

1. Generating Functions

- (a) Find the generating function for the sequence $1, 1, 1, 1, \dots$ in closed form.

Solution:

- (b) What's the coefficient of x^{2005} in the generating function $G(x) = \frac{1}{(1+x)^2(1-x)^2}$?

Solution:

- (c) We have 20 bags, each bag containing a 5 dollar coin and a 7 dollar coin. If we can use at most one coin from each bag, in how many ways can we make 17 dollars, assuming that all coins are distinguishable (i.e. the 5 dollar coin from the first bag is considered to be different from that in the second bag, and so on)?

Solution:

- (d) Using generating functions, find a_n in terms of n :

$$a_0 = 2 \text{ and } a_{n+1} = 3a_n \text{ for } n \geq 0.$$

Solution:

2. Nim with Stones

We play in a line of squares labeled $0, 1, 2, \dots$ with several stones are placed in some square such that no two stones are placed in a same square. One move consists of moving one stone to its left onto any empty square and not passing any other stone.

The game ends when a player cannot make any legal moves, since all the stone are jammed at the left-end of the strip.

Find the P and N positions of this game.

Solution:

3. **Nim with Arrows**

Given a horizontal line of N arrows with some arrows pointing in the right direction, and some pointing in the wrong direction. Each turn, a player have to flip one arrow from wrong to right, and in the same time (if he/she wants), flip one more arrow to the left of it.

Find the P and N positions of this game.

Solution: