Worksheet: Asymptotics & Hashtables

15-211 Recitation #2

July 7, 2010

Part I
Asymptotics

Exercise 1. Find $T(n) \in \Omega(n^3)$ s.t. $T(n) \notin O(n^4 \log n)$

$T(n) = n^5$ works.

Exercise 2. Prove that $(\forall s, t \in \mathbb{N}\backslash\{1\})[\log_s n \in \Theta(\log_t n)]$

Let $s, t \in \mathbb{N}\backslash\{1\}$. Choose $c \in \mathbb{R}$ s.t. $c = \frac{\log t}{\log s}$. We know that $\forall n > 2$, $\log n \leq \log n$, so $c \log n \leq c \log n$.

Thus, $\frac{\log t}{\log s} \log n \leq c \log n$.

Thus $\frac{1}{\log s} \log n \leq \frac{c}{\log t} \log n$

Thus $\log_s n \leq c \log_t n$

Thus $(\exists c > 0)(\exists n_0 \in \mathbb{N})(\forall n \in \mathbb{N} : n > n_0)[\log_s n \leq c \log_t n]$

Thus, $\log_s n \in O(\log_t n)$.

By generality, $\log_t n \in O(\log_s n)$.

Thus, $\log_s n \in \Theta(\log_t n)$.

Exercise 3. Find a bound for $T(n) = 2T(\frac{n}{3}) + 1$

A good bound is $\Theta(n^{\log_3 2})$, found using master method.

Exercise 4. Find a theta bound for $T(n) = nT(n-1)$

This evaluates to $T(n) = n!$ (given the right base case), and thus because we have an exact solution, we have the theta bound $\Theta(n!)$

Exercise 5. Find a closed form for $T(n) = n + T(n-1)$

This is simply $T(n) = \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ (well known identity). This can be proven with induction. On an exam or hw I would expect you to provide the proof by induction.