Computer Science 355
Modern Computer Algebra

Assignment 1

Due date: Jan 27
Objective: Celine's algorithm

Your name:

Problem 1 (Programming) (20 pts)

Let $d(n)$ be defined as the sum of proper divisors of n (numbers less than n which divide evenly into n). If $d(a) = b$ and $d(b) = a$, where $a \neq b$, then a and b are an amicable pair and each of a and b are called amicable numbers.

For example, the proper divisors of 220 are 1, 2, 4, 5, 10, 11, 20, 22, 44, 55 and 110; therefore $d(220) = 284$. The proper divisors of 284 are 1, 2, 4, 71 and 142; so $d(284) = 220$.

Your task is to find all the amicable numbers below given n and return their sum. *Mathematica* functiuons: Divisors, Most, Sum.

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AmicableSum[n_Integer?Positive] :=
 Module[{},

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Problem 2 (Difference equations) (20 pts)

Solve the recurrence

$$a_n - 5\,a_{n-1} + 7\,a_{n-2} - 3\,a_{n-3} = 0$$
$$a_0 = 1,\ a_1 = 2,\ a_2 = 3$$

using the characteristic equation. Verify your solution with *Mathematica's* RSolve[].

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Problem 3 (Gamma Function) (20 pts)

Using the functional properties of the Gamma functions (see the Lecture 2), compute

$$\frac{\Gamma\left(\frac{5}{3}\right)}{\Gamma\left(\frac{11}{3}\right)}, \quad \frac{\Gamma\left(-\frac{3}{4}\right)}{\Gamma\left(\frac{5}{4}\right)}, \quad \Gamma\left(-\frac{3}{2}\right)$$

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Problem 4 (Celine's algorithm) (40 pts)

Evaluate the sum using Sister Celine' s algorithm

$$\sum_{k=0}^{n} \binom{k+n}{k} 2^{-k}$$

Demonstrate each step of the algorithm (using *Mathematica*). Find a recurrence for the summand, then sum that recurence over the range to find a recurrence that is satisfied by the sum. Finally, solve that recurrence.

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