Reinforcement Learning

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see “Machine Learning” – Tom Mitchell, chapter 13 on RL

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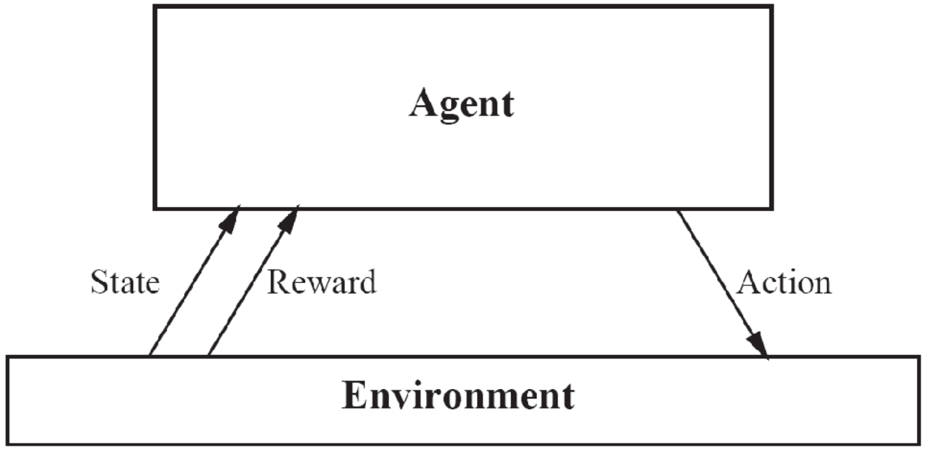
Different Aspects of “Machine Learning”

- Supervised learning
  - Classification - concept learning
  - Learning from labeled data
  - Function approximation
- Unsupervised learning
  - Data is not labeled
  - Data needs to be *grouped*, *clustered*
  - We need distance metric
- Control and action model learning
  - Learning to select actions efficiently
  - Feedback: goal achievement, failure, *reward*
  - Control learning, reinforcement learning

Goal Achievement - Rewards

- "Reward" today versus future (promised) reward
- Future rewards not worth as much as current.
- \$100K + \$100K + \$100K + ...
  INFINITE sum
- Assume reality ...: discount factor , say γ.
- \$100K + γ \$100K + γ^2 \$100K + ...
  CONVERGES.

Reinforcement Learning Problem



$$s_0 \xrightarrow[r_0]{a_0} s_1 \xrightarrow[r_1]{a_1} s_2 \xrightarrow[r_2]{a_2} \dots$$

Goal: Learn to choose actions that maximize

$r_0 + \gamma r_1 + \gamma^2 r_2 + \dots$, where $0 \leq \gamma < 1$

Learning Conditions

- Assume world can be modeled as a Markov Decision Process, with rewards as a function of state and action.
- *Markov assumption:*
  New states and rewards are a function only of the current state and action, i.e.,
  - $s_{t+1} = \delta(s_t, a_t)$
  - $r_t = r(s_t, a_t)$
- *Unknown and uncertain environment:*
  Functions δ and r may be nondeterministic and are not necessarily known to learner.

Control Learning Task

- Execute actions in world,
- Observe state of world,
- Learn action policy $\pi : S \rightarrow A$
- Maximize expected reward

$$E[r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \ldots]$$

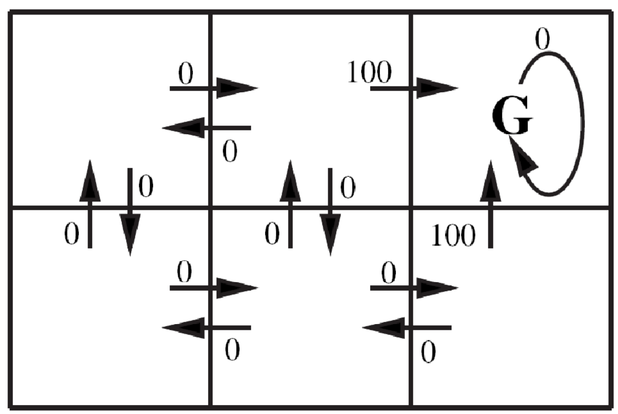
from any starting state in S.

– $0 \leq \gamma < 1$, discount factor for future rewards

Statement of Learning Problem

- We have a target function to learn $\pi : S \rightarrow A$
- We have **no** training examples of the form $\langle s, a \rangle$
- We have training examples of the form $\langle \langle s, a \rangle, r \rangle$

  (rewards can be *any* real number)



immediate reward values $r(s,a)$

Policies

- There are *many possible* policies, of course not necessarily *optimal*, i.e., with maximum expected reward
- There can be also *several OPTIMAL* policies.

Value Function

- For each possible policy π, define an *evaluation function over states* (deterministic world)

$$V^{\pi}(s) \equiv r_t + \gamma r_{t+1} + \gamma^2 r_{t+1} + \ldots$$
$$\equiv \sum_{i=0}^{\infty} \gamma^i r_{t+i}$$

where r_t, r_{t+1},... are generated by following policy π starting at state s

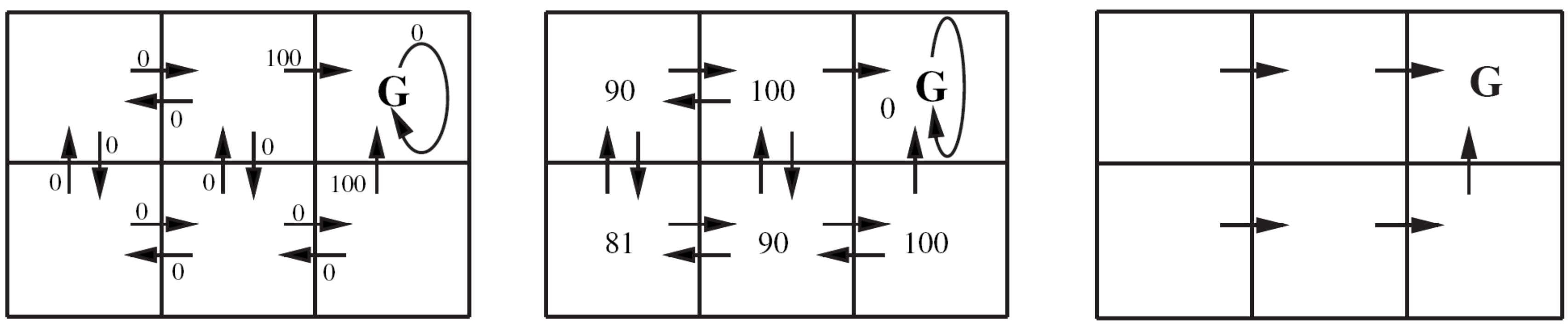
- Learning task: Learn OPTIMAL policy

$$\boxed{\pi^* \equiv \mathrm{argmax}_{\pi} V^{\pi}(s), (\forall s)}$$

Learn Value Function

- Learn the evaluation function $V^{\pi*}$ - V^*.
- Select the optimal action from any state s, i.e., have an optimal policy, by using V^* with one step lookahead:

$$\pi^*(s) = \arg\max_a \left[r(s,a) + \gamma V^*(\delta(s,a)) \right]$$



rewards $\qquad$ $V^*(s)$ values $\qquad$ **ONE** optimal policy

Optimal Value to Optimal Policy

$$\boxed{\pi^*(s) = \operatorname{argmax}_a[r(s,a) + \gamma V^*(\delta(s,a))]}$$

A problem:

- This works well if agent knows $\delta : S \times A \rightarrow S$, and $r : S \times A \rightarrow \Re$
- When it doesn't, it can't choose actions this way

Q Function

- Define new function very similar to V^*

$$Q(s,a) \equiv r(s,a) + \gamma V^*(\delta(s,a))$$

Learn Q function – Q-learning

- If agent learns Q, it can choose optimal action even without knowing δ or r.

$$\pi^*(s) = \arg\max_a \left[r(s,a) + \gamma V^*(\delta(s,a)) \right]$$

$$\pi^*(s) = \arg\max_a Q(s,a)$$

Q-Learning

Note that Q and V^* are closely related:

$$V^*(s) = \max_{a'} Q(s, a')$$

Which allows us to write Q recursively as

$$\begin{aligned} Q(s_t, a_t) &= r(s_t, a_t) + \gamma V^*(\delta(s_t, a_t)) \\ &= r(s_t, a_t) + \gamma \max_{a'} Q(s_{t+1}, a') \end{aligned}$$

Q-learning actively generates examples.
It "processes" examples by updating its Q values.
While learning, Q values are approximations.

Training Rule to Learn Q

Let $\hat{Q}$ denote current approximation to Q.

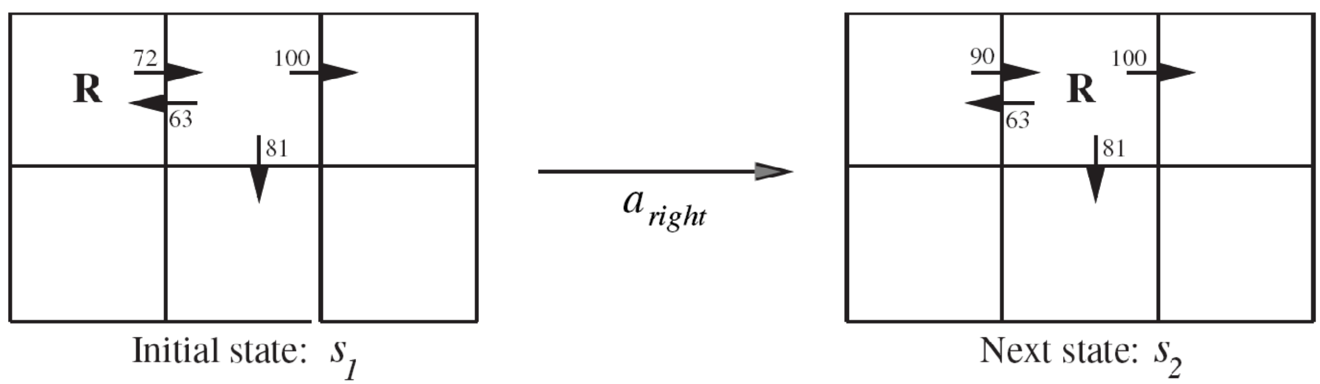
Then Q-learning uses the following **training rule**:

$$\boxed{\hat{Q}(s,a) \leftarrow r + \gamma \max_{a'} \hat{Q}(s',a')}$$

where s' is the state resulting from applying action a in state s,

and r is the reward that is returned.

Example - Updating $\hat{Q}$



$$
\begin{aligned}
\hat{Q}(s_1, a_{right}) &\leftarrow r + \gamma \max_{a'} \hat{Q}(s_2, a') \\
&\leftarrow 0 + 0.9 \max\{63, 81, 100\} \\
&\leftarrow 90
\end{aligned}
$$

Q Learning for Deterministic Worlds

For each s, a initialize table entry $\hat{Q}(s,a) \leftarrow 0$

Observe current state s

Do forever:

- Select an action a and execute it
- Receive immediate reward r
- Observe the new state s'
- Update the table entry for $\hat{Q}(s,a)$ as follows:

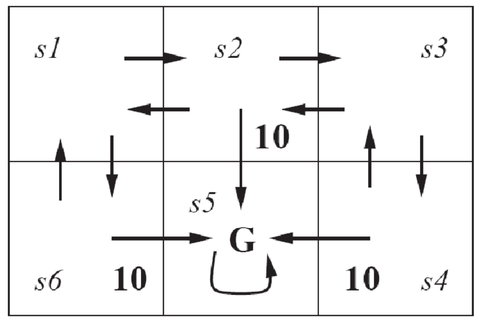
$$\hat{Q}(s,a) \leftarrow r + \gamma \max_{a'} \hat{Q}(s',a')$$

- $s \leftarrow s'$

Q Learning Iterations

Starts at bottom left corner – moves clockwise around perimeter;

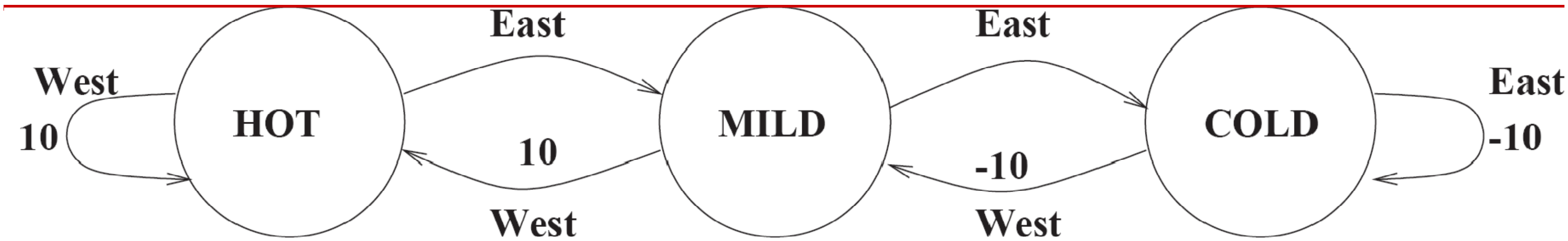
Initially $Q(s,a)$ = 0; $\gamma = 0.8$



$$\hat{Q}(s,a) \leftarrow r + \gamma \max_{a'} \hat{Q}(s',a')$$

Q(s1,E)	Q(s2,E)	Q(s3,S)	Q(s4,W)
0	0	0	r + γ max{Q(s5,loop)}= 10 + 0.8 . 0 = 10
0	0	r + γ max{Q(s4,W),Q(s4,N)}= 0 + 0.8 max{10,0}= 8	10
0	r + γ max{Q(s3,W),Q(s3,S)}= 0 + 0.8 max{0,8}= 6.4	8	10

Problem - Deterministic

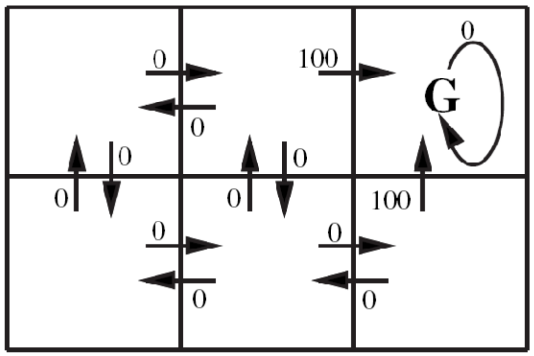


How many possible **policies** are there in this 3-state, 2-action deterministic world?

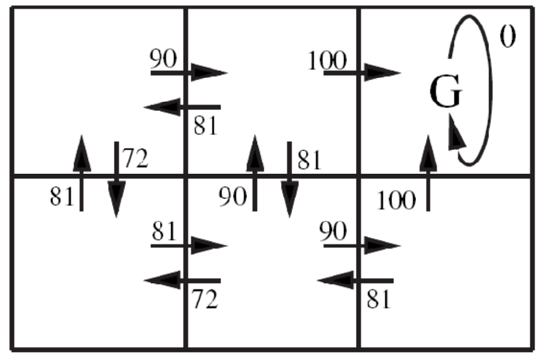
A robot starts in the state Mild. It moves for 4 steps choosing actions **West, East, East, West**. The initial values of its Q-table are 0 and the discount factor is $\gamma = 0.5$

	Initial State: MILD		Action: West New State: HOT		Action: East New State: MILD		Action: East New State: COLD		Action: West New State: MILD	
	East	West	East	West	East	West	East	West	East	West
HOT	0	0	0	0	**5**	0	5	0	5	0
MILD	0	0	0	**10**	0	10	**0**	10	0	10
COLD	0	0	0	0	0	0	0	0	0	**-5**

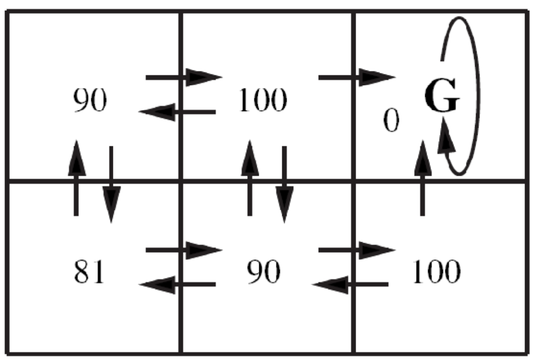
Another Deterministic Example



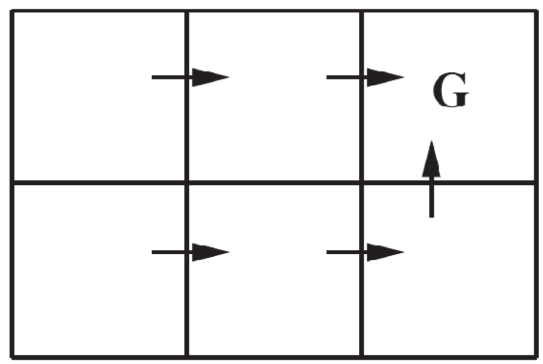
$r(s, a)$ values



$Q(s, a)$ values



$V^*(s)$ values



One optimal policy

Nondeterministic Case

What if reward and next state are non-deterministic?

We redefine V, Q by taking expected values

$$V^{\pi}(s) \equiv E\left[r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \ldots\right]$$
$$\equiv E\left[\sum_{i=0}^{\infty} \gamma^i r_{t+i}\right]$$

$$Q(s,a) \equiv E\left[r(s,a) + \gamma V^*(\delta(s,a))\right]$$

Nondeterministic Case

Q learning generalizes to nondeterministic worlds

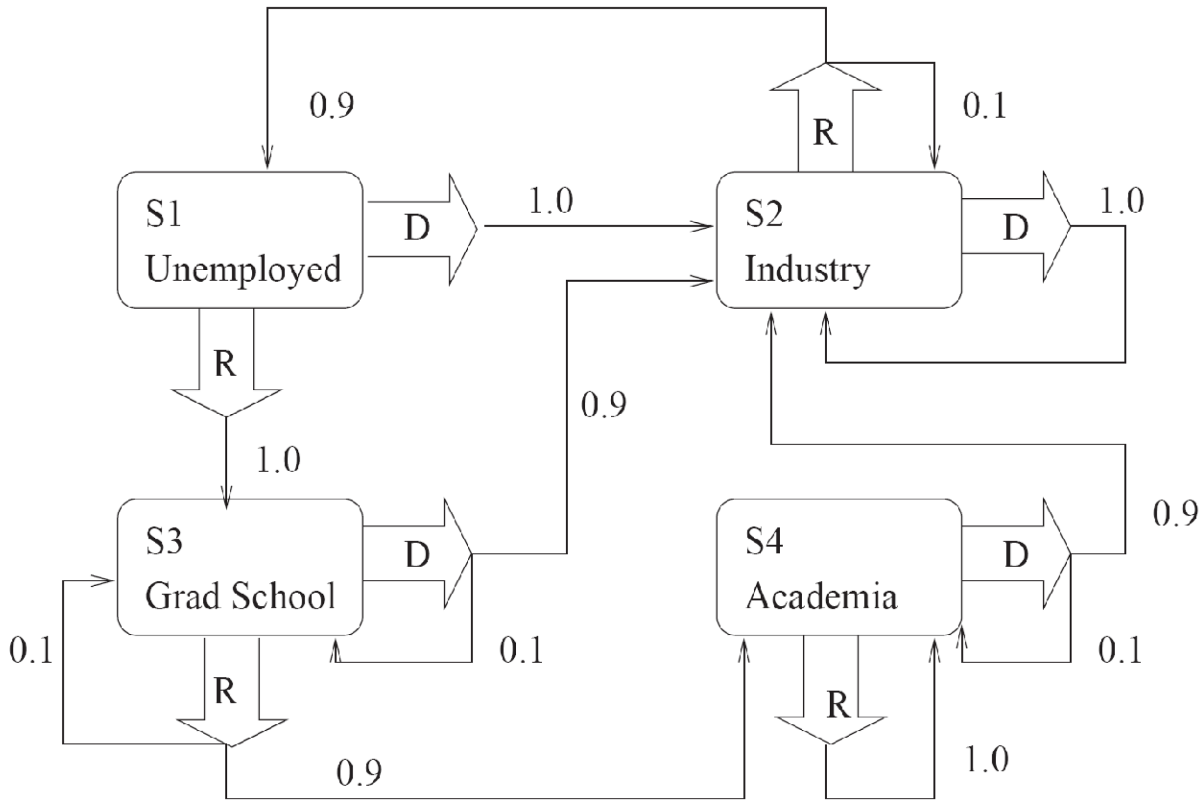
Alter training rule to

$$\hat{Q}_n(s,a) \leftarrow (1-\alpha_n)\hat{Q}_{n-1}(s,a) + \alpha_\mathrm{n}\left[r + \gamma \max_{a'} \hat{Q}_{n-1}(s',a')\right],$$

where $\alpha_n = \frac{1}{1+visits_n(s,a)}$, and $s' = \delta(s,a)$.

$\hat{Q}$ still converges to Q^* (Watkins and Dayan, 1992)

Nondeterministic Example



REWARD

S1	S2	S3	S4
0	100	0	10

Nondeterministic Example

$\pi^*(s) = D$, for any s= S1, S2, S3, and S4, $\gamma = 0.9$.

```
V*(S2) = r(S2,D) + 0.9 (1.0 V*(S2))
V*(S2) = 100 + 0.9 V*(S2)
V*(S2) = 1000.

V*(S1) = r(S1,D) + 0.9 (1.0 V*(S2))
V*(S1) = 0 + 0.9 x 1000
V*(S1) = 900.

V*(S3) = r(S3,D) + 0.9 (0.9 V*(S2) + 0.1 V*(S3))
V*(S3) = 0 + 0.9 (0.9 x 1000 + 0.1 V*(S3))
V*(S3) = 81000/91.

V*(S4) = r(S4,D) + 0.9 (0.9 V*(S2) + 0.1 V*(S4))
V*(S4) = 40 + 0.9 (0.9 x 1000 + 0.1 V*(S4))
V*(S4) = 85000/91.
```

Nondeterministic Example

What is the Q-value, Q(S2,R)?

```
Q(S2,R) = r(S2,R) + 0.9 (0.9 V*(S1) + 0.1 V*(S2))
Q(S2,R) = 100 + 0.9 (0.9 x 900 + 0.1 x 1000)
Q(S2,R) = 100 + 0.9 (810 + 100)
Q(S2,R) = 100 + 0.9 x 910
Q(S2,R) = 919.
```

Discussion

- How should the learning agent use the *intermediate* Q values?
  - Exploration
  - Exploitation

- Scaling up in the size of the state space
  - Function approximator (neural net instead of table)
  - Generalization
  - Reuse, use of macros
  - Abstraction, learning substructure

Ongoing Research

- Partially observable state
- Continuous action, state spaces
- Learn state abstractions
- Optimal exploration strategies
- Learn and use $\hat{\delta} : S \times A \rightarrow S$
- Multiple learners - Multi-agent reinforcement learning

Summary

- Markov model for state/action transitions.
- Value, policy iteration
- Q-learning
  - Deterministic, non-deterministic update rule
- Exploration, exploitation