# ACM International Collegiate Programming Contest 2000 East Central Regional Contest Case Western Reserve University University of Cincinnati November 11, 2000

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#### <u>Rules:</u>

- 1. All questions require you to read the test data from standard input and write results to standard output. You cannot use files for input or output. Additional input and output specifications can be found in the General Information Sheet.
- 2. All programs will be re-compiled prior to testing with the judges' data.
- 3. Non-standard libraries cannot be used in your solutions. The Standard Template Library (STL) and C++ string libraries are allowed.
- 4. Programming style is not considered in this contest. You are free to code in whatever style you prefer. Documentation is not required.
- 5. All communication with the judges will be handled by the  $PC^2$  environment.
- 6. The allowed programming languages are C, C++ and Java.
- 7. Judges' decisions are to be considered final. No cheating will be tolerated.
- 8. There are eight questions to be completed in five hours.

## Problem A: Picnic Planning

The Contortion Brothers are a famous set of circus clowns, known worldwide for their incredible ability to cram an unlimited number of themselves into even the smallest vehicle. During the off-season, the brothers like to get together for an Annual Contortionists Meeting at a local park. However, the brothers are not only tight with regard to cramped quarters, but with money as well, so they try to find the way to get everyone to the party which minimizes the number of miles put on everyone's cars (thus saving gas, wear and tear, etc.). To this end they are willing to cram themselves into as few cars as necessary to minimize the total number of miles put on all their cars together. This often results in many brothers driving to one brother's house, leaving all but one car there and piling into the remaining one. There is a constraint at the park, however: the parking lot at the picnic site can only hold a limited number of cars, so that must be factored into the overall miserly calculation. Also, due to an entrance fee to the park, once any brother's car arrives at the park it is there to stay; he will not drop off his passengers and then leave to pick up other brothers. Now for your average circus clan, solving this problem is a challenge, so it is left to you to write a program to solve their milage minimization problem.

#### Input

Input will consist of one problem instance. The first line will contain a single integer n indicating the number of highway connections between brothers or between brothers and the park. The next n lines will contain one connection per line, of the form name1 name2 dist, where name1 and name2 are either the names of two brothers or the word Park and a brother's name (in either order), and dist is the integer distance between them. These roads will all be 2-way roads, and dist will always be positive. The maximum number of brothers will be 20 and the maximum length of any name will be 10 characters. Following these n lines will be one final line containing an integer s which specifies the number of cars which can fit in the parking lot of the picnic site. You may assume that there is a path from every brother's house to the park and that a solution exists for each problem instance.

#### Output

Output should consist of one line of the form

Total miles driven: xxx

where xxx is the total number of miles driven by all the brothers' cars.

## Sample Input 1

### 10

Alphonzo Bernardo 32 Alphonzo Park 57 Alphonzo Eduardo 43 Bernardo Park 19 Bernardo Clemenzi 82 Clemenzi Park 65 Clemenzi Herb 90 Clemenzi Eduardo 109 Park Herb 24 Herb Eduardo 79 3

### Sample Output 1

Total miles driven: 183

## Sample Input 2

10 Alphonzo Bernardo 32 Alphonzo Park 57 Alphonzo Eduardo 43 Bernardo Park 19 Bernardo Clemenzi 82 Clemenzi Park 65 Clemenzi Herb 90 Clemenzi Eduardo 109 Park Herb 24 Herb Eduardo 79 1

### Sample Output 2

Total miles driven: 255

## Problem B: Poly-polygonal Numbers

A *polygonal* number is a number which can be represented by a regular geometrical arrangement of equally spaced points, where the arrangement forms a regular polygon. Some examples are shown in the figure below.



The first figure shows the first 4 triangular numbers 1, 3, 6, 10. The next three show the first four square, pentagonal and hexagonal numbers, respectively. In general, k-gonal numbers are those whose points define a regular k-gon (hence triangular numbers are 3-gonal, square numbers are 4-gonal, etc.). We will define k as an index of the polygonal number. For this problem, you are to find numbers which are k-gonal for two or more values of k. We will call these numbers poly-polygonal.

#### Input

Input will consist of multiple problem instances. Each instance will consist of 3 lines. The first line will be a non-negative integer  $n \leq 50$  indicating the number of types of polygonal numbers of interest in this problem. Note that this line may be longer than 80 characters. The next line will contain n integers indicating the indices of these polygonal numbers (all distinct and in increasing order). For example, if the first line contained the value 3, and the next line contained the values 3 6 10, then that problem instance would be interested in 3-gonal, 6-gonal and 10-gonal numbers. Each index k will lie in the range  $3 \leq k \leq 1000$ . The last line of the problem instance will consist of a single positive integer  $s \leq 10000$ , which serves as a starting point for the search for poly-polygonal numbers. A value of n = 0 terminates the input.

#### Output

For each problem instance, you should output the next 5 poly-polygonal numbers which are greater than or equal to s. Each number should be on a single line and conform to the following format:

num:k1 k2 k3 ...

where num is the poly-polygonal number, and k1, k2, k3 ... are the indices (in increasing order) of the poly-polygonal number equal to num. A single space should separate each index, and you should separate each problem instance with a single blank line. The judges input will be such that the maximum value for any poly-polygonal number will fit in a long variable.

## Sample Input

```
10
6 7 8 9 10 11 12 13 14 15
1000
5
3 4 13 36 124
1
0
```

## Sample Output

## **Problem C:** Rational Approximation

A polynomial p(x) of degree n can be used to approximate a function f(x) by setting the coefficients of p(x) to match the first n coefficients of the power series of f(x) (expanded about x = 0). For example,

$$rac{1}{(1-x)}pprox 1+x+x^2+\ldots+x^n.$$

Unfortunately, polynomials are "nice" and they do not work well when they are used to approximate functions that behave poorly (e.g. those with singularities). To overcome this problem, we can instead approximate functions by rational functions of the form p(x)/q(x), where p(x) and q(x) are polynomials. You have been asked by Approximate Calculation Machinery to solve this problem, so they can incorporate your solution into their approximate calculation software.

Given m, n, and the first m + n coefficients of the power series of f(x), we wish to compute two polynomials p(x) and q(x) of degrees at most m - 1 and n - 1, respectively, such that the power series expansion of  $q(x) \cdot f(x) - p(x)$  has 0 as its first m + n - 1 coefficients, and 1 as its coefficient corresponding to the  $x^{m+n-1}$  term. In other words, we want to find p(x) and q(x) such that

$$q(x) \cdot f(x) - p(x) = x^{m+n-1} + \dots$$

where ... contains terms with powers of x higher than m+n-1. From this, f(x) can be approximated by p(x)/q(x).

#### **Background Definitions**

A polynomial p(x) of degree *n* can be written as  $p_0 + p_1 x + p_2 x^2 + \ldots + p_n x^n$ , where  $p_i$ 's are integers in this problem.

A power series expansion of f(x) about 0 can be written as  $f_0 + f_1 x + f_2 x^2 + \ldots$ , where  $f_i$ 's are integers in this problem.

#### Input

The input will consist of multiple cases. Each case will be specified on one line, in the form  $m n f_0$  $f_1 \dots f_{m+n-1}$  where  $f_i$  is the coefficient of  $x^i$  in the power series expansion of f. You may assume that  $1 \leq m, 1 \leq n \leq 4, 2 \leq m+n \leq 10$ , and  $f_i$  are integers such that  $|f_i| \leq 5$ . The end of input will be indicated by a line containing m = n = 0, and no coefficients for f. You may assume that there is a unique solution for the given input.

#### Output

For each test case, print two lines of output. Print the polynomial p(x) on the first line, and then q(x) on the second line. The polynomial p(x) should be printed as a list of pairs  $(p_i, i)$  arranged in ascending order in *i*, such that  $p_i$  is a non-zero coefficient for the term  $x^i$ . Each non-zero coefficient  $p_i$  should be printed as a/b, where b > 0 and a/b is the coefficient expressed in lowest terms. In addition, if b = 1 then print only a (and omit b). If p(x) = 0, print a line containing only (0,0). Separate the pairs in the list by one space. The polynomial q(x) should be printed in the same manner. Insert a blank line between cases.

## Sample Input

## Sample Output

(0,0) (1,1) (-4/33,0) (-1/11,1) (-2/33,2) (-1/33,3) (-4/33,0) (5/33,1) (2/3,0) (1/3,0) (25/6,0) (-5/6,0) (1/3,2) (-1/6,3)

## **Problem D:** Stacking Cubes

Consider the following pattern of positive integers:

331 31 2

Note that each row is left-justified and no longer than its preceding row. Also, the entries in each row, when read left to right, are non-increasing and the entries in each column, when read top to bottom are non-increasing. We will call such a pattern a *stacking pattern* (SP) because such a pattern can represent a way of stacking cubes in a corner in the following way: if you consider placing the topmost row and leftmost column against walls, then the SP gives a bird's-eye view of how many cubes are stacked vertically. The SP above represents the following corner stacking:

We will call the wall against the topmost row the *right wall*, and the wall against the leftmost column the *left wall*. Here is another SP and the corner stacking it represents:

Note that if you rotate a corner stacking so the left wall becomes the floor and the floor becomes the right wall, you still have a corner stacking. (We will call this a *left rotation*.) Likewise, you would still have a corner stacking if you rotate so the right wall becomes the floor and the floor becomes the left wall. (We will call this a *right rotation*.) So the SP of the left and right rotations of the first SP given above are

3	2	1	3	3	2	
2	1	1	2	1	1	
2	1		1			

You should check that both the left and right rotations of the second example SP are identical to the original SP.





#### Input

This problem will consist of multiple problem instances. Each problem instance will consist of a positive integer  $n \leq 11$  indicating the number of rows in the SP that follows. (n = 0 indicates the end of input.) The rows of the SP will follow, one per line with entries separated by single spaces, delimited by a trailing 0. (The trailing 0 is, of course, not part of the input data proper and you may assume that each row given has at least one cube.) Each entry in the pattern proper will be a positive integer less than or equal to 20 and there will be no more than 20 entries in any row.

### Output

For each input SP you should produce two stacking patterns corresponding to the left rotation and the right rotation (in that order). Rows of the SP should be left-justified with entries separated by a single space. One blank line should separate the left and right rotations of the given SP and two blank lines should separate output for different problem instances.

#### Sample Input

#### Sample Output

## Problem E: Checker's Check

The game of checkers is played on an 8 by 8 red-black board using alternate squares. Two players (Red and White) each start with 12 pieces which are set up in the starting position shown below:



Rules for movement and capture are as follows (Note: a forward move is one in which a piece moves towards the opponent's side of the board):

- 1. Players alternate moves.
- 2. A piece may move one square forward diagonally to any empty square.
- 3. A piece may jump forward over an opposing player's piece if the opposing piece is adjacent to the piece and the square directly beyond the opposing piece is empty. After a jump, the opposing piece is removed from the board (captured). If after a jump, the jumping piece can make another jump, it must do so. This continues until it can make no other jumps. This is called a multiple jump.
- 4. A player must make a jump if possible. If several jumps are possible, the player may choose any, even choosing a single jump over a multiple jump. (However, if a multiple jump is chosen, it must be completed.)
- 5. The last row on the opponent's side of the board (the row where the piece can make no more forward moves) is the player's *promotion row*. When a piece reaches the promotion row it is promoted to a king and may now move and capture backward as well as forward. Once a piece is promoted, its move ends—it cannot start to jump or continue a multiple jump after becoming a king until the next turn.
- 6. A piece may be jumped at most once during a move (only a consideration when a king is doing the jumping).

Games are recorded using the square numbering shown above. For example, a simple forward move for White might be 22-18; a single jump for Red might be 14-23 (capturing a White piece at square 18); and a multiple jump for a White king might be 22-31-24-15 (capturing Red pieces at 26, 27 and 19).

For this problem, you will be given a position of a game in progress and a set of moves to be applied starting at that position, and you must determine if all the moves are legal by writing an Advanced Checkers Machine.

#### Input

Input will consist of multiple problem instances. Each instance will start with two integers r and w, indicating the number of Red and White pieces on the board (values of r = w = 0 indicates end of input, otherwise  $1 \le r, w \le 12$ ). The next line will contain r square numbers indicating the Red piece positions and the next line will contain w square numbers for the White piece positions. Positive square values will indicate that a normal piece lies on that square, while a negative value -sq will indicate that a promoted piece lies on square sq. The next line will contain a single integer  $m \ge 1$  indicating the number of moves to make, followed by a space and then a single character (either R or W) indicating whose move it is. The next m lines will contain the m moves, using the notation described above. (You may assume that there are no more than 13 square numbers listed in any one move.)

All board positions will be legal positions (e.g., there will never be two pieces occupying the same square). You may assume that pieces that have advanced to their promotion row are indeed promoted; that is, there will be no pieces on their promotion row that are not kings.

### Output

For each problem instance, output either All moves valid or Move n is invalid, where n=1 corresponds to the first move in the problem instance. If there are multiple illegal moves, you should list only the first such move.

#### Sample Input

### Sample Output

All moves valid Move 5 is invalid

## Problem F: To Bet or Not To Bet

Alexander Charles McMillan loves to gamble, and during his last trip to the casino he ran across a new game. It is played on a linear sequence of squares as shown below.

Start	•••	End
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A chip is initially placed on the Start square. The player then tries to move the chip to the End square through a series of turns, at which point the game ends. In each turn a coin is flipped: if the coin is heads the chip is moved one square to the right and if the coin is tails the chip is moved two squares to the right (unless the chip is one square away from the End square, in which case it just moves to the End square). At that point, any instruction on the square the coin lands on must be followed. Each instruction is one of the following:

- 1. Move right n squares (where n is some positive integer)
- 2. Move left n squares (where n is some positive integer)
- 3. Lose a turn
- 4. No instruction

After following the instruction, the turn ends and a new one begins. Note that the chip only follows the instruction on the square it lands on after the coin flip. If, for example, the chip lands on a square that instructs it to move 3 spaces to the left, the move is made, but the instruction on the resulting square is ignored and the turn ends. Gambling for this game proceeds as follows: given a board layout and an integer T, you must wager whether or not you think the game will end within T turns.

After losing his shirt and several other articles of clothing, Alexander has decided he needs professional help—not in beating his gambling addiction, but in writing a program to help decide how to bet in this game.

#### Input

Input will consist of multiple problem instances. The first line will consist of an integer n indicating the number of problem instances. Each instance will consist of two lines: the first will contain two integers m and T ( $1 \le m \le 50$ ,  $1 \le T \le 40$ ), where m is the size of the board *excluding* the Start and End squares, and T is the target number of turns. The next line will contain instructions for each of the m interior squares on the board. Instructions for the squares will be separated by a single space, and a square instruction will be one of the following: +n, -n, L or 0 (the digit zero). The first indicates a right move of n squares, the second a left move of n squares, the third a lose-a-turn square, and the fourth indicates no instruction for the square. No right or left move will ever move you off the board.

#### Output

Output for each problem instance will consist of one line, either

Bet for. x.xxxx

if you think that there is a greater than 50% chance that the game will end in T or fewer turns, or

Bet against. x.xxxx

if you think there is a less than 50% chance that the game will end in T or fewer turns, or

Push. 0.5000

otherwise, where x.xxxx is the probability of the game ending in T or fewer turns rounded to 4 decimal places. (Note that due to rounding the calculated probability for display, a probability of 0.5000 may appear after the Bet for. or Bet against. message.)

### Sample Input

#### Sample Output

Bet for. 0.9375 Bet against. 0.0000 Push. 0.5000 Bet for. 0.7500 Bet for. 0.8954

## Problem G: BSP Trees

When rendering a scene with multiple objects onto a screen, the order in which the objects are drawn is very important. In general, the farther an object is from the screen, the earlier it should be drawn allowing later, closer objects to be drawn on top of them. If two objects do not overlap, the order of drawing is immaterial. A binary space-partitioning (BSP) tree is one type of data structure which attempts to simplify the determination of the ordering of objects. It works as follows. Assume that the screen lies in the xy-plane centered on the z-axis and that the z-axis points away from the user looking at the screen. (For our purposes, assume the user lies near  $-\infty$  on the z-axis.) We also assume that all the objects lie on the opposite side of the screen (z > 0). The BSP tree is built by placing a series of planes parallel to the y-axis. The first plane divides space into two regions: a region containing the viewer and a region not containing the viewer. We partition all objects in space according to which of these two regions they lie in, and observe that all objects in the region containing the viewer should be drawn after all the objects in the other region. The BSP tree can be viewed at this point as a root with only two children, each child containing one of the partitions. We can now add a second plane, which subdivides the space again. We split each of the two partitions from the first plane in two, making a total of 4 partitions, and the resulting BSP tree now has three levels, with the partitions in the leaves (note that some of these partitions may contain several objects and some may contain none). This process is continued until each partition has at most one object in it, or until some predetermined number of planes has been used. The diagram below gives an example of using 1, 2 and 3 planes (looking down along the y-axis). For simplicity we assume that all objects lie parallel to the z-axis, so we need only deal this 2-d image to determine the BSP tree.



Assuming you have split the partitions correctly, a simple traversal of the BSP tree will give you an appropriate ordering for which to render the objects in the scene. Note in the example above that once a node contains just one object it need not be split as additional planes are added.

#### Input

Input will consist of one problem instance. The first line will contain a positive integer  $n \leq 20$  indicating the number of objects in the scene. The next n lines will contain a description of these objects using the format  $m \ x1 \ z1 \ x2 \ z2 \ ... \ xm \ zm$ , where m is the number of vertices in the object and the remaining values are the vertices of the intersection of the object with the xz-plane. All objects will have between 3 and 6 vertices. Objects are assumed to be labeled "A", "B", "C", ... in the order they are defined. Next in the input file will be a positive integer  $p \leq 10$  indicating the number of planes used to create the BSP tree. The last p input lines will contain a description of each plane of the form  $x1 \ z1 \ x2 \ z2$ representing two points on the intersection line of the plane and the xz-plane. You may assume that no line will intersect any object (including edges and vertices) and that no plane is parallel to the z-axis. All coordinates will be integers.

### Output

Output will consist of a single line containing the names of the objects in the order that they should be rendered for the specified BSP tree. In the case when some partition contains two or more objects, you should list the objects in alphabetical order.

### Sample Input

Sample Output

BCGEJFIHDA

## Problem H: Double Trouble

Alice Catherine Morris and her sister Irene Barbara frequently send each other e-mails. Ever wary of interceptions and wishing to keep their correspondence private, they encrypt their messages in two steps. After removing all nonalphabetic characters and converting all letters to upper case, they: 1) replace each letter by the letter s positions after it in the alphabet  $(1 \le s \le 25)$ —we call this a *shift* by s—and then, 2) divide the result of step 1 into groups of m letters and reverse the letters in each group  $(5 \le m \le 20)$ . If the length of the message is not divisible by m, then the last k (less than m) letters are reversed. For example, suppose s = 2 and m = 6. If the plaintext were

Meet me in St. Louis, Louis.

after removing unwanted characters and changing to upper case we get

#### MEETMEINSTLOUISLOUIS

We will call this the *modified plaintext*. We then shift each letter by 2 (Y would be replaced with A and Z would be replaced by B, here), getting the intermediate result:

#### OGGVOGKPUVNQWKUNQWKU

And finally reverse every group of 6 letters:

#### GOVGGOQNVUPKWQNUKWUK

Note the last two letters made up the last reversed group. As is customary, we write the result in groups of 5 letters. So the ciphertext would be:

GOVGG OQNVU PKWQN UKWUK

Alas, it's not so hard to find the values for s and m when the ciphertext is intercepted. In fact it's even easier if you know a *crib*, which is a word in the modified plaintext. In the above example, LOUIS would be a crib. Your job here is to find s and m when presented with a ciphertext and a crib.

#### Input

Input will consist of multiple problem instances. The first line of input will contain a positive integer indicating the number of problem instances. The input for each problem will consist of multiple lines. The first line of input for a problem will contain the integer n ( $20 \le n \le 500$ ) which is equal to the number of characters in the ciphertext. The following lines will contain the ciphertext, all upper case in groups of 5 letters separated by a single space. (The last group of letters may contain fewer than 5 letters.) There will be 10 groups of letters per line, except possibly for the last line of ciphertext. The input line following the last line of ciphertext will contain the crib; a single word consisting of between 4 and 10 (inclusive) upper case characters.

### Output

Output will be two integers, s and m on a line, separated by a single space, indicating the encryption key that produces the crib, where s is the shift and m is the reversed group size. If there is more than one solution, output the one with smallest s. If there is more than one with the same s, output the one with smallest m. If no such s and m exist, output the message Crib is not encrypted.

#### Sample Input

4 83 FIQMF IISFN QMFIB EOPFH FNQMV PSFIU IZNGP UPEUS BFPEP PEPPE PPEPN QMFIP EOPIS FIQMF IBSFN QMFBE OPI RHONDA 105 VDBMN DQDGS LNQEM ZLZRZ RNGVX ZALNA TERZV CZDGD MZQHZ GENKK KONSC DJHKC KKZAD RZAXZ SNMRH GBHGV RZVDG XZRNS XZKOS ZCNNF SHFMH BOMBAY 50 QFNWX YQFNW YSAQX FYNWY XQFNW SXYQF FXNYS AXYQF NASXY QFNAX HEAVEN 20 GOVGG OQNVU PKWQN UKWUK LOUIS

### Sample Output

1 6 25 6 Crib is not encrypted. 2 6