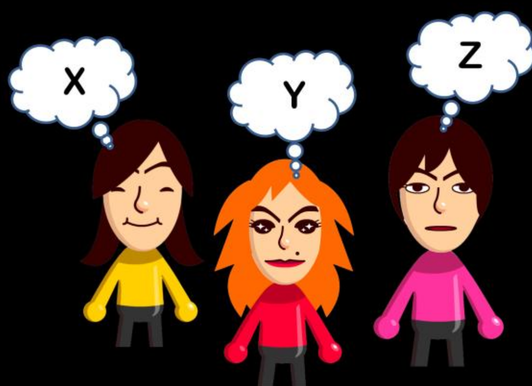
Auctions



How much is 10 dollars worth (we should all know)

Luis is auctioning off a 10 dollar bill. The winner pays his or her bid and gets the 10 dollar bill. The runner up pays his or her bid and gets nothing http://www.swoopo.com/new.html?cm_sp_o=ZBFw%20VzTwCjC8zfgt%20ebpwBCjCPzEEwy

How swoopo works. They make money as losers still have to pay for their bids..

This is called an all-pay auction, as everyone pays. The auctioneer makes bank, but all bidders but one get hosed.

Types of Auctions

Ascending Bid or English Auctions

Descending Bid or Dutch Auctions

First Price Sealed Bid Auctions

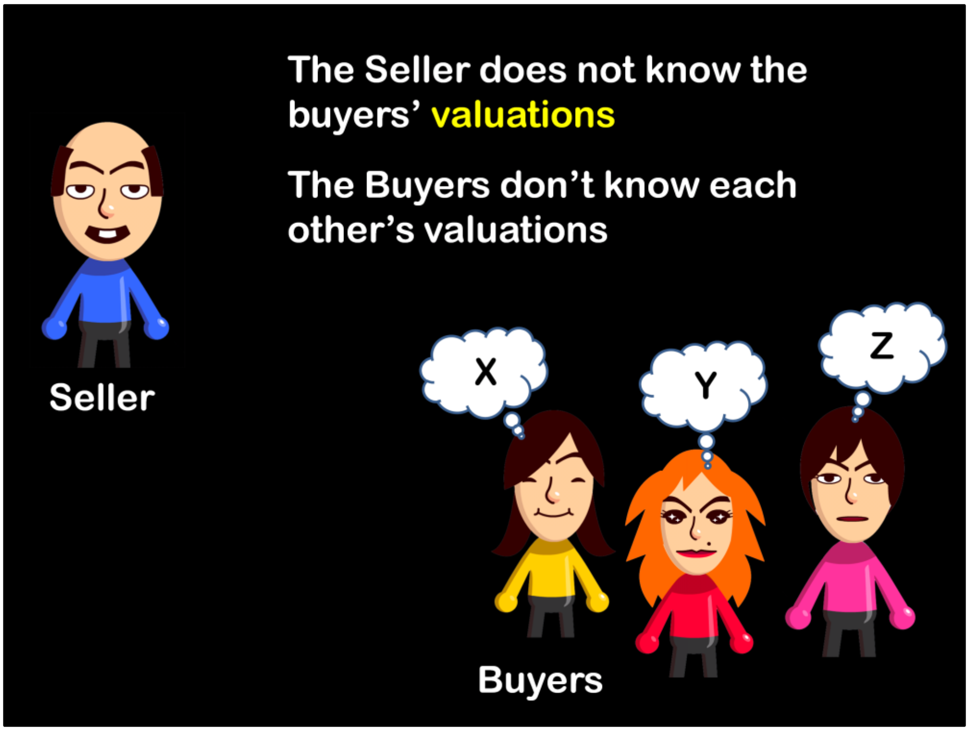
Second Price Sealed Bid or Vickrey Auctions

English (Ascending): what you usually see in live auctions (everyone knows the current bid and can bid higher)

Dutch (Descending): auctioneer starts at a really high amount and descends- the first person to agree buys.

First price sealed bid: Send your bid, they pick the highest one. House buying is a First Price Sealed Bid. You make an offer for the house, and the seller gives it to the person offering the highest price.

Second price sealed bid: Send your bid, they pick the highest one, but the winner pays the second highest bid. (more honesty inducing, as shown later in the slides)



Valuation is how much someone values an object (not necessarily how much they will bid, though)

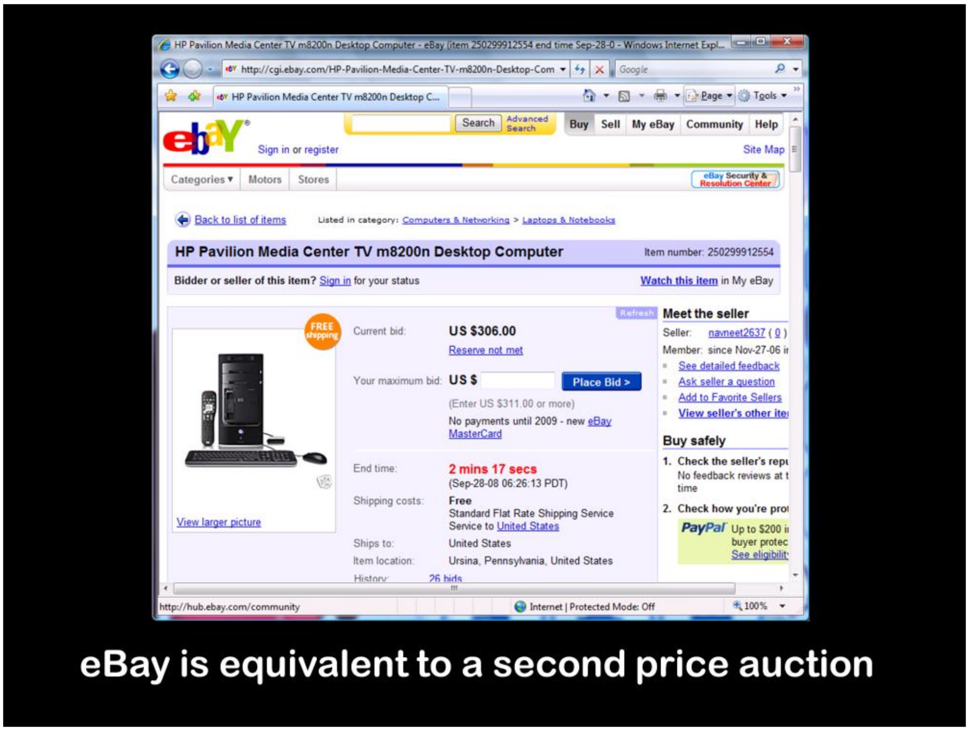
Descending bid and first price sealed bid are essentially equivalent from the buyer's perspective

In both cases if someone bids higher than you then you lose, and you don't know what everyone else will bid.

In an ascending bid auction, each buyer will want to stay in the auction until the precise moment when the price reaches his value.

As long as the price is less than your validation, you’re still making a profit if you win. However, as long as other people are bidding you still have to bid to get it.


Second Price Auctions
The highest bidder wins, but pays the what the second highest bidder bid
Seller
3
5
23
10
Buyers



You only need to pay 1 more cent than the second highest bidder.

Bidding your true value is a dominant strategy in a second price sealed bid auction

v_i = bidder i's value for the object

b_i = bidder i's bid for the object

A bidder's **strategies** are bids as functions of their values

The payoff to bidder i with value v_i and bid b_i is:

$$\begin{cases} v_i - \max\limits_{j \neq i} b_j & \text{if } b_i > \max\limits_{j \neq i} b_j \\ 0 & \text{otherwise} \end{cases}$$

Reminder: Dominant strategy means there is no better strategy.

v_i = bidder i's value for the object

b_i = bidder i's bid for the object

$$\text{Payoff} = \begin{cases} v_i - \max\limits_{j \neq i} b_j & \text{if } b_i > \max\limits_{j \neq i} b_j \\ 0 & \text{otherwise} \end{cases}$$

Theorem: Bidding $b_i = v_i$ is a dominant strategy

If $b_i > v_i$ bidder i could get object and pay more than what she values it for (and thus go negative)

If $b_i < v_i$ bidder i could fail to obtain the object; obtaining the object can get her positive payoff

v_i = bidder i's value for the object

b_i = bic … e object

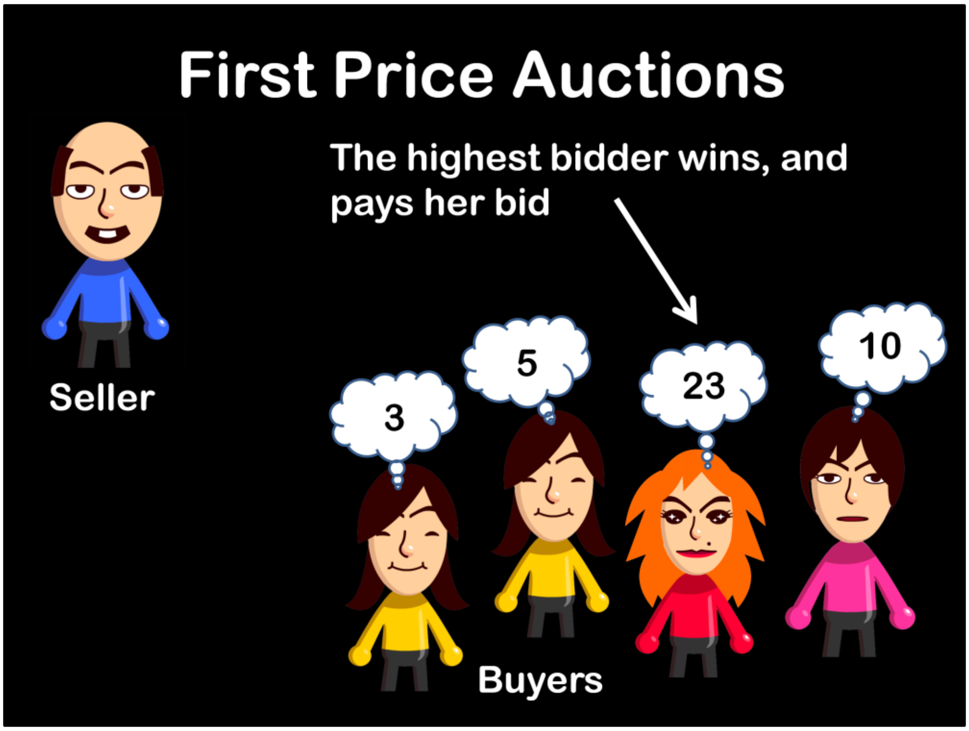
… max b_j … i

In a second price auction, your bid does not affect how much you pay; it just affects whether you get the object or not

The … y

If $b_i > v_i$ … more than what … egative)

If $b_i < v_i$ bidder i … n the object; obtaining the object can get him positive payoff


First Price Auctions
The highest bidder wins, and pays her bid
Seller
3
5
23
10
Buyers

Bidding your true value is NOT a dominant strategy in a first price sealed bid auction

Since your bid also affects what you pay, you will tend to underbid

First Price Auctions

Suppose all bids v_i are uniformly distributed between [0,1] and there are N other bidders

Suppose all bidders bid $s(v_i)$ where s( ) is a strictly increasing function

What is the probability that bidder who bids $s(v_i)=b_i$ will win? $(v_i)^{N-1}$

What is the net payoff for value v_i and bid $s(v_i)=b_i$ if they win? (v_i-b_i)

Expected payoff to bidder i with value v_i and bid $s(v_i)=b_i$: $(v_i)^{N-1}(v_i-s(v_i))$

Expected payoff to bidder i with value v_i and bid $s(v_i)=b_i$: $(v_i)^{N-1}(v_i-s(v_i))$

Bidders could pretend their value is some u_i instead of v_i

For s( ) to be an equilibrium, this deviation must make i worse off:

$$(v_i)^{N-1}(v_i-s(v_i)) \geq (u_i)^{N-1}(v_i-s(u_i))$$

It can be shown that s(v) = v(N-1)/N is an equilibrium strategy

E.g., For N=2:

$v_i(v_i-v_i/2) \geq u_i(v_i-u_i/2)$ is true for all v_i, u_i

Seller Revenue

Which is better for the Seller: first or second price auctions?

In first price auctions the bidders pay less than their true valuation

In second price auctions they only pay the second largest valuation

Assume N valuations uniformly distributed in [0,1]

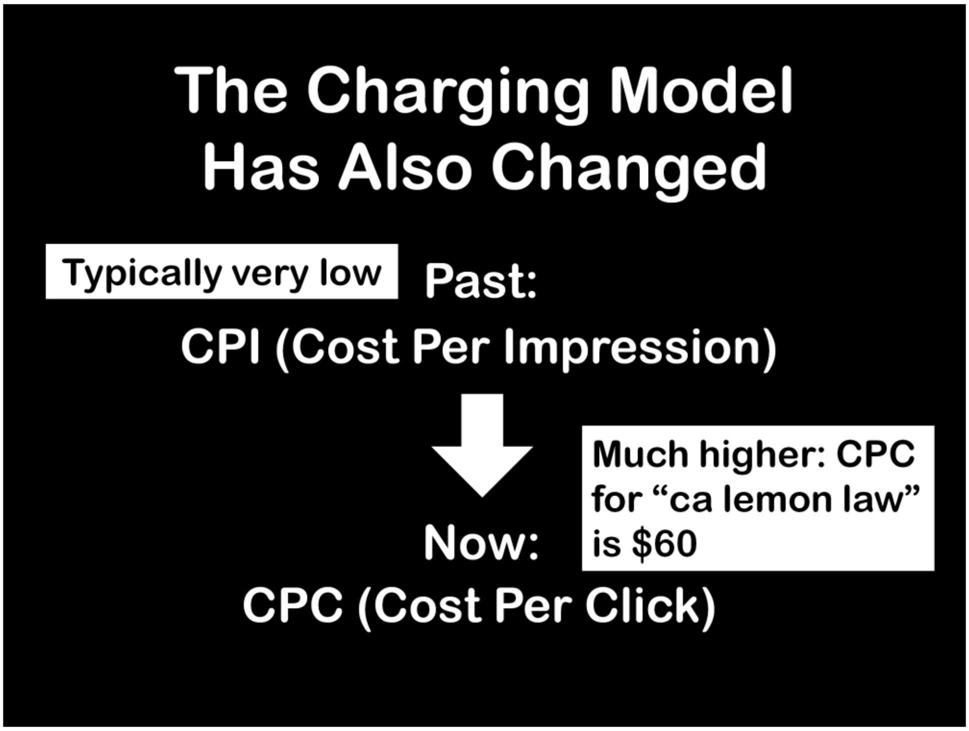
In expectation, the highest value is N/(N+1) and the second highest value is (N-1)/(N+1)

So, seller revenue is the same in both cases

Web 1.0 Advertising

Advertisers paid per "impression" and ads were not targeted

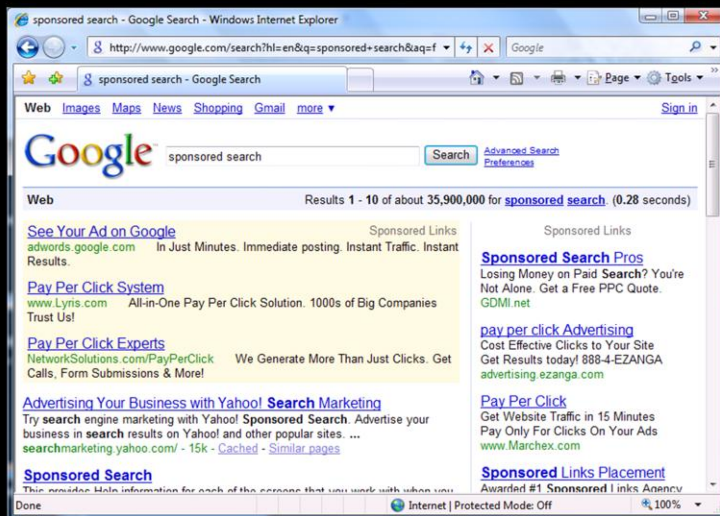
However, Overture first used a first-price auction, so people began to underbid



The search engine folks’ commodity is the ad impressions, so they would like to charge on cost-per-impression– as in, they want to get paid every time they show an ad. On the other hand, the ad-buyer wants to pay based on when someone buys their stuff because of an ad. Using “cost per click” is a compromise-- it’s also information available to both parties (search engine knows clicks, and the ad buyer can use backtracks on their page to make sure the search engine isn’t lying)

This also makes it so that Google has a high incentive to serve relevant ads (so they get more clicks), and to some extent, for ad buyers to make sure that people won’t click on their ad and decide not to buy– that users are in the right “frame of mind”.

How should Google decide how much to charge?



Here’s Exactly How Much Google Charges:

http://services.google.com/awp/en_us/breeze/3004832/index.html

So there’s a second-price auction. Google charges based on what position you are on the page and how relevant you are, based on a “quality score” that nobody really knows except Google.

There’s enough “hidden” things in Google’s ad system that it’s hard for people to game. Also, note that since Google only gets paid for the clicks, there’s some incentive for them to make sure there are relevant ads on search pages, instead of just selling their impression willy-nilly.

Generalized Second Price Auction

Each advertiser j announces a bid b_j

Slot i is assigned to the i^{th} highest bidder at a price per click equal to the $(i+1)^{st}$ highest bidder's bid

Truthful Bidding is Not Necessarily an Equilibrium!

(and therefore also not a dominant strategy)

	True Revenue Per Click		Click-Through Rate
Bidder A	7	Slot 1	10
Bidder B	6	Slot 2	4
Bidder C	1	Slot 3	0

If each bidder bids their true valuation, then A gets Slot 1 and their payoff is 7*10-6*10=10

Truthful Bidding is Not Necessarily an Equilibrium!

(and therefore also not a dominant strategy)

	True Revenue Per Click		Click-Through Rate
Bidder A	7	Slot 1	10
Bidder B	6	Slot 2	4
Bidder C	1	Slot 3	0

If A were to bid 4, then its payoff would be 7*4-1*4 = 24, which is higher than 10

Google just tries to make it hard to game, by using quality score, etc.

True Revenue Per Click		Click-Through Rate	
Bidder A	7	Slot 1	10
Bidder B	6	Slot 2	4
Bidder C	1	Slot 3	0

Bidder A bids 5, B bids 4 and C bids 1 is an equilibrium

Bidder A bids 3, B bids 5 and C bids 1 is also an equilibrium (and it's not socially optimal, since it assigns B the highest Slot g2g

ttyl