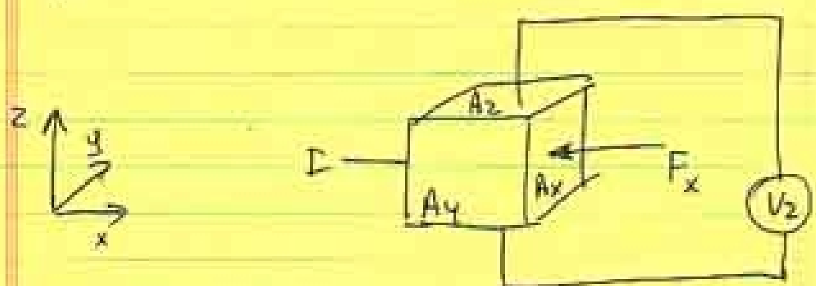
Piezoelectricity. Detection of Cantilever Deflection.

Appearance of opposite charges on the the surfaces of certain nonconducting crystals <u>with no center of symmetry</u> when they are subjected to mechanical pressure.



Force acting in x will give rise to the following charges:

on A_z : $Q_z = d_{zx} F_x$

A_y : $Q_y = d_{yx} F_x$

A_x : $Q_x = d_{xx} F_x$

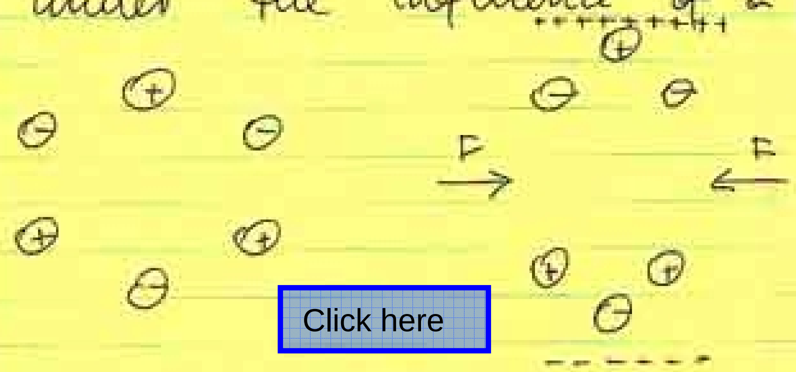
in general

$$\boxed{Q_i = d_{ij} F_j} \quad \text{where } i,j = x,y,z$$

d_{ij} – charge sensitivity tensor

$$d_{ij} = \begin{pmatrix} d_{xx} & d_{xy} & d_{zy} \\ d_{yx} & d_{yy} & d_{yz} \\ d_{zx} & d_{zy} & d_{zz} \end{pmatrix}$$

The source of piezoelectricity is the redistribution of charges in the unit cell of a crystal under the influence of a pressure



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Notice that the effect will occur only if the original arrangement of charges is non centrosymmetric!

Typical values of d_{ij} are in the pC/N range.

Example 1

What is the voltage arising on z faces of a piezoelectric cube with dimensions 1 cm × 1 cm × 1 cm under the influence of force $F_x = 1N$?

The charge sensitivity coefficient of material $d_{zx} = 110\ pC/N$, its Young's modulus $E = 8.3 \times 10^{10} \frac{N}{m^2}$

its relative dielectric permittivity $\varepsilon_r = 1200$.

Charge appearing on the surface:

$Q_z = d_{zx} \cdot F_x$

Voltage is related to charge through $C_z = \frac{Q_z}{V_z}$

where C_z – is capacitance.
For a parallel plate system

$C_z = \varepsilon_r \varepsilon_0 \frac{A_z}{z}$, where ε_0 – dielectric permittivity of vacuum

$\varepsilon_0 = 8.854 \times 10^{-12} \frac{F}{m}$

Thus:

$$V_z = \frac{Q_z}{C_z} = \frac{d_{zx} \cdot F_x \cdot z}{\varepsilon_r \, \varepsilon_0 \, A_z} =$$

$$= \frac{110 \times 10^{-12} \frac{C}{N} \cdot 1N \cdot 10^{-2} m}{1.2 \times 10^3 \cdot 8.85 \times 10^{-12} \frac{F}{m} \times 10^{-4} m^2} =$$

$$\approx 1V$$

Example 2

What is the change in length in z in the same element in response to 100 mV voltage applied along x?

In order to calculate the change in length in z, we need to determine the stress acting on A_z

$$\sigma_z = E \epsilon_z$$

$$\frac{F_z}{A_z} = E \frac{\Delta l_z}{l_z}$$

$$\Delta l_z = \frac{F_z l_z}{A_z E} =$$

$$= \frac{V_x C_x l_z}{d_{zx} A_z E} = \frac{V_x \varepsilon_0 \varepsilon_r \frac{A_z}{l_z} \cdot l_z}{d_{zx} A_z E} =$$

$$= V_x \frac{\varepsilon_0 \varepsilon_r}{d_{zx} E} = 100 \times 10^{-3} V \frac{8.85 \times 10^{-12} \frac{F}{m} \cdot 1.2 \times 10^{3}}{110 \times 10^{-12} \frac{C}{N} \cdot 8.3 \cdot 10^{10} \frac{N}{m^2}} =$$

$$\approx \underline{1.2 \times 10^{-10}\ m = 1.2\ \mathring{A}}$$

Further reading (click inside boxes):

Fundamentals of Piezoelectricity

http://www.physikinstrumente.com/tutorial/4_15.html

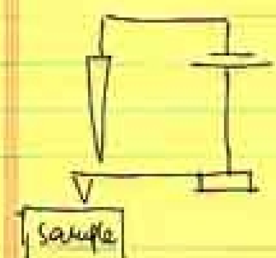
The AFM scanner

A chapter from ThermoMicroscope's "Practical Guide to Scanning Probe Microscopy"

http://cmm.mrl.uiuc.edu/instruments/AFM/PracticalGuide.pdf

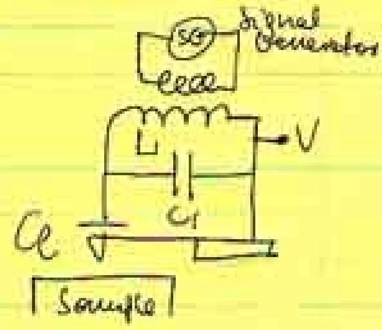
Various Deflection Detection Systems

Tunneling



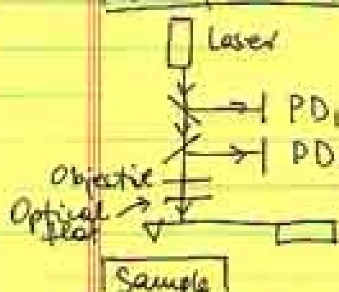
Cantilever deflection modulates the tunneling current

Capacitance



Measure changes of capacitance of a capacitor formed by the ~~tip~~ lever and reference plate. Small changes in Ce are detected by detuning the high-Q-tuned circuit excited by signal generator SG

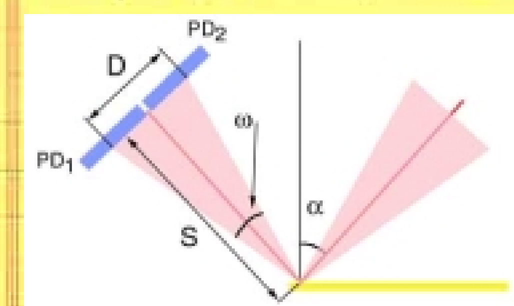
Interferometry

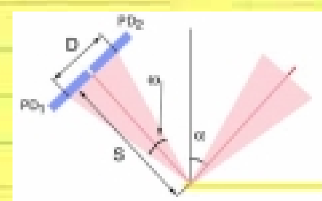


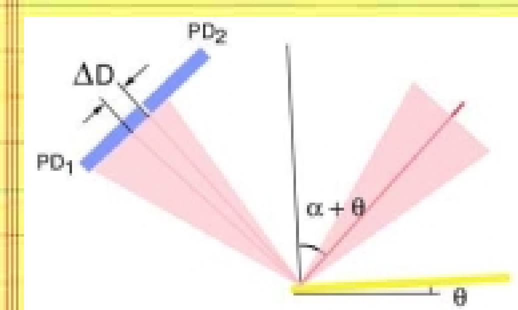
Detect interference modulation of a polarized laser beam passing through the Fabry Perot element formed by the optical flat and the backside of a cantilever. The returning beam is directed towards photodetector PD2 and its intensity is compared with the reference beam.

Laser Beam Deflection Method

This method is most widely used nowadays and will be analyzed in more detail.
A laser beam is reflected off the cantilever onto two photodetectors (PD_1 and PD_2). Deflection of a cantilever changes the position of a laser spot on PD_1 and PD_2, giving rise to a difference in photocurrents which is directly proportional to the deflection of a lever.



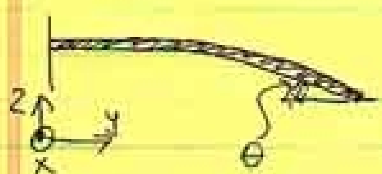




The angle θ, by which the cantilever tilts under the influence of force F can be obtained from equation

$$z = \frac{Fy^2}{6Ek_2^2A}(y-3a)$$

by noting that $\tan\theta = -\frac{\partial z}{\partial y}$



and approximating $\tan\theta = \theta$

Thus

$$\theta = \frac{Fya}{Ek_2^2A} - \frac{1}{2}\frac{Fy^2}{Ek_2^2A}$$

For a force acting at the end of a cantilever ($a=\ell$) the ~~angle~~ tilt at the end ($y=\ell$) is equal to

$$\theta = \frac{F\ell^2}{2Ek_2^2A}$$

since $Ek_2^2A = k\ell^3/3$

and $F = k\cdot z$

$$\theta = \frac{3}{2}\frac{z}{\ell}$$

The displacement of a laser beam on photodetectors

$$\delta D = 2\theta \cdot s$$

where s is the distance between the end of a cantilever and photodetector, thus

$$\delta D = 3\frac{s}{l} \cdot z$$

For simplicity we assume that for $\theta = 0$, the beam is perfectly centered on the detector. Thus, the optical power on each detector

$$P_1 = P_2 = \frac{1}{2}P$$

where P is the total power in the spot.
The power on each photodetector after the spot is shifted by δD is equal to

$$P_1' = P_1 + \frac{\delta D}{D} \cdot P$$

$$P_2' = P_2 - \frac{\delta D}{D} P$$

and thus

$$\Delta P = 2P\frac{dD}{D}.$$

The difference in photocurrents between P_1 and P_2

$$\Delta i = \Delta P \cdot \eta$$

where η is the quantum efficiency of each photodetector. The measured voltage difference

$\Delta U = \Delta i \cdot R$, where R is the load resistance.

Thus, the measured signal is equal to

$$\Delta U = 6\,\eta P R \frac{s}{l D} \cdot z,$$

i.e. it is directly proportional to z and inversely proportional to the length of a cantilever.

It might appear, that the response of a detector might be increased by increasing the length of the "optical lever" s, however the effect is canceled due to divergence of a beam, since

$$D = \omega \cdot s \quad ,$$

and thus

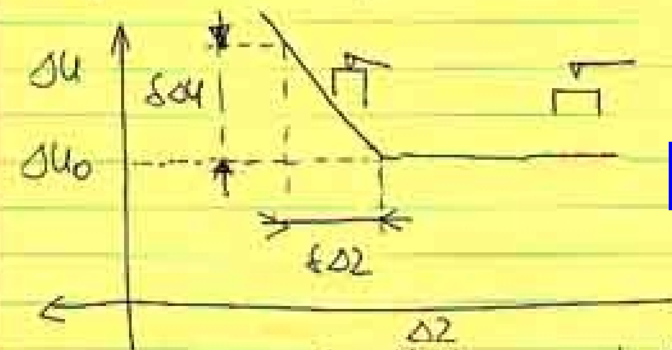
$$\boxed{\Delta U = 6 \eta P R \frac{1}{\omega e} \cdot z} \; .$$

Typical value of sensitivity of AFM detectors

$\frac{\Delta U}{z}$ is in the range of $\frac{100\,mV}{nm}$

Calibration of The Laser Beam Deflection

Calibration procedure involves displacing the rigid sample mounted on a piezo scanner towards the tip, causing its upward deflection



vertical displacement of a scanner

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calibration coefficient:

$$\alpha_{cal} = \frac{\delta \Delta U}{\delta \Delta Z}$$

Thus, if cantilever spring constant k_s is known, the force with which the tip is pushing the sample

$$\boxed{F = k_s \cdot \frac{1}{\alpha_{cal}} \cdot (\delta U - \Delta U_0)}$$

where ΔU_0 corresponds to undeflected position of a cantilever.

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The first integration gives

$$\frac{dz}{dy} = \frac{1}{2}\frac{F}{Ek_2^2A}y^2 - \frac{F}{Ek_2^2A}ay + C_1$$

from the boundary condition

$$\frac{dz}{dy} = 0 \quad \text{for } y=0$$

we obtain $C_1 = 0$.

The second integration gives

$$\boxed{z = \frac{Fy^2}{6Ek_2^2A}(y-3a)} + C_2 .$$

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Thus, if the force F is applied to the free end of the cantilever $(a = \ell)$, the z position of its free end $(y = \ell)$ is given by

$$z = -\frac{\ell^3}{3Ek_2^2A}\cdot F$$