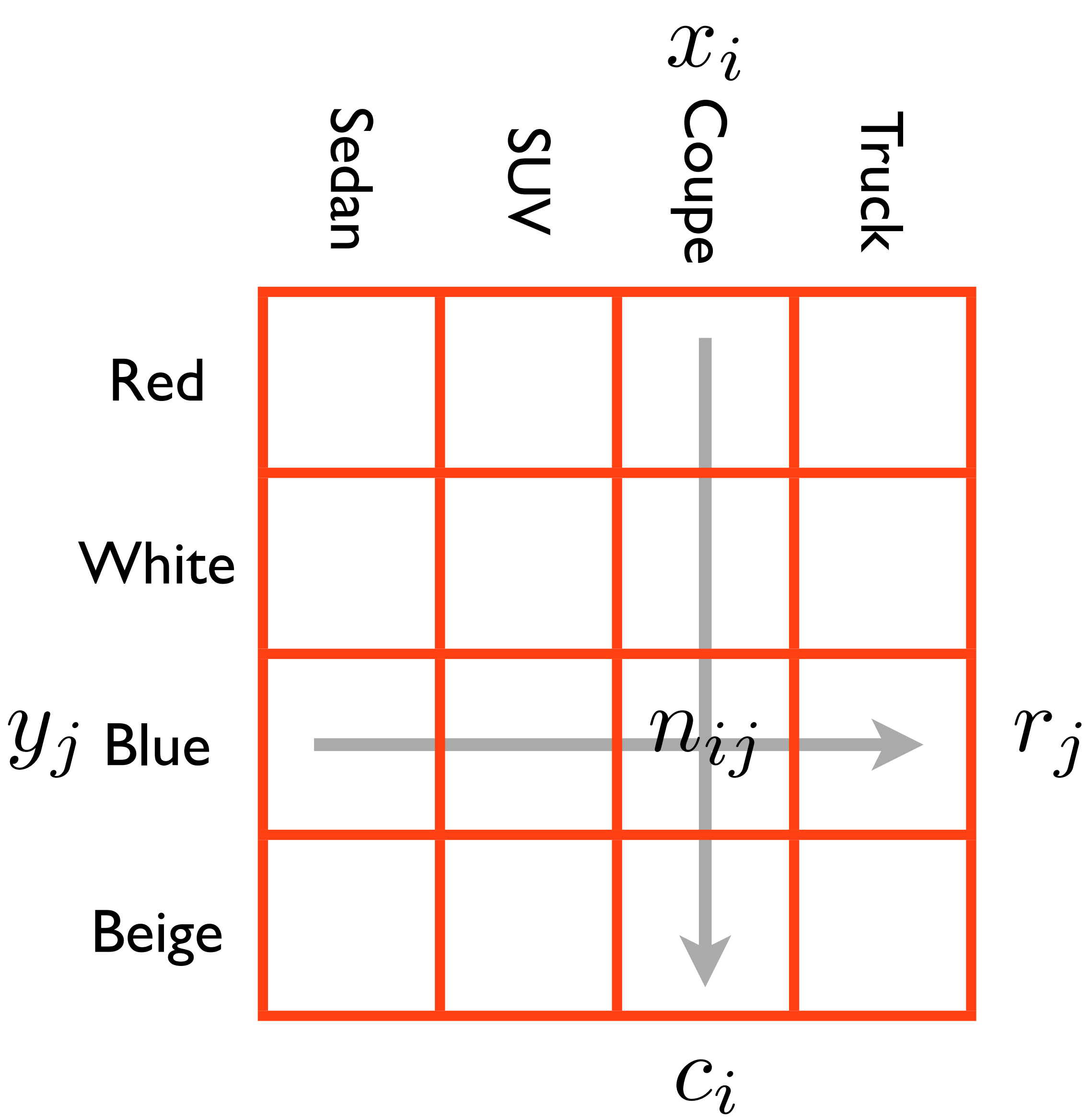
$$\boxed{p(X) = \sum_Y p(X, Y)}$$

Given that the car is a coupe, what is the probability that it is blue?

PRODUCT RULE

DISCRETE

$$X \in \{\text{Sedan}, \text{SUV}, \text{Coupe}, \text{Truck}\}$$
$$Y \in \{\text{Red}, \text{White}, \text{Blue}, \text{Beige}\}$$

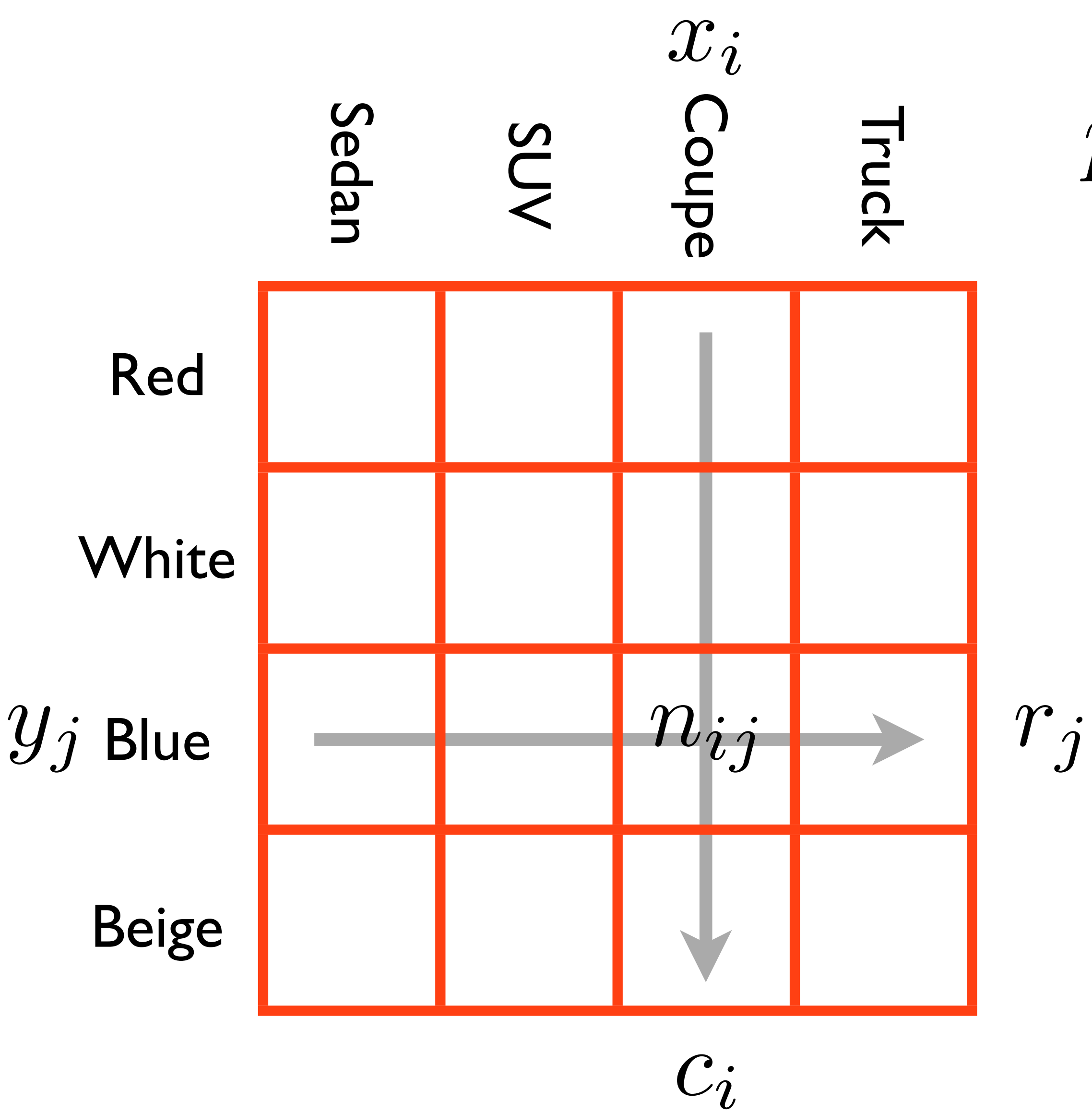


PRODUCT RULE

DISCRETE

$$X \in \{\text{Sedan}, \text{SUV}, \text{Coupe}, \text{Truck}\}$$

$$Y \in \{\text{Red}, \text{White}, \text{Blue}, \text{Beige}\}$$

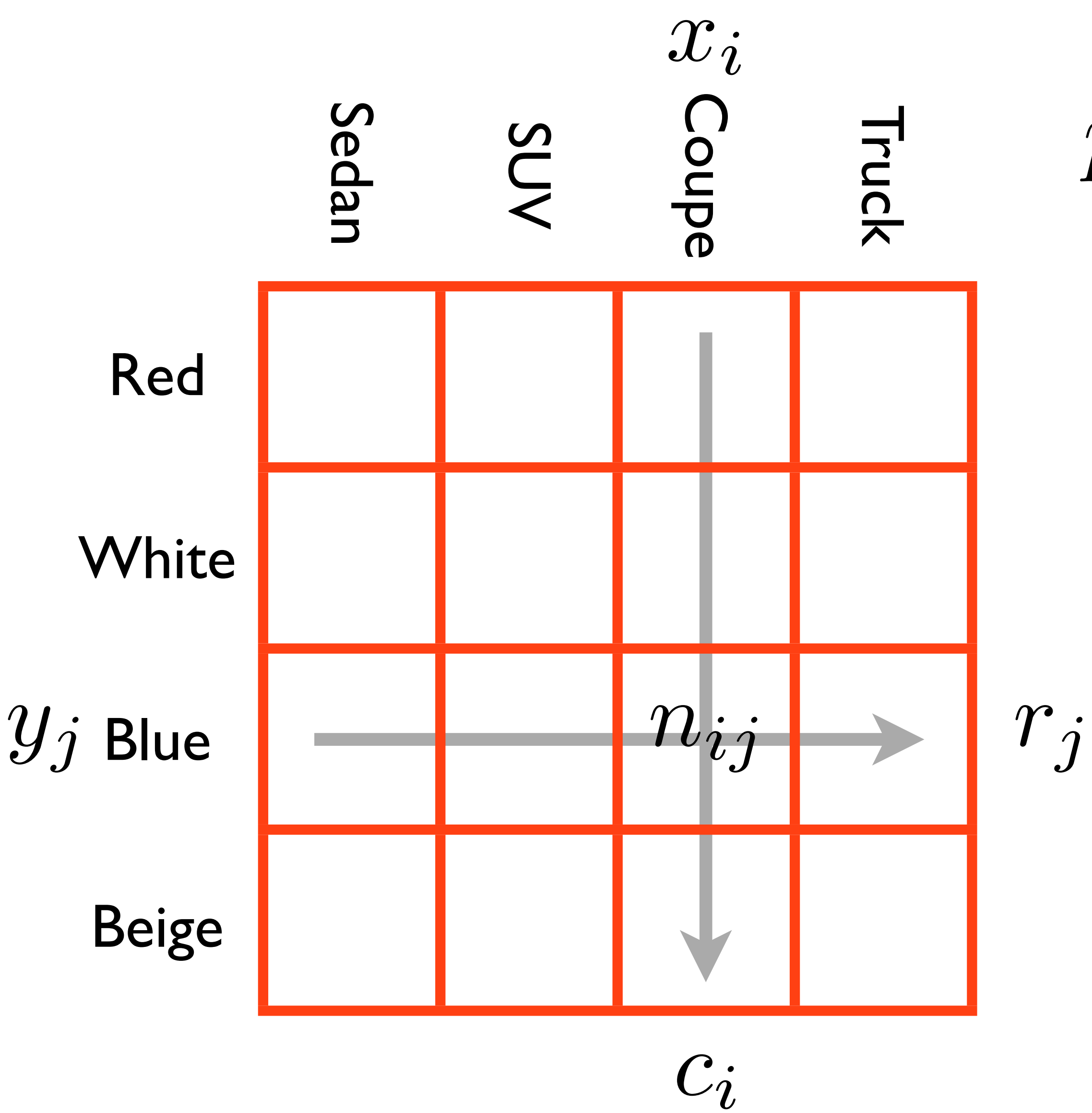


$$p(Y = y_j | X = x_i) = \frac{n_{ij}}{c_i}$$

PRODUCT RULE

DISCRETE

$$X \in \{\text{Sedan}, \text{SUV}, \text{Coupe}, \text{Truck}\}$$
$$Y \in \{\text{Red}, \text{White}, \text{Blue}, \text{Beige}\}$$



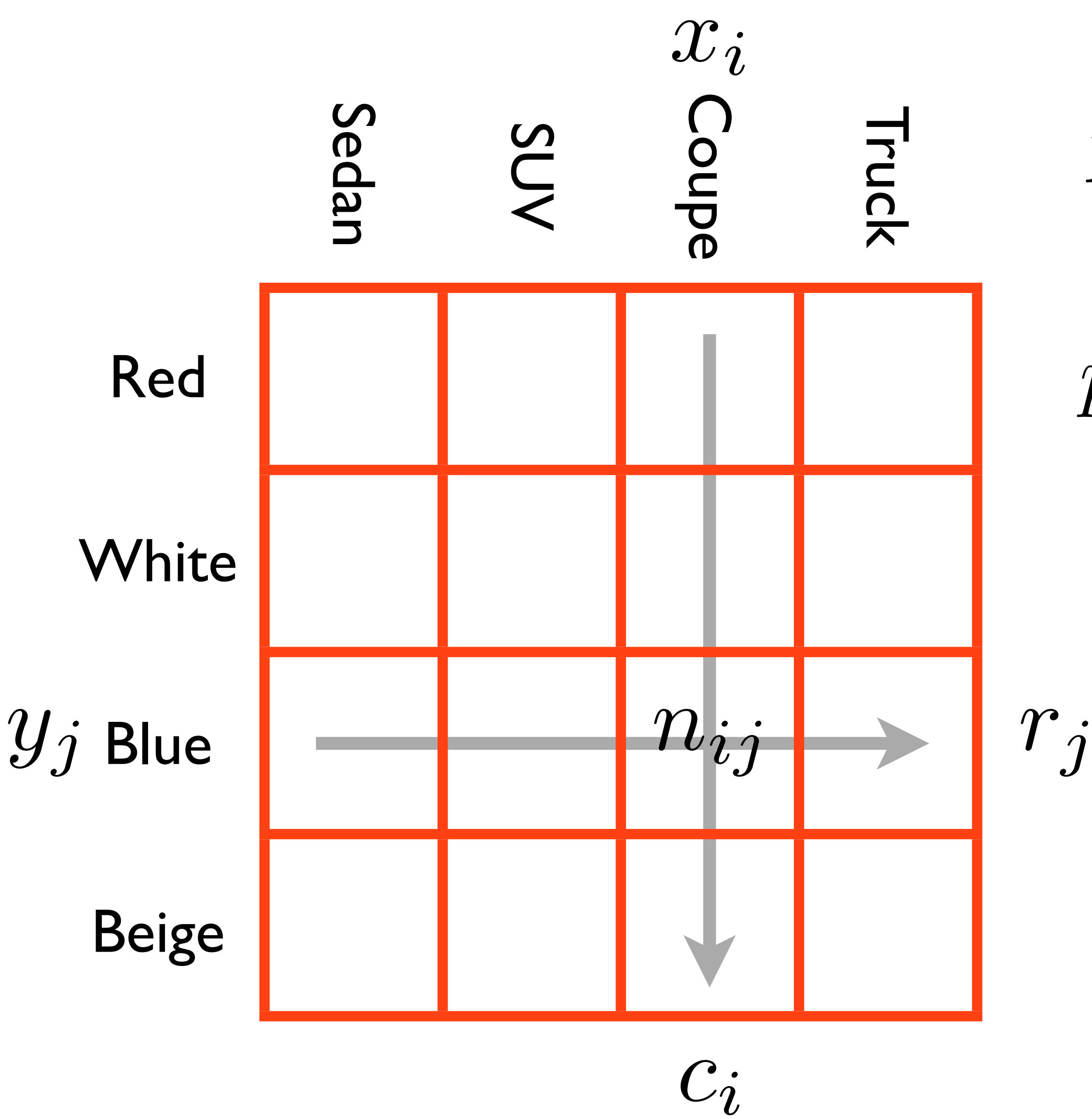
Conditional Probability

$$p(Y = y_j | X = x_i) = \frac{n_{ij}}{c_i}$$

PRODUCT RULE

DISCRETE

$$X \in \{\text{Sedan}, \text{SUV}, \text{Coupe}, \text{Truck}\}$$
$$Y \in \{\text{Red}, \text{White}, \text{Blue}, \text{Beige}\}$$



Conditional Probability

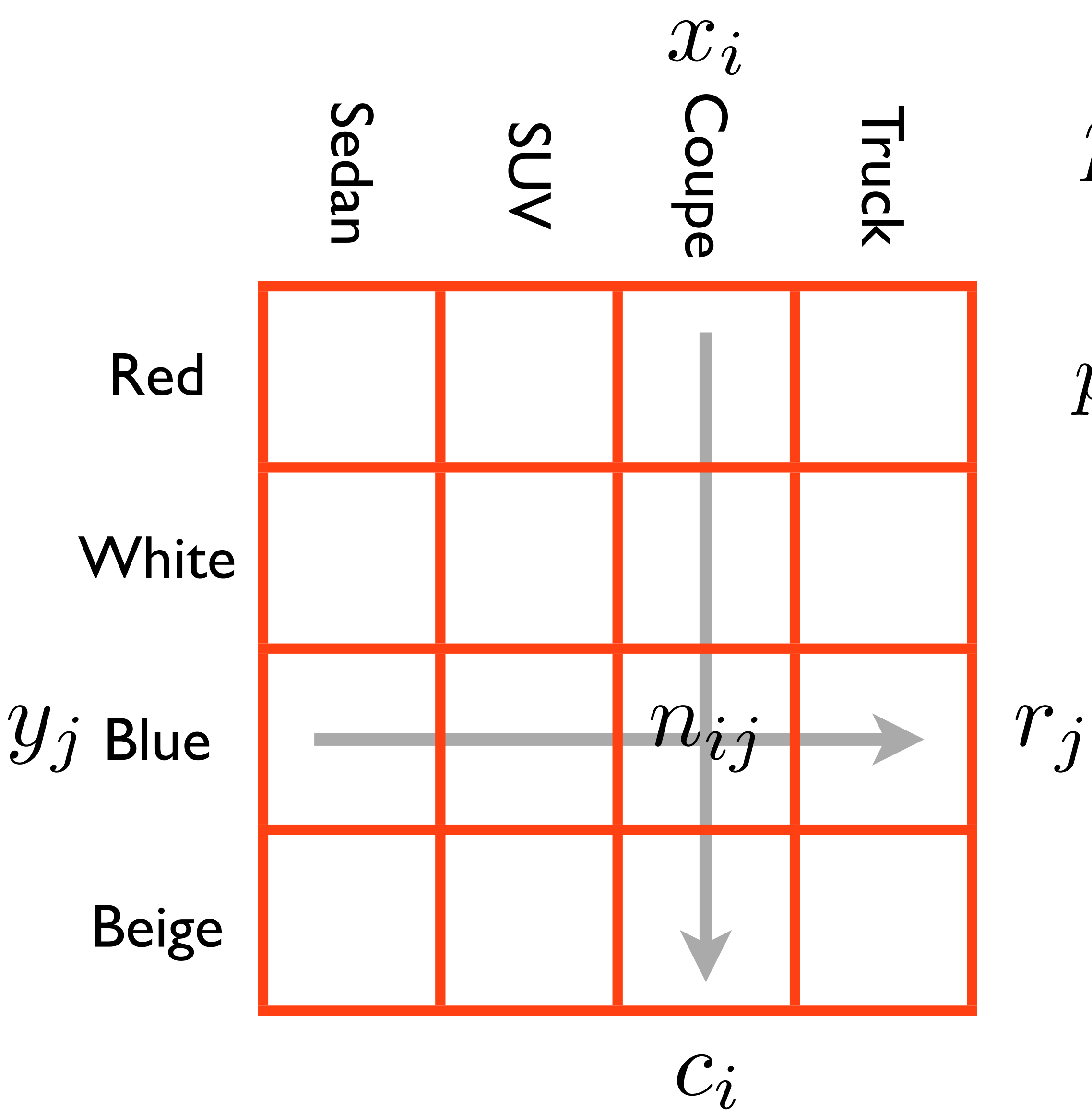
$$p(Y = y_j | X = x_i) = \frac{n_{ij}}{c_i}$$

$$p(X = x_i, Y = y_j) = \frac{n_{ij}}{N}$$

PRODUCT RULE

DISCRETE

$$X \in \{\text{Sedan}, \text{SUV}, \text{Coupe}, \text{Truck}\}$$
$$Y \in \{\text{Red}, \text{White}, \text{Blue}, \text{Beige}\}$$



Conditional Probability

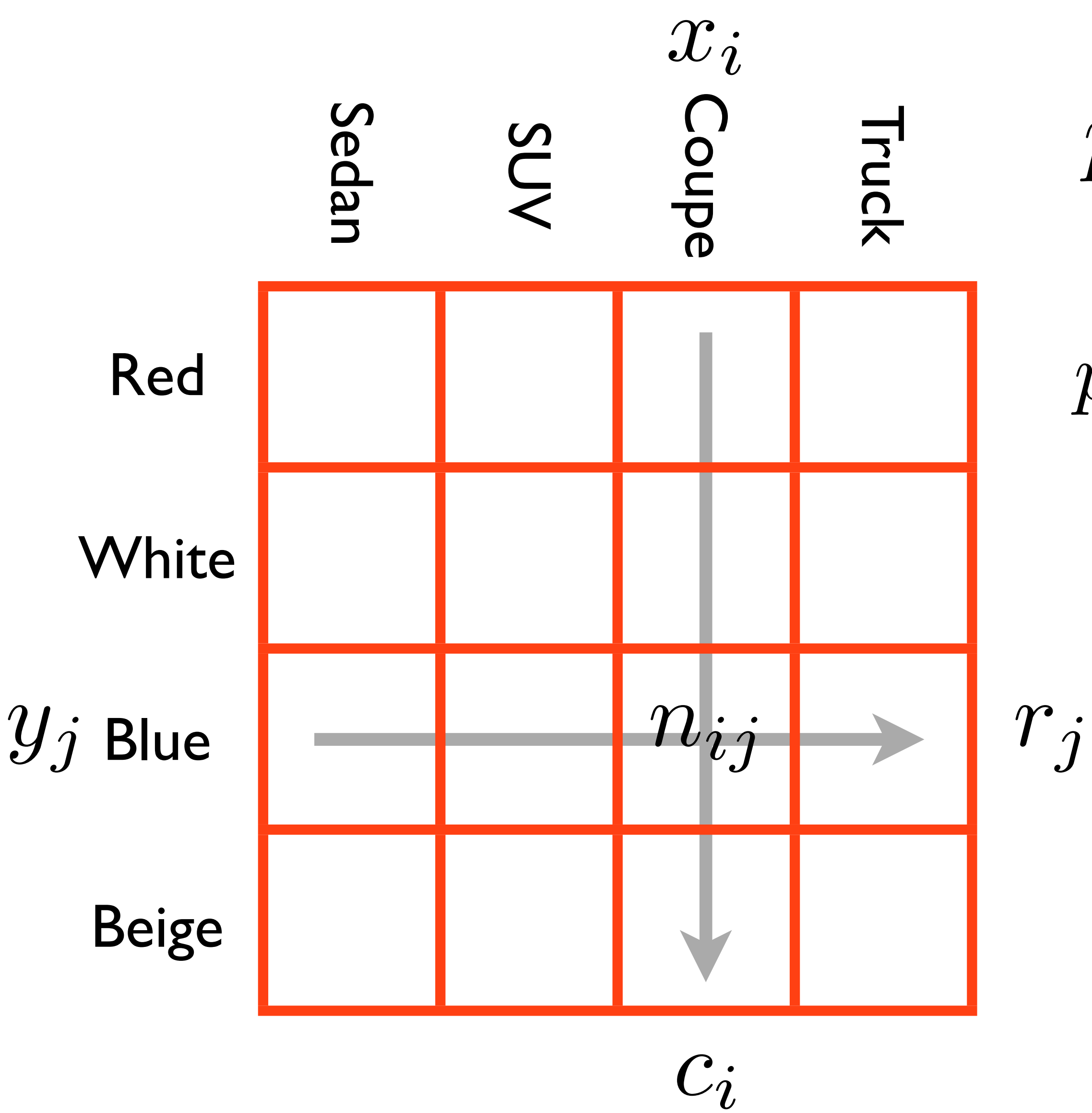
$$p(Y = y_j | X = x_i) = \frac{n_{ij}}{c_i}$$

$$p(X = x_i, Y = y_j) = \frac{n_{ij}}{N} = \frac{n_{ij}}{c_i} \cdot \frac{c_i}{N}$$

PRODUCT RULE

DISCRETE

$$X \in \{\text{Sedan}, \text{SUV}, \text{Coupe}, \text{Truck}\}$$
$$Y \in \{\text{Red}, \text{White}, \text{Blue}, \text{Beige}\}$$



Conditional Probability

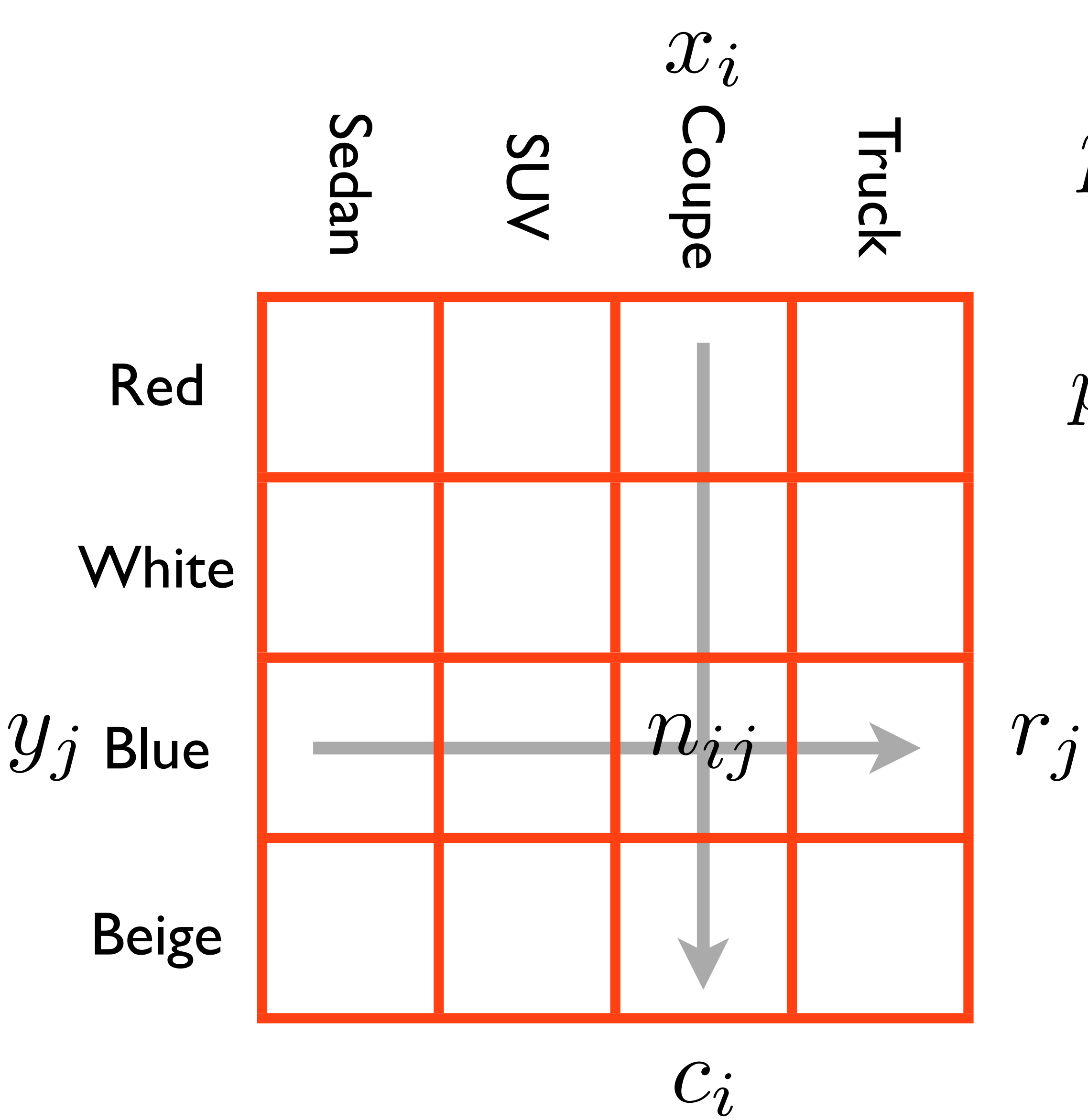
$$p(Y = y_j | X = x_i) = \frac{n_{ij}}{c_i}$$

$$p(X = x_i, Y = y_j) = \frac{n_{ij}}{N} = \frac{n_{ij}}{c_i} \cdot \frac{c_i}{N}$$
$$= p(Y = y_j | X = x_i) p(X = x_i)$$

PRODUCT RULE

DISCRETE

$$X \in \{\text{Sedan}, \text{SUV}, \text{Coupe}, \text{Truck}\}$$
$$Y \in \{\text{Red}, \text{White}, \text{Blue}, \text{Beige}\}$$



Conditional Probability

$$p(Y = y_j | X = x_i) = \frac{n_{ij}}{c_i}$$

$$p(X = x_i, Y = y_j) = \frac{n_{ij}}{N} = \frac{n_{ij}}{c_i} \cdot \frac{c_i}{N}$$

$$= p(Y = y_j | X = x_i) p(X = x_i)$$

$$\boxed{p(X = x_i, Y = y_j) = p(Y = y_j | X = x_i) p(X = x_i)}$$

PRODUCT RULE

DISCRETE

PRODUCT RULE

DISCRETE

$$p(X = x_i, Y = y_j) = p(Y = y_j | X = x_i) p(X = x_i)$$

PRODUCT RULE

DISCRETE

$$p(X = x_i, Y = y_j) = p(Y = y_j | X = x_i) p(X = x_i)$$

$$p(X = x_i, Y = y_j) = p(X = x_i | Y = y_j) p(Y = y_j)$$

PRODUCT RULE

DISCRETE

$$p(X = x_i, Y = y_j) = p(Y = y_j | X = x_i) p(X = x_i)$$

$$p(X = x_i, Y = y_j) = p(X = x_i | Y = y_j) p(Y = y_j)$$

$$\boxed{p(X, Y) = p(Y|X) p(X)}$$

RULES OF PROBABILITY

SUM RULE $$p(X) = \sum_Y p(X, Y)$$

PRODUCT RULE $$p(X, Y) = p(Y|X)p(X)$$

RULES OF PROBABILITY

Joint Probability

SUM RULE $$p(X) = \sum_Y p(X,Y)$$

PRODUCT RULE $$p(X,Y) = p(Y|X)p(X)$$

RULES OF PROBABILITY

Marginal Probability

Joint Probability

SUM RULE

$$p(X) = \sum_Y p(X, Y)$$

PRODUCT RULE

$$p(X, Y) = p(Y|X)p(X)$$

RULES OF PROBABILITY

Marginal Probability

Joint Probability

SUM RULE $$p(X) = \sum_Y p(X, Y)$$

PRODUCT RULE $$p(X, Y) = p(Y|X)p(X)$$

Conditional Probability

BAYES' THEOREM

$$p(X,Y) = p(Y|X)p(X)$$

BAYES' THEOREM

$$p(X|Y)p(Y) = p(X,Y) = p(Y|X)p(X)$$

BAYES' THEOREM

$$p(X|Y)p(Y) = \cancel{p(X,Y)} = p(Y|X)p(X)$$

$$p(X|Y)p(Y) = p(Y|X)p(X)$$

BAYES' THEOREM

$$p(X|Y)p(Y) = \cancel{p(X,Y)} = p(Y|X)p(X)$$

$$p(X|Y)p(Y) = p(Y|X)p(X)$$

$$p(X|Y) = \frac{p(Y|X)p(X)}{p(Y)}$$

BAYES' THEOREM

$$p(X|Y)p(Y) = \cancel{p(X,Y)} = p(Y|X)p(X)$$

$$p(X|Y)p(Y) = p(Y|X)p(X)$$

Prior Probability

$$p(X|Y) = \frac{p(Y|X)p(X)}{p(Y)}$$

BAYES' THEOREM

$$p(X|Y)p(Y) = \cancel{p(X,Y)} = p(Y|X)p(X)$$

$$p(X|Y)p(Y) = p(Y|X)p(X)$$

Prior Probability

Posterior Probability

$$p(X|Y) = \frac{p(Y|X)p(X)}{p(Y)}$$

BAYES' THEOREM

$$p(X|Y)p(Y) = \cancel{p(X,Y)} = p(Y|X)p(X)$$

$$p(X|Y)p(Y) = p(Y|X)p(X)$$

Likelihood

Prior Probability

Posterior Probability

$$p(X|Y) = \frac{p(Y|X)p(X)}{p(Y)}$$

BAYES' THEOREM

$$p(X|Y)p(Y) = \cancel{p(X,Y)} = p(Y|X)p(X)$$

$$p(X|Y)p(Y) = p(Y|X)p(X)$$

Likelihood

Prior Probability

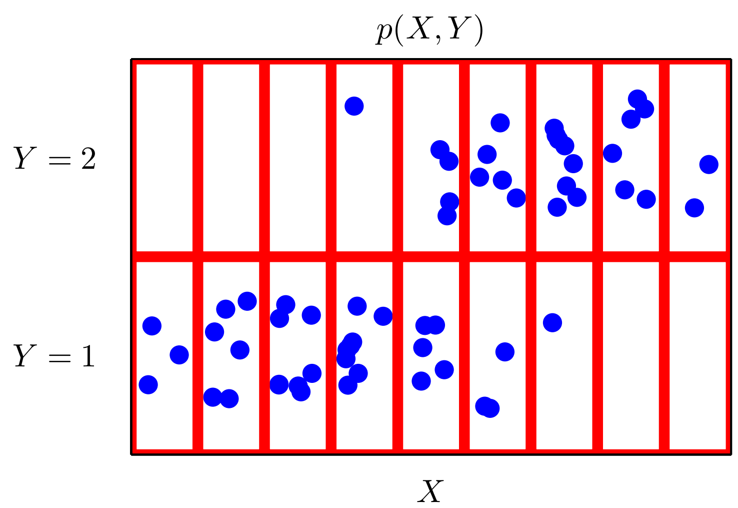
Posterior Probability

$$p(X|Y) = \frac{p(Y|X)p(X)}{p(Y)}$$

Evidence

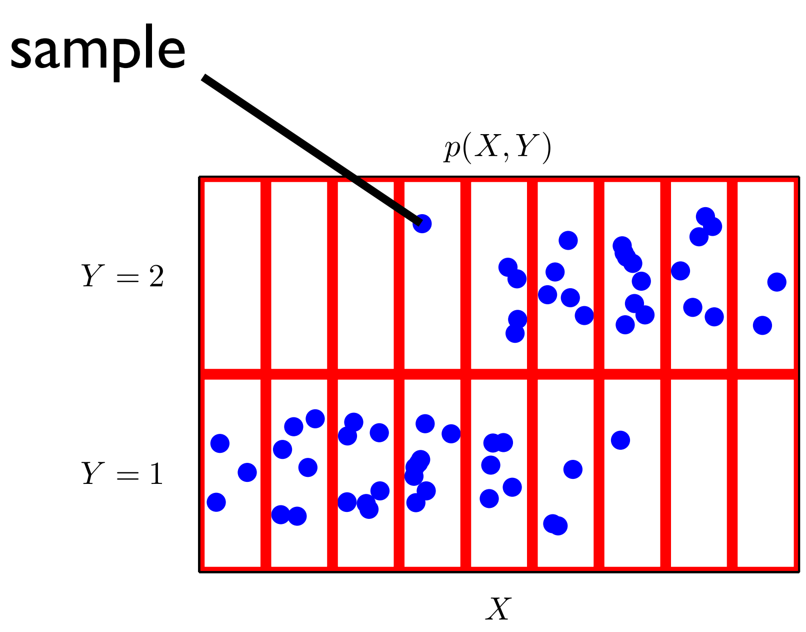
MODELING DISTRIBUTIONS

HISTOGRAMS



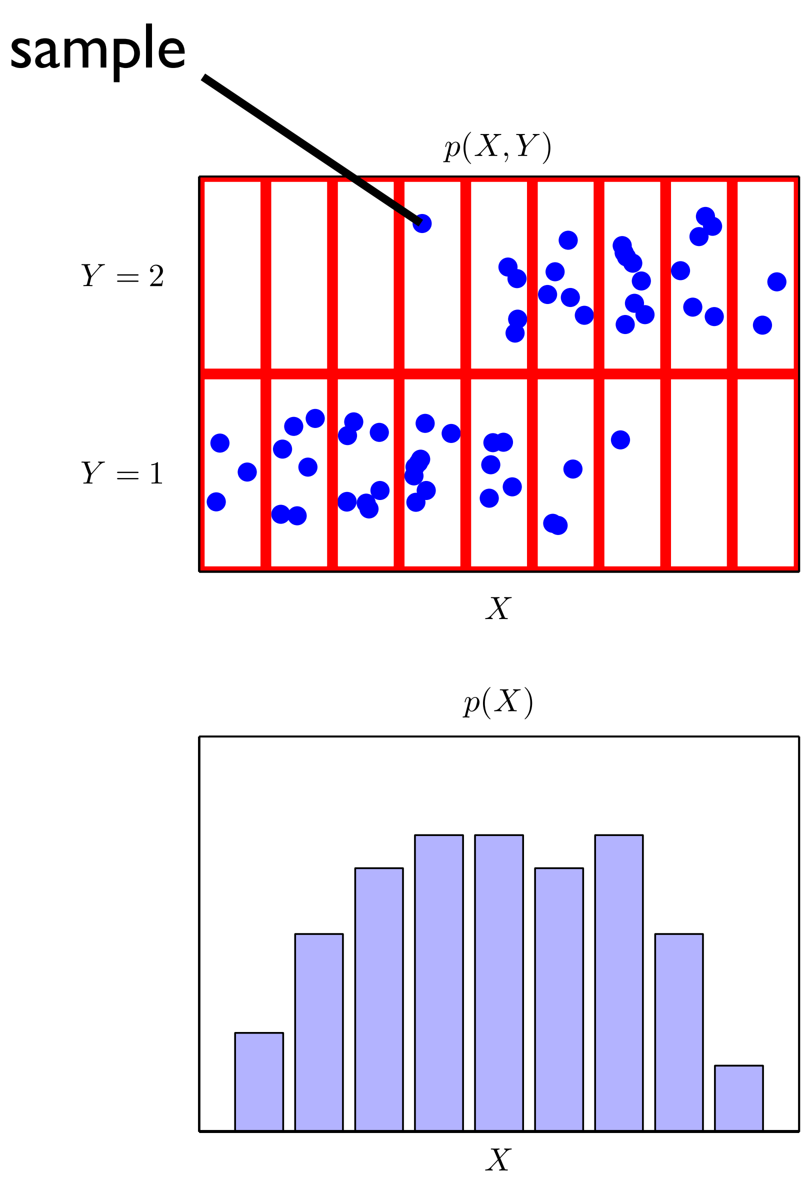
MODELING DISTRIBUTIONS

HISTOGRAMS



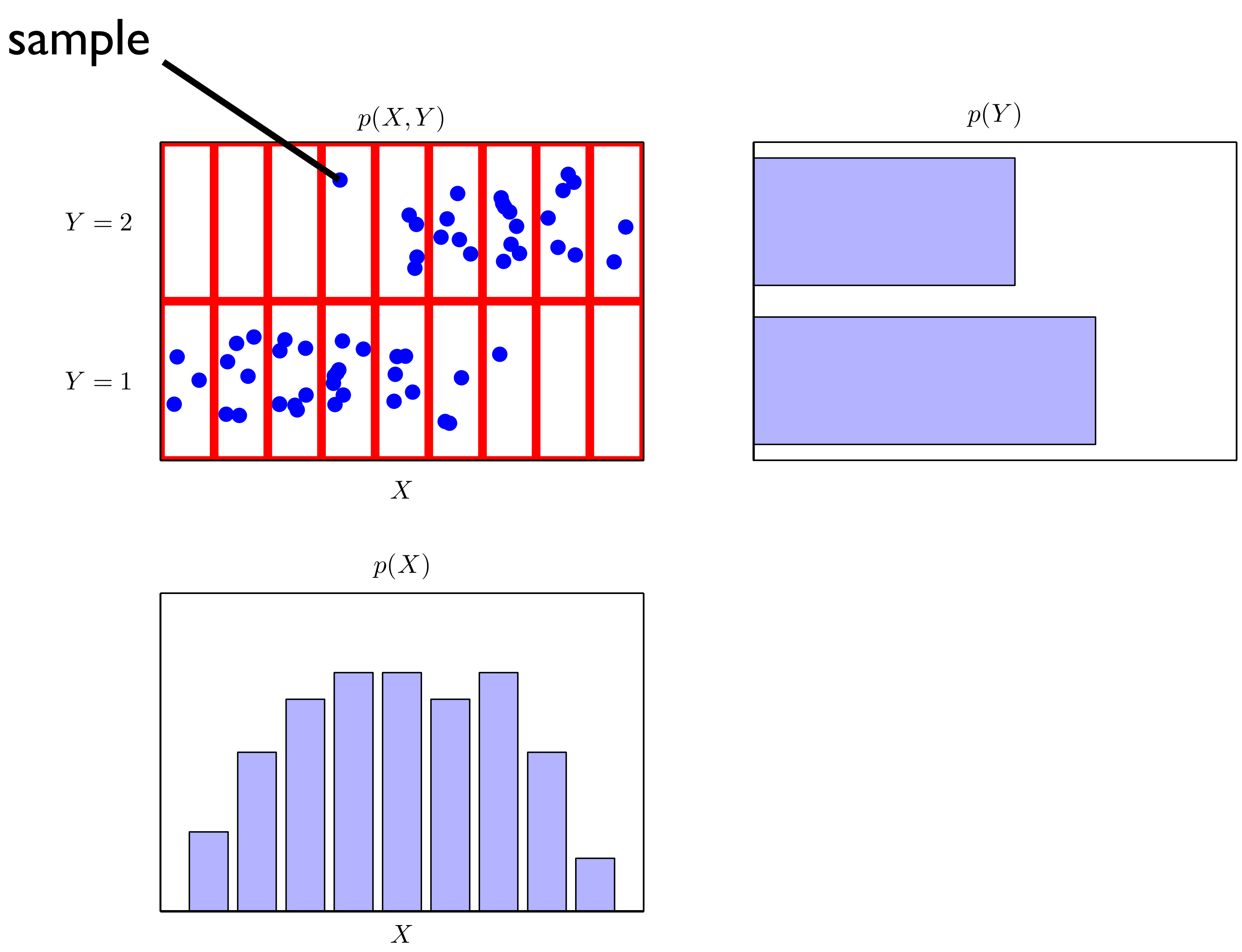
MODELING DISTRIBUTIONS

HISTOGRAMS



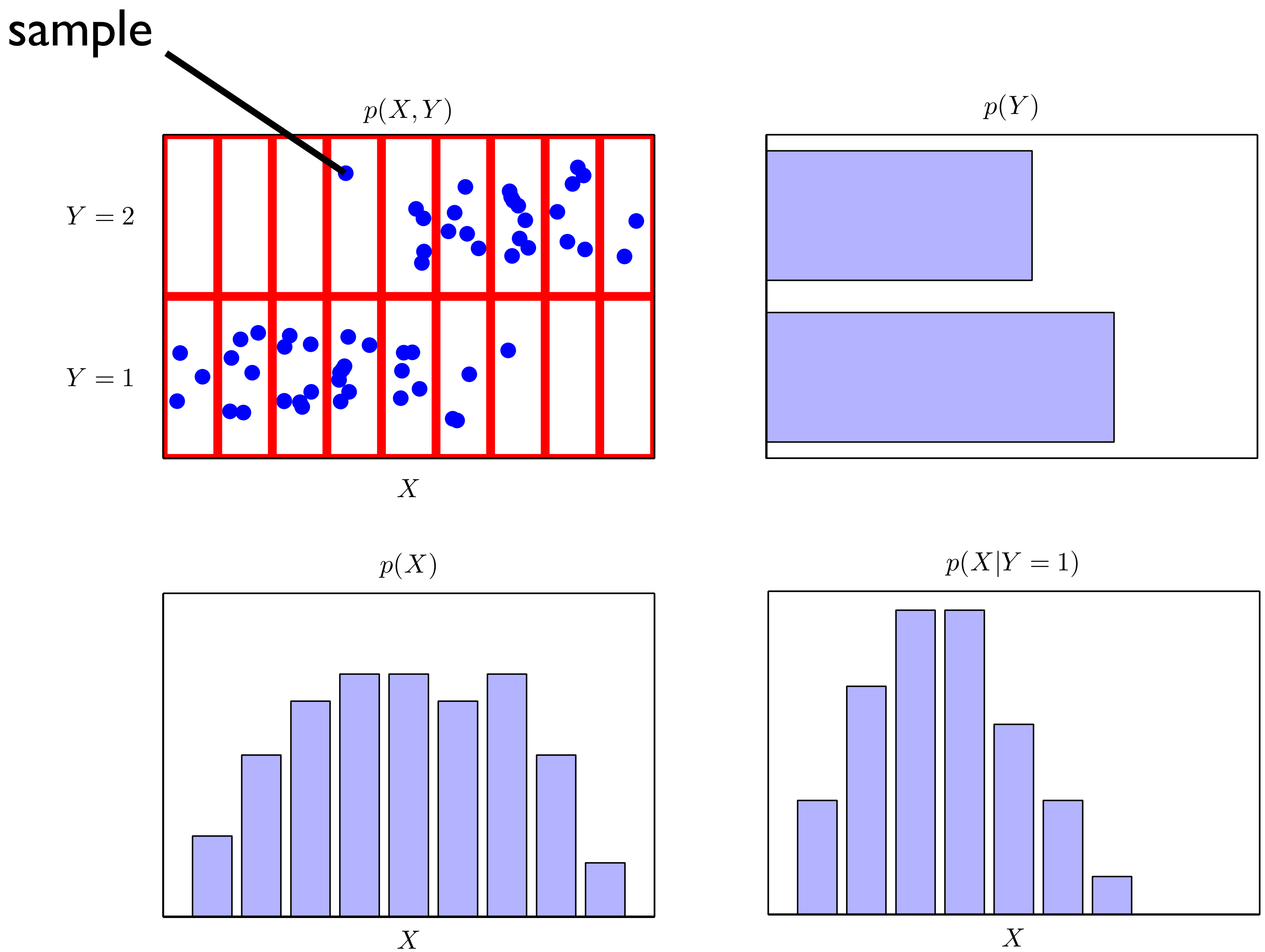
MODELING DISTRIBUTIONS

HISTOGRAMS



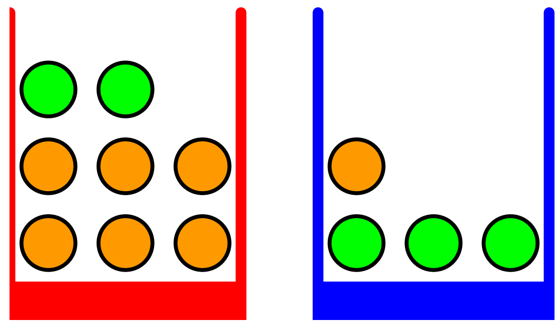
MODELING DISTRIBUTIONS

HISTOGRAMS



RANDOM VARIABLE

DISCRETE



$$\text{Box} : B \in \{r, b\}$$

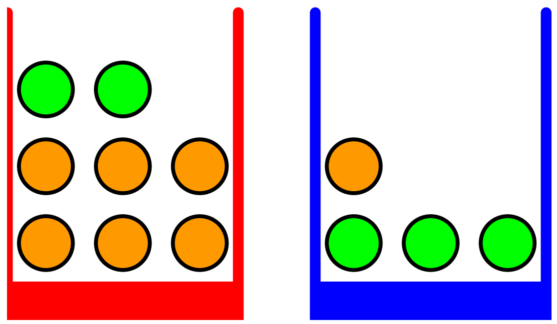
$$\text{Fruit} : F \in \{a, o\}$$

$$p(B = r) = 0.4$$
$$p(B = b) = 0.6$$

$$p(B = b) + p(B = r) = 1$$

RANDOM VARIABLE

DISCRETE



$$\text{Box} : B \in \{r, b\}$$

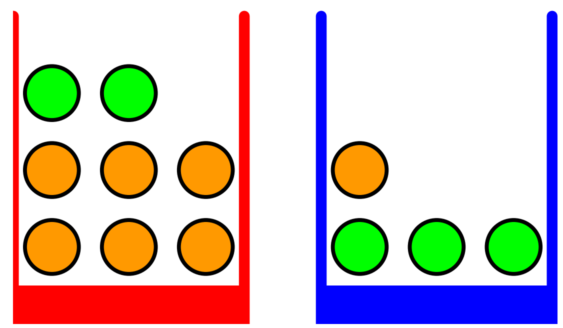
$$\text{Fruit} : F \in \{a, o\}$$

$$p(B = r) = 0.4$$
$$p(B = b) = 0.6$$

$$p(B = b) + p(B = r) = 1$$

RANDOM VARIABLE

DISCRETE



$$\text{Box} : B \in \{r, b\}$$

$$\text{Fruit} : F \in \{a, o\}$$

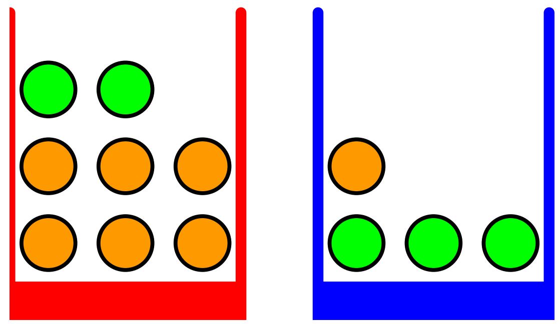
$$p(B = r) = 0.4$$
$$p(B = b) = 0.6$$

$$p(B = b) + p(B = r) = 1$$

$$p(F = a | B = r) =$$
$$p(F = o | B = r) =$$
$$p(F = a | B = b) =$$
$$p(F = o | B = b) =$$

RANDOM VARIABLE

DISCRETE



$$\text{Box} : B \in \{r, b\}$$

$$\text{Fruit} : F \in \{a, o\}$$

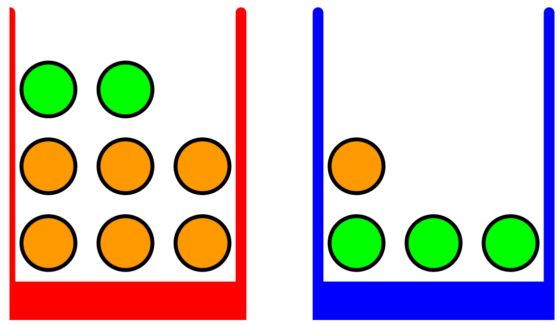
$$p(B = r) = 0.4$$
$$p(B = b) = 0.6$$

$$p(B = b) + p(B = r) = 1$$

$$p(F = a | B = r) = 0.25$$
$$p(F = o | B = r) = 0.75$$
$$p(F = a | B = b) = 0.75$$
$$p(F = o | B = b) = 0.25$$

RANDOM VARIABLE

DISCRETE



$$\text{Box} : B \in \{r, b\}$$

$$\text{Fruit} : F \in \{a, o\}$$

$$p(B = r) = 0.4$$
$$p(B = b) = 0.6$$

$$p(B = b) + p(B = r) = 1$$

$$p(F = a | B = r) = 0.25$$
$$p(F = o | B = r) = 0.75$$
$$p(F = a | B = b) = 0.75$$
$$p(F = o | B = b) = 0.25$$

$$p(F = a | B = r) + p(F = o | B = r) = 1$$
$$p(F = a | B = b) + p(F = o | B = b) = 1$$

RANDOM VARIABLE

DISCRETE

WHAT IS THE PROBABILITY THAT THE SELECTION PROCEDURE WILL PICK AN APPLE?

RANDOM VARIABLE

DISCRETE

$$p(B = r) = 0.4$$
$$p(B = b) = 0.6$$
$$p(F = a|B = r) = 0.25$$
$$p(F = o|B = r) = 0.75$$
$$p(F = a|B = b) = 0.75$$
$$p(F = o|B = b) = 0.25$$

WHAT IS THE PROBABILITY THAT THE SELECTION PROCEDURE WILL PICK AN APPLE?

RANDOM VARIABLE

DISCRETE

$p(B = r) = 0.4$
$p(B = b) = 0.6$

$p(F = a|B = r) = 0.25$
$p(F = o|B = r) = 0.75$
$p(F = a|B = b) = 0.75$
$p(F = o|B = b) = 0.25$

WHAT IS THE PROBABILITY THAT THE SELECTION PROCEDURE WILL PICK AN APPLE?

$$p(F = a)$$

$p(B = r) = 0.4$
$p(B = b) = 0.6$

$p(F = a|B = r) = 0.25$
$p(F = o|B = r) = 0.75$
$p(F = a|B = b) = 0.75$
$p(F = o|B = b) = 0.25$

RANDOM VARIABLE

DISCRETE

WHAT IS THE PROBABILITY THAT THE SELECTION PROCEDURE WILL PICK AN APPLE?

$$p(F = a) = \sum_B p(F = a, B)$$ **SUM RULE**

RANDOM VARIABLE

DISCRETE

$p(B = r) = 0.4$
$p(B = b) = 0.6$

$p(F = a|B = r) = 0.25$
$p(F = o|B = r) = 0.75$
$p(F = a|B = b) = 0.75$
$p(F = o|B = b) = 0.25$

WHAT IS THE PROBABILITY THAT THE SELECTION PROCEDURE WILL PICK AN APPLE?

$$p(F = a) = \sum_B p(F = a, B) \quad \textbf{SUM RULE}$$

$$= p(F = a, B = r) + p(F = a, B = b)$$

$p(B = r) = 0.4$
$p(B = b) = 0.6$
$p(F = a|B = r) = 0.25$
$p(F = o|B = r) = 0.75$
$p(F = a|B = b) = 0.75$
$p(F = o|B = b) = 0.25$

RANDOM VARIABLE

DISCRETE

WHAT IS THE PROBABILITY THAT THE SELECTION PROCEDURE WILL PICK AN APPLE?

$$p(F = a) = \sum_{B} p(F = a, B)$$ **SUM RULE**

$$= p(F = a, B = r) + p(F = a, B = b)$$

$$= p(F = a|B = r)p(B = r) + p(F = a|B = b)p(B = b)$$

PRODUCT RULE

$p(B = r) = 0.4$
$p(B = b) = 0.6$
$p(F = a|B = r) = 0.25$
$p(F = o|B = r) = 0.75$
$p(F = a|B = b) = 0.75$
$p(F = o|B = b) = 0.25$

RANDOM VARIABLE

DISCRETE

WHAT IS THE PROBABILITY THAT THE SELECTION PROCEDURE WILL PICK AN APPLE?

$$p(F = a) = \sum_B p(F = a, B)$$ **SUM RULE**

$$= p(F = a, B = r) + p(F = a, B = b)$$

$$= p(F = a|B = r)p(B = r) + p(F = a|B = b)p(B = b)$$

PRODUCT RULE

$$= 0.25 \times 0.4 + 0.75 \times 0.6 = 0.55$$

$$p(B = r) = 0.4$$
$$p(B = b) = 0.6$$
$$p(F = a|B = r) = 0.25$$
$$p(F = o|B = r) = 0.75$$
$$p(F = a|B = b) = 0.75$$
$$p(F = o|B = b) = 0.25$$

RANDOM VARIABLE

DISCRETE

GIVEN THAT AN ORANGE WAS SELECTED WHAT IS THE PROBABILITY THAT THE BOX WE CHOSE WAS BLUE?

$p(B = r) = 0.4$
$p(B = b) = 0.6$

$p(F = a|B = r) = 0.25$
$p(F = o|B = r) = 0.75$
$p(F = a|B = b) = 0.75$
$p(F = o|B = b) = 0.25$

RANDOM VARIABLE

DISCRETE

GIVEN THAT AN ORANGE WAS SELECTED WHAT IS THE PROBABILITY THAT THE BOX WE CHOSE WAS BLUE?

$$p(B = b|F = o)$$

$p(B = r) = 0.4$
$p(B = b) = 0.6$
$p(F = a|B = r) = 0.25$
$p(F = o|B = r) = 0.75$
$p(F = a|B = b) = 0.75$
$p(F = o|B = b) = 0.25$

RANDOM VARIABLE

DISCRETE

GIVEN THAT AN ORANGE WAS SELECTED WHAT IS THE PROBABILITY THAT THE BOX WE CHOSE WAS BLUE?

$$p(B = b|F = o) = \frac{p(F = o|B = b)p(B = b)}{p(F = o)}$$

BAYES' THEOREM

$p(B = r) = 0.4$
$p(B = b) = 0.6$
$p(F = a|B = r) = 0.25$
$p(F = o|B = r) = 0.75$
$p(F = a|B = b) = 0.75$
$p(F = o|B = b) = 0.25$

RANDOM VARIABLE

DISCRETE

GIVEN THAT AN ORANGE WAS SELECTED WHAT IS THE PROBABILITY THAT THE BOX WE CHOSE WAS BLUE?

$$p(B = b|F = o) = \frac{p(F = o|B = b)p(B = b)}{p(F = o)}$$

BAYES' THEOREM

$$= \frac{0.25 \times 0.6}{0.45} = 0.33$$

$p(B=r)=0.4$
$p(B=b)=0.6$
$p(F=a|B=r)=0.25$
$p(F=o|B=r)=0.75$
$p(F=a|B=b)=0.75$
$p(F=o|B=b)=0.25$

RANDOM VARIABLE

DISCRETE

GIVEN THAT AN ORANGE WAS SELECTED WHAT IS THE PROBABILITY THAT THE BOX WE CHOSE WAS BLUE?

$$p(B=b|F=o) = \frac{p(F=o|B=b)p(B=b)}{p(F=o)}$$

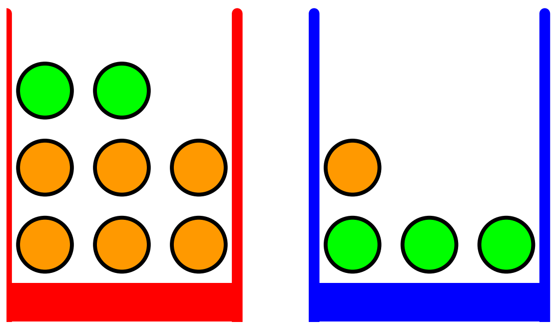
BAYES' THEOREM

$$= \frac{0.25 \times 0.6}{0.45} = 0.33$$

$$p(B=r|F=o) = 1 - p(B=b|F=o) = 0.66$$

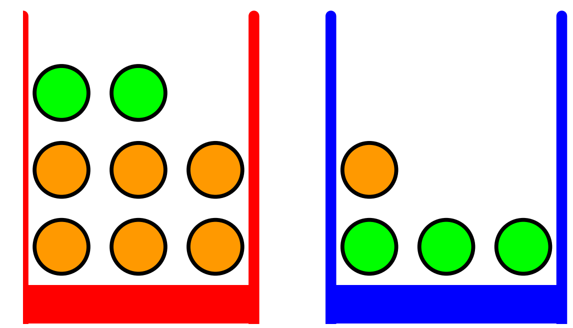
BAYES' THEOREM

PRIOR TO POSTERIOR



BAYES' THEOREM

PRIOR TO POSTERIOR



Likelihood

Prior Probability

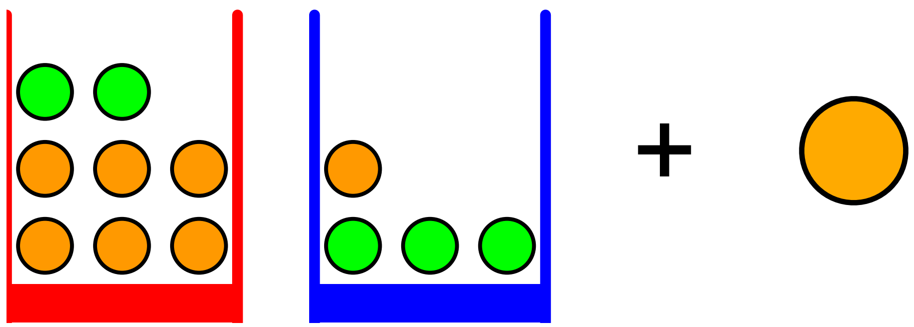
Posterior Probability

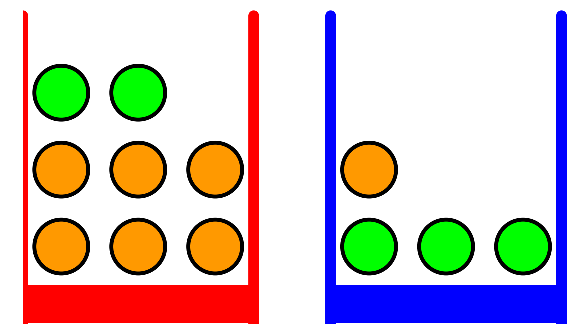
$$p(X|Y) = \frac{p(Y|X)p(X)}{p(Y)}$$

Evidence

BAYES' THEOREM

PRIOR TO POSTERIOR





Likelihood

Prior Probability

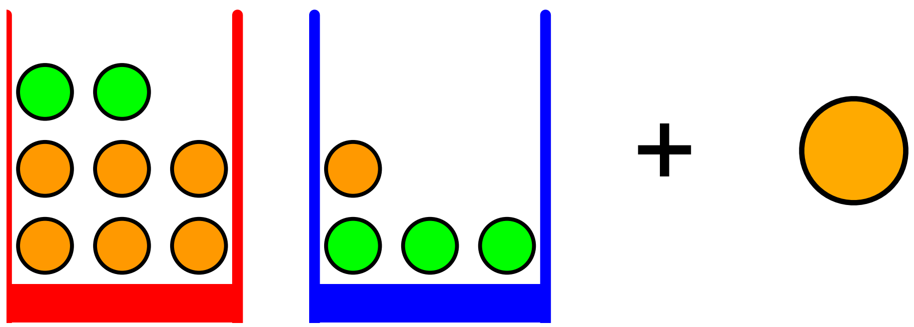
Posterior Probability

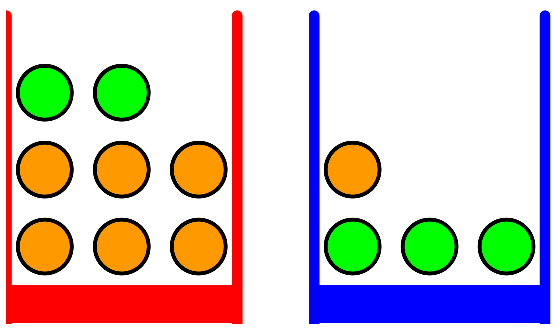
$$p(X|Y) = \frac{p(Y|X)p(X)}{p(Y)}$$

Evidence

BAYES' THEOREM

PRIOR TO POSTERIOR





Likelihood

Prior Probability

Posterior Probability

$$p(X|Y) = \frac{p(Y|X)p(X)}{p(Y)}$$

Evidence

	prior	posterior
red	0.4	0.66
blue	0.6	0.33

INDEPENDENCE

$$p(X, Y) = p(X)p(Y)$$

$$p(Y|X) = p(Y)$$

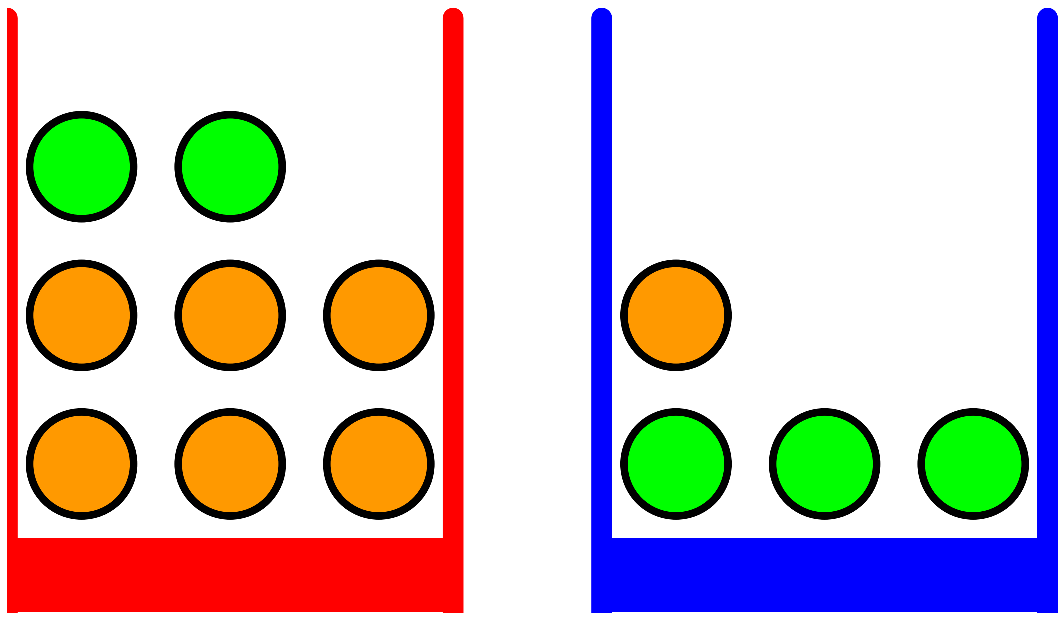
X and *Y* are **independent**

INDEPENDENCE

$$p(X, Y) = p(X)p(Y)$$

$$p(Y|X) = p(Y)$$

X and *Y* are **independent**

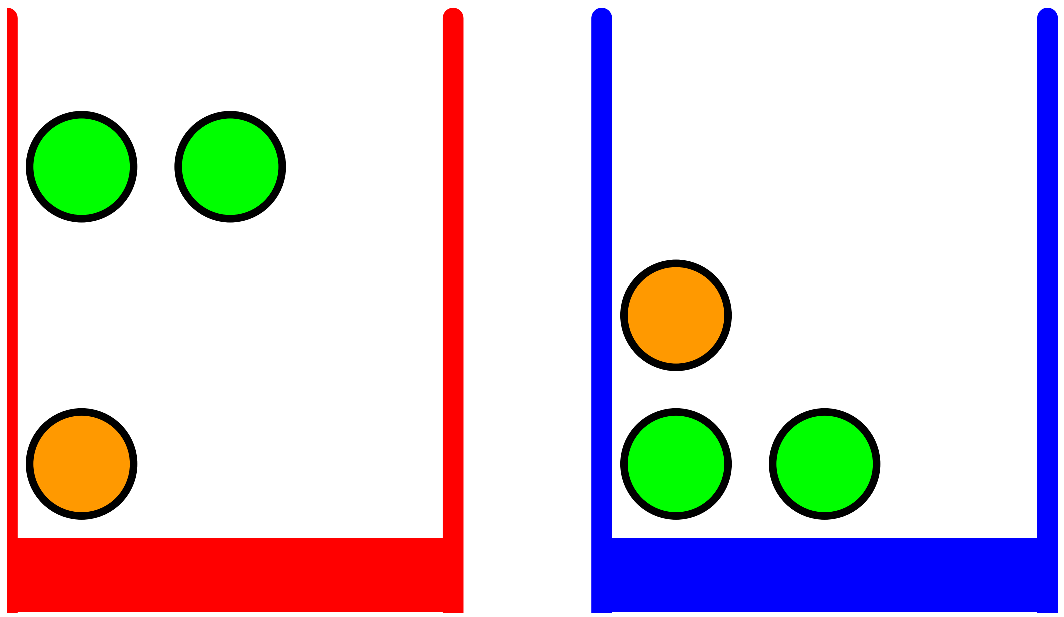


INDEPENDENCE

$$p(X,Y) = p(X)p(Y)$$

$$p(Y|X) = p(Y)$$

X and *Y* are **independent**

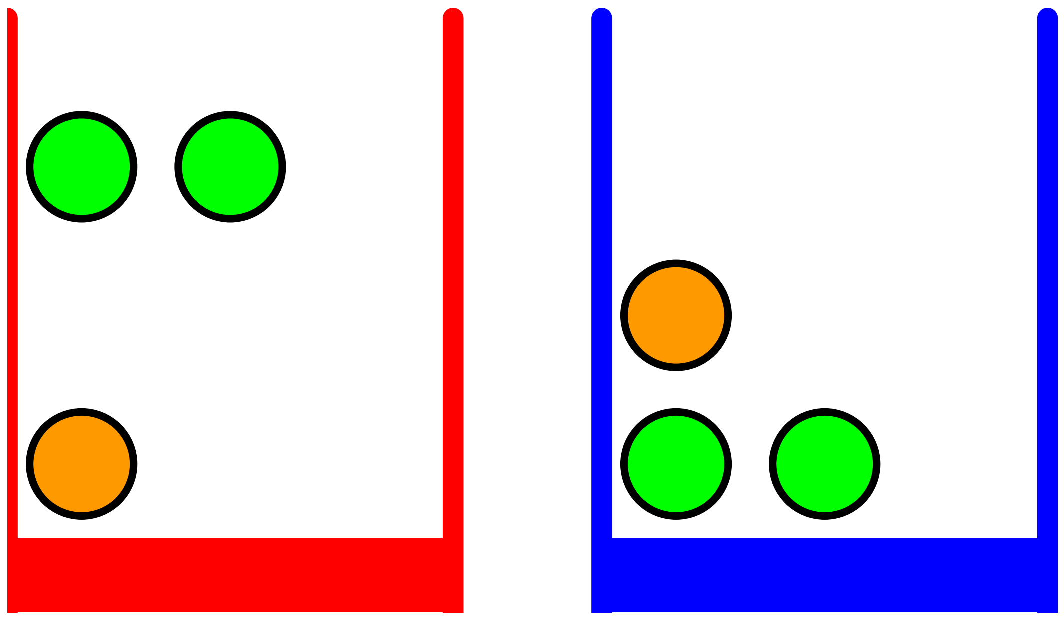


INDEPENDENCE

$$p(X, Y) = p(X)p(Y)$$

$$p(Y|X) = p(Y)$$

X and *Y* are **independent**



$$p(F|B) = p(F)$$

CONDITIONAL INDEPENDENCE

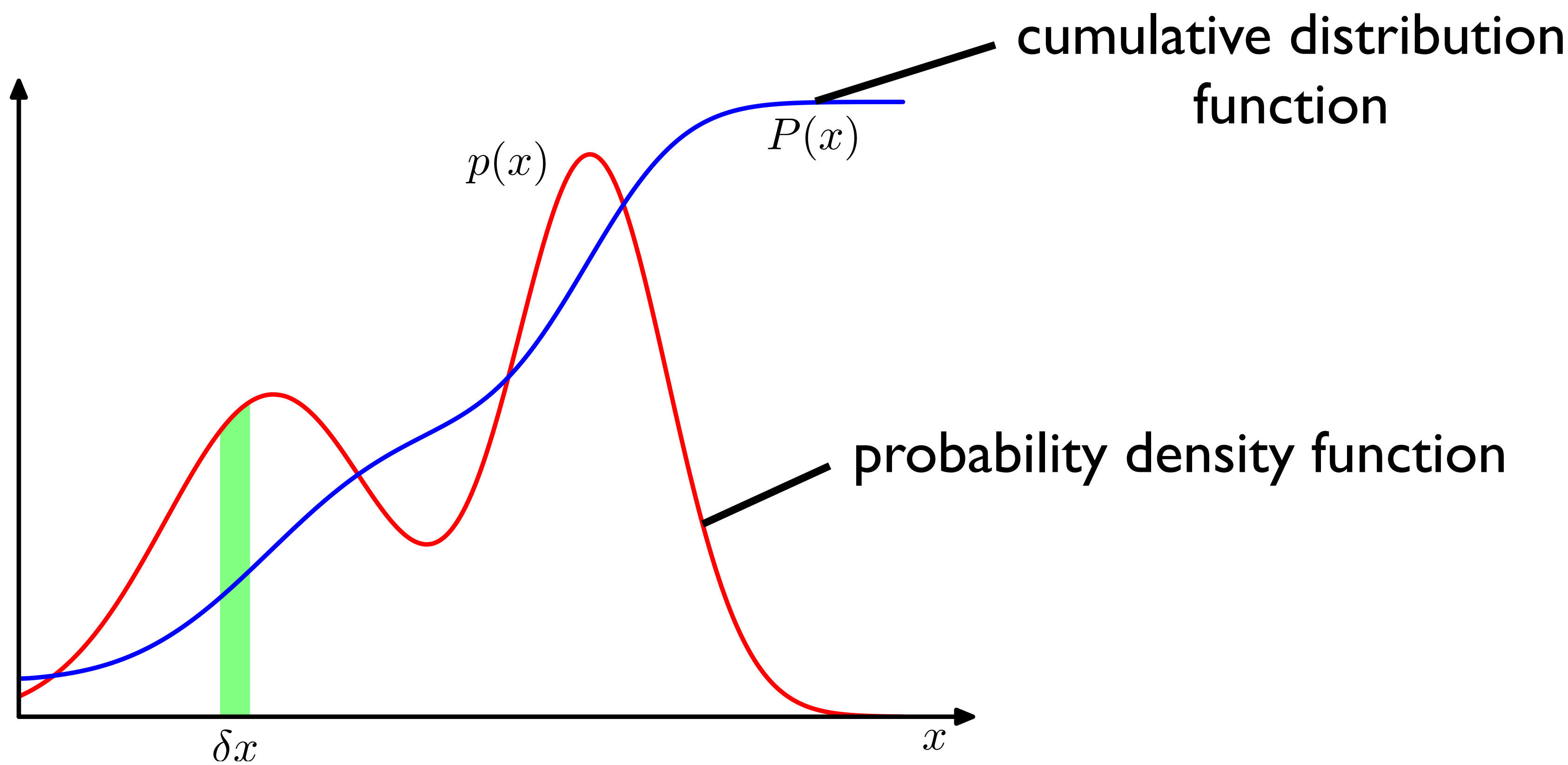
$$P(X|Y,Z) = P(X|Z)$$

$$P(Y|X,Z) = P(Y|Z)$$

X and *Y* are **conditionally independent** given *Z*

PROBABILITY

CONTINUOUS



$$p(x) \geq 0$$

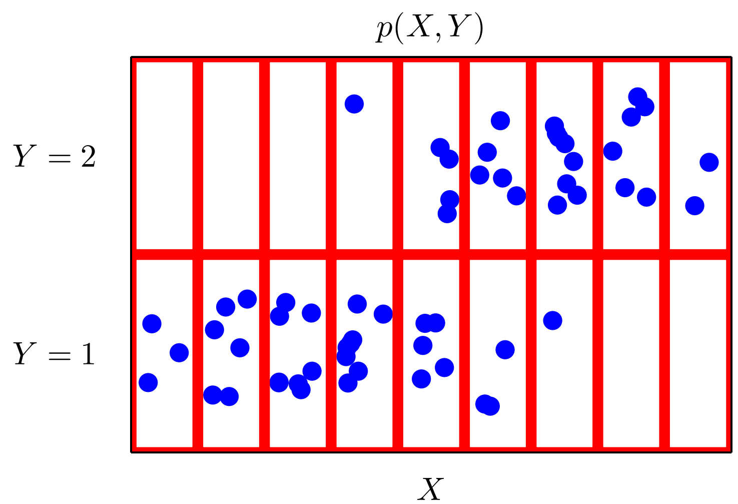
$$\int_{-\infty}^{\infty} p(x)dx = 1$$

$$p(x \in (a,b)) = \int_a^b p(x)dx$$

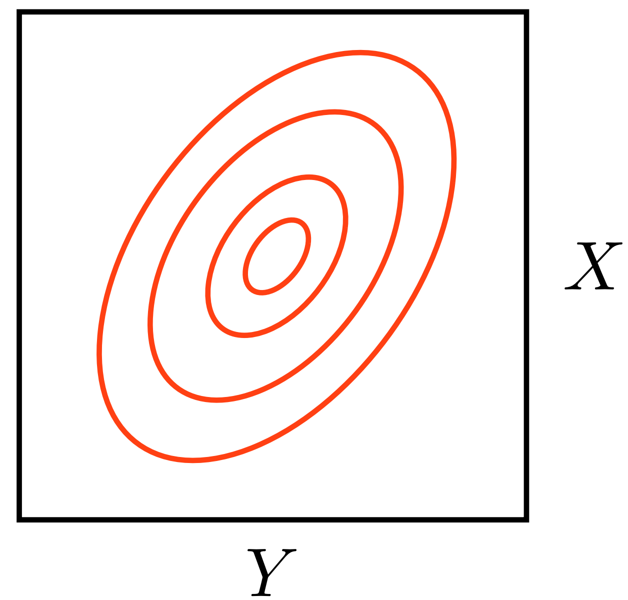
JOINT PROBABILITY

CONTINUOUS

HEIGHT vs WEIGHT IN 15-381?



DISCRETE



CONTINUOUS

JOINT PROBABILITY

CONTINUOUS

height weight age

vital stats

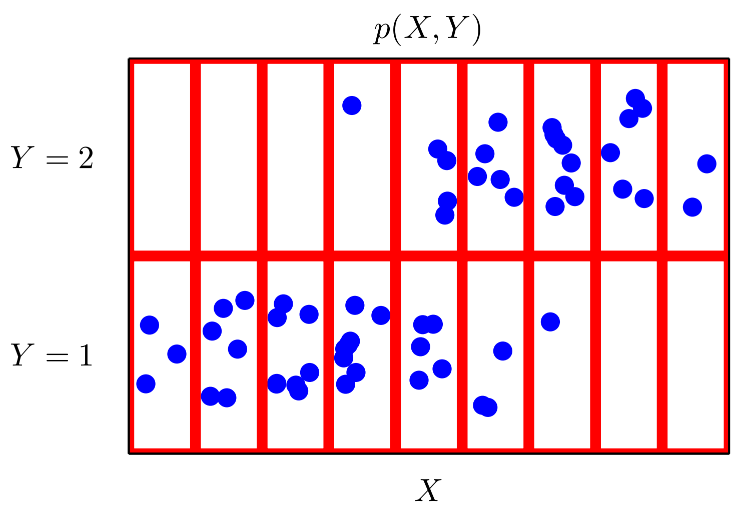
$$\mathbf{x} = [x_1, x_2, \cdots, x_n]$$

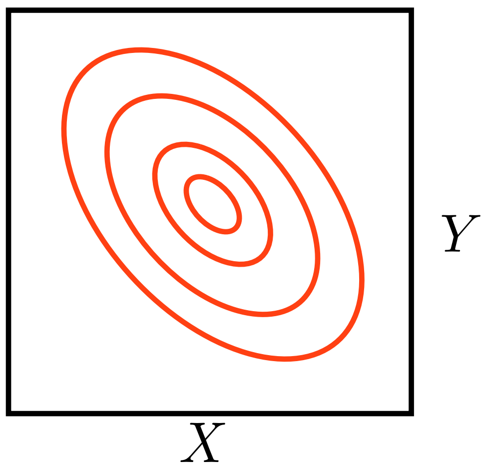
$$p(\mathbf{x}) \geq 0$$

$$\int p(\mathbf{x}) = 1$$

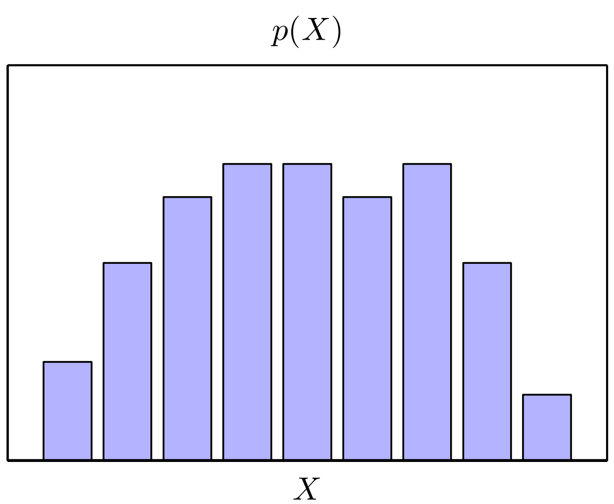
SUM RULE

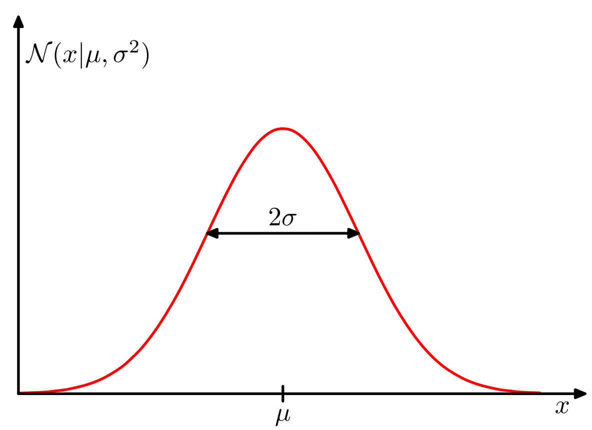
CONTINUOUS





CONTINUOUS

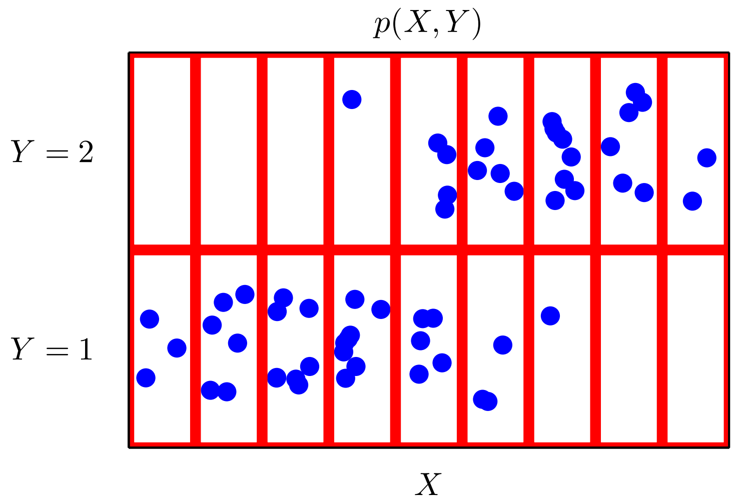


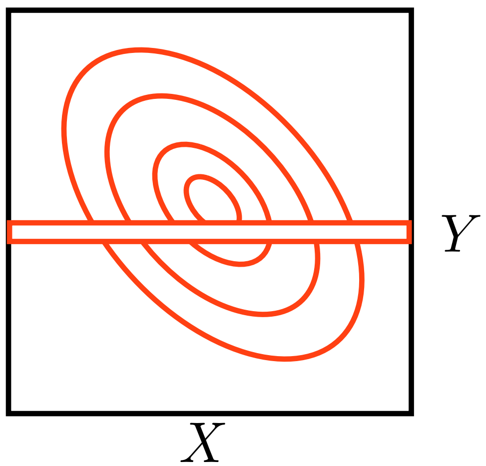


$$p(X) = \sum_{Y} p(X,Y)$$

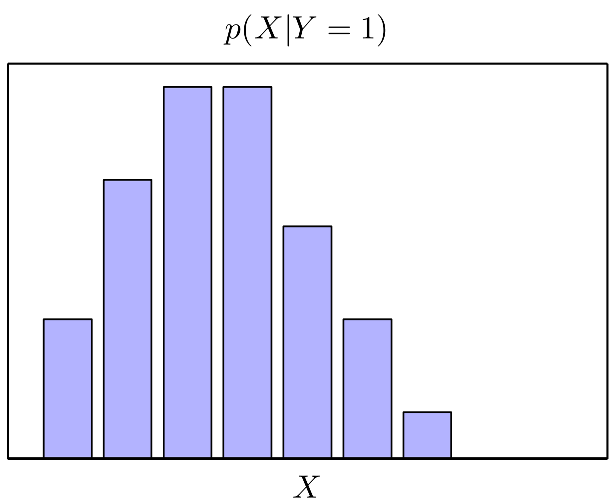
$$p(x) = \int_{y} p(x,y)\,dy$$

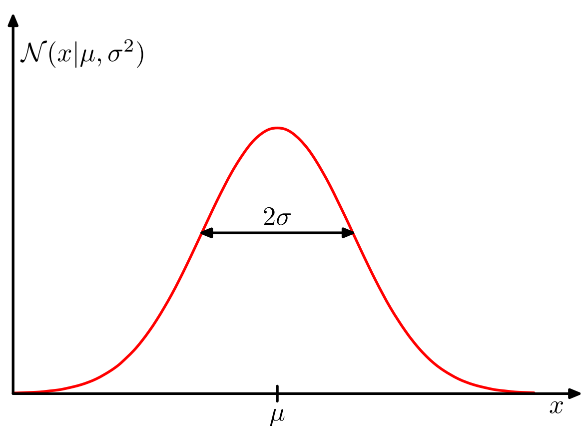
CONDITIONAL DISTRIBUTION





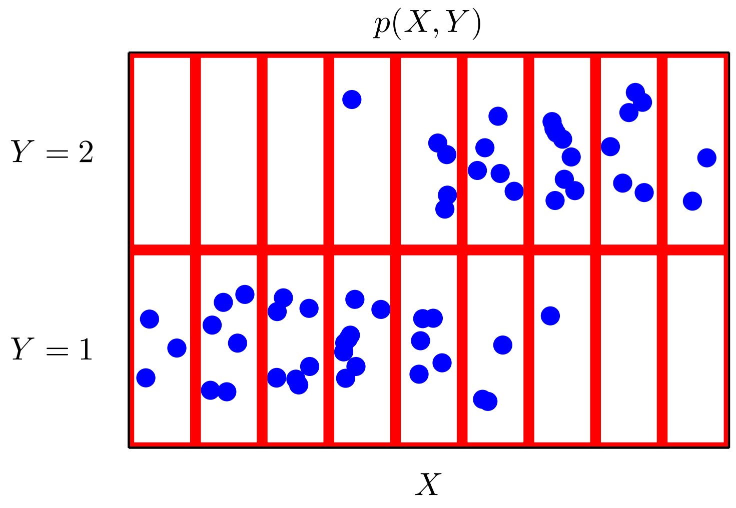
CONTINUOUS

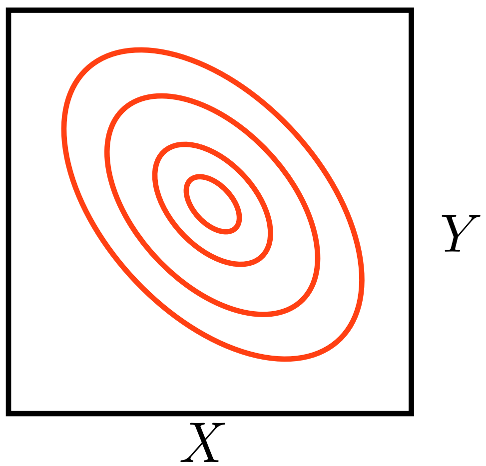




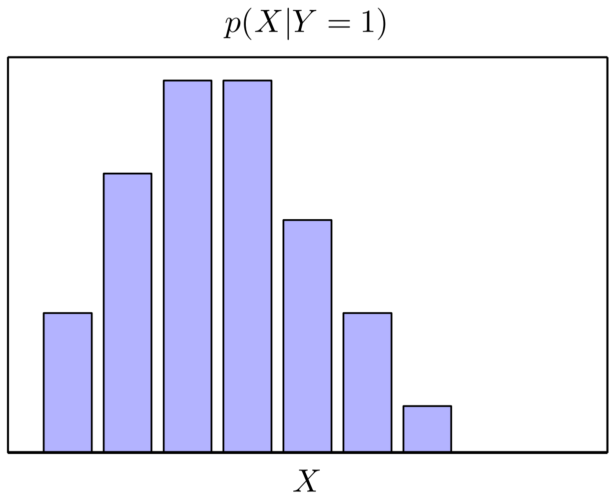
PRODUCT RULE

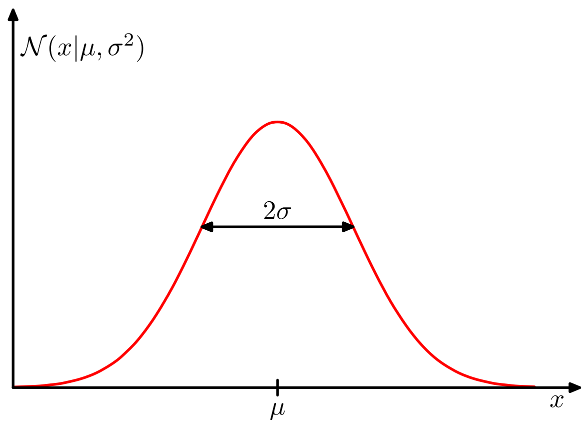
CONTINUOUS





CONTINUOUS





$$p(X,Y) = p(Y|X)p(X)$$

$$p(x,y) = p(y|x)p(x)$$

BAYES' THEOREM

CONTINUOUS

$$p(x|y) = \frac{p(y|x)p(x)}{p(y)}$$

WHAT IS PROBABILITY?

Frequentists	**Bayesians**
Frequency of Event	Degree of Belief

"Probability theory could be regarded as an extension of Boolean logic to situations involving uncertainty."

--- E.T. Jaynes