1. **Not Pirates and Not Gold**

(a) In lecture we developed a solution to this question:

How many nonnegative integer solutions are there to the equation

$$x_1 + x_2 + x_3 + x_4 + x_5 = 40$$

Solution:

(b) How many nonnegative integer solutions are there to the equation

$$x_1 + x_2 + x_3 + x_4 + x_5 \leq 40$$

Solution:

(c) How many nonnegative integer solutions are there to the equation

$$x_1 + x_2 + x_3 + x_4 + x_5 = 40$$

if we must satisfy $x_1 \geq 1, x_2 \geq 2, x_3 \geq 3, x_4 \geq 4, x_5 \geq 5$?

Solution:

(d) How many nonnegative integer solutions are there to the equation

$$x_1 + x_2 + x_3 + x_4 + x_5 = 40$$

if we must satisfy $x_1 \leq 20$?

Solution:

2. **There must be one...**

Let $S \subset \{0, 1, 2, 3, \ldots 99\}$ and $|S| = 10$.

Show that there must be two distinct subsets $A, B \subset S$ such that

$$\sum_{x \in A} x = \sum_{y \in B} y$$

.

Solution:

3. **Manhattaning Walks**

Consider the grid of points from $(0,0)$ to (n,n). Let $(a,x),(b,y),(c,z)$ be three points such that $0 < a < b < c < n$ and $0 < x < y < z < n$.

How many Manhattan walks are there from $(0,0)$ to (n,n) that don't go through any of the points $(a,x),(b,y),(c,z)$?

Solution: