Computer Science 355
Modern Computer Algebra

Assignment 5

Due date: Mar. 27
Objective: Applications of Grobner Bases

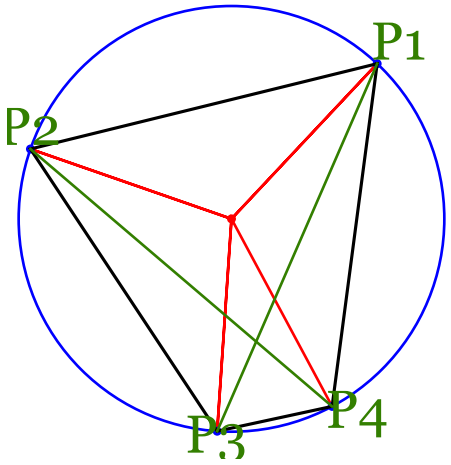
write your name here

Problem 1

Let a quadrilateral be inscribed in a circle. Then the product of its two diagonals equals to the sum of the products of the two pairs of opposite sides:

P1P3 * P2P4 = P1P2 * P3P4 + P1P4 * P2P3

Prove this result by using the Gröbner bases technique.



Problem 2

Given a square marix $A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$, find another square matrix B, such that $A = B * B$.

In analogy to the square root of a number, this matrix B is called the square root of the matrix A. You have to find all squre roots of A. Use Groebner bases technique.

Problem 3

Prove this identity using the Gröbner bases technique.

$$\frac{\sin(2\pi/7)}{\sin^2(3\pi/7)} - \frac{\sin(\pi/7)}{\sin^2(2\pi/7)} + \frac{\sin(3\pi/7)}{\sin^2(\pi/7)} = \sqrt{28}$$

Problem 4

Given Boolean variables X_i and a number of Boolean clauses each with three literals, i.e., clauses of the form

$$Y_j \bigvee Y_k \bigvee Y_i, \qquad \text{where} \qquad (Y_j, Y_k, Y_i) \in \{X_1, \ldots, X_n, \neg X_1, \ldots, \neg X_n\},$$

3-SAT is the problem of deciding whether there exists a Boolean assignment to the X_i's that makes all the clauses true simultaneously. Deescribe how you would solve 3-SAT problem using the Gröbner Bases.