Computer Science 355

Modern Computer Algebra

Assignment 4

Due date: Mar. 14

Objective: Polynomial Ideals and Gröbner Bases

Your name:

Problem 1 (10 pts)

Let $X = C([0, 1])$ be the set of all continuous functions with domain $[0, 1]$, and range $\mathbb{R}$. Further impose a ring structure on this set by making $*$ and $+$ function multiplication and addition. Prove that the set $M = \{f \in X \mid f(1) = 0\}$ is an ideal in this ring.

Problem 2 (15 pts)

Given two ideals I and J of a ring R such that $I + J = R$. Show that there is some $x \in R$ such that $x - a \in I$ and $x - b \in J$ for all $a,\ b \in R$.

Problem 3 (15 pts)

Determine whether a given polynomial is in an ideal

1. $x^3 + 2x^2 + 2x + 1, \quad I = \{x + 1\}$
2. $x^2 - 4x + 4, \quad I = \{x^3 - 6x^2 + 12x - 8,\ 2x^3 - 10x^2 + 16x - 8\}$
3. $x^3 - 1, \quad I = \{x^9 - 1,\ x^5 + x^3 - x^2 - 1\}$

Problem 4 (15 pts)

Working in $Q[x,\ y,\ z]$ and using the lex order with $x > y > z$, prove that $\{x-y,\ y^2+z\}$ is a Gröbner basis

Problem 5 (20 pts)

Find a Gröbner basis for $< z + y\,x^2,\; z\,x + y >$ with respect to lex with $x < y < z$.

Problem 6 (25 pts)

Find a Gröbner basis for $< x^2 y + z,\ x z + y >$ with respect to grlex with $x > y > z$.