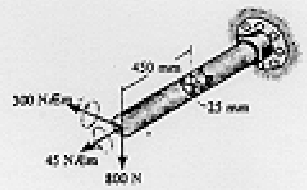
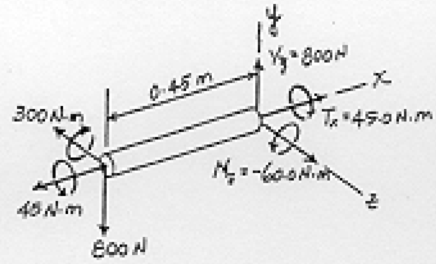
9–90. The solid shaft is subjected to a torque, bending moment, and shear force as shown. Determine the principal stresses acting at points A and B and the absolute maximum shear stress.





Internal Forces and Moment : As shown on FBD.

Section Properties :

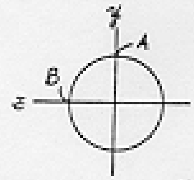
$$I_z = \frac{\pi}{4}\left(0.025^4\right) = 0.306796\left(10^{-6}\right)\ \text{m}^4$$

$$J = \frac{\pi}{2}\left(0.025^4\right) = 0.613592\left(10^{-6}\right)\ \text{m}^4$$

$$(Q_A)_y = 0$$

$$(Q_B)_y = \bar{y}'A'$$

$$= \frac{4(0.025)}{3\pi}\left[\frac{1}{2}(\pi)\left(0.025^2\right)\right] = 10.417\left(10^{-6}\right)\ \text{m}^3$$

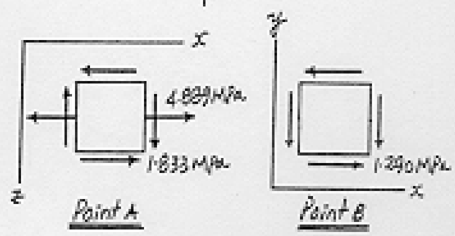


Normal Stress : Applying the flexure formula.

$$\sigma = -\frac{M_z y}{I_z}$$

$$\sigma_A = -\frac{-60.0(0.025)}{0.306796(10^{-6})} = 4.889\ \text{MPa}$$

$$\sigma_B = -\frac{-60.0(0)}{0.306796(10^{-6})} = 0$$



Shear Stress : Applying the torsion formula for point A.

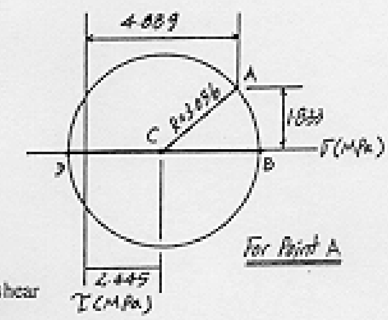
$$\tau_A = \frac{Tc}{J} = \frac{45.0(0.025)}{0.613592(10^{-6})} = 1.833\ \text{MPa}$$

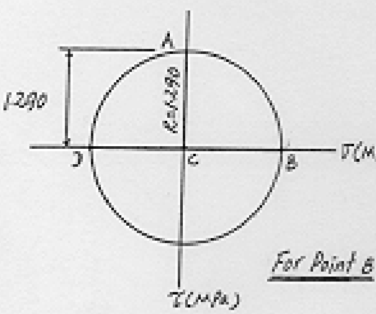
The transverse shear stress in the y direction and the torsional shear stress can be obtained using shear formula and torsion formula, $\tau_V = \frac{VQ}{It}$ and $\tau_{twist} = \frac{T\rho}{J}$, respectively.

$$\tau_B = (\tau_V)_y - \tau_{twist}$$

$$= \frac{800\left[10.417(10^{-6})\right]}{0.306796(10^{-6})(0.05)} - \frac{45.0(0.025)}{0.613592(10^{-6})}$$

$$= -1.290\ \text{MPa}$$





Construction of the Circle : $\sigma_x = 4.889$ MPa, $\sigma_z = 0$, and $\tau_{xz} = -1.833$ MPa for point A. Hence,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_z}{2} = \frac{4.889 + 0}{2} = 2.445\ \text{MPa}$$

The coordinates for reference points A and C are $A(4.889,\ -1.833)$ and $C(2.445,\ 0)$.

The radius of the circle is

$$R = \sqrt{(4.889 - 2.445)^2 + 1.833^2} = 3.056\ \text{MPa}$$

$\sigma_x = \sigma_y = 0$ and $\tau_{xy} = -1.290$ MPa for point B. Hence,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_z}{2} = 0$$

The coordinates for reference points A and C are $A(0,\ -1.290)$ and $C(0,\ 0)$.

The radius of the circle is $R = 1.290$ MPa

In-Plane Principal Stresses : The coordinates of points B and D represent σ_1 and σ_2, respectively. For point A

$$\sigma_1 = 2.445 + 3.056 = 5.50\ \text{MPa}$$

$$\sigma_2 = 2.445 - 3.056 = -0.611\ \text{MPa}$$

For point B,

$$\sigma_1 = 0 + 1.290 = 1.29\ \text{MPa}$$

$$\sigma_2 = 0 - 1.290 = -1.290\ \text{MPa}$$

Three Mohr's Circles : From the results obtained above, the principal stresses for point A are

$$\sigma_{max} = 5.50\ \text{MPa} \qquad \sigma_{int} = 0 \qquad \sigma_{min} = -0.611\ \text{MPa} \quad \textbf{Ans}$$

And for point B

$$\sigma_{max} = 1.29\ \text{MPa} \qquad \sigma_{int} = 0 \qquad \sigma_{min} = -1.29\ \text{MPa} \quad \textbf{Ans}$$

Absolute Maximum Shear Stress : For point A,

$$\tau_{\substack{abs\\max}} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{5.50 - (-0.611)}{2} = 3.06\ \text{MPa} \quad \textbf{Ans}$$

For point B,

$$\tau_{\substack{abs\\max}} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{1.29 - (-1.29)}{2} = 1.29\ \text{MPa} \quad \textbf{Ans}$$