

09-723 Proximal probe techniques.

Name \_\_\_\_\_

**Homework #4.**

Due by 6:30 PM, Thursday, September 29, 2004

Download the PowerEnergy.zip file which contains all necessary Matlab files by clicking on it and choosing to save it on your computer (e.g. on your desktop). Open the archive on your computer and extract it to some folder. Make sure that a path is set in Matlab to this folder. To set a path, choose the set path option in the file menu of Matlab. Click the Add with subfolders button and locate the folder to be added to the path in the pop-up window and click OK. Then click Save and Close.

**As usual, please contact me if you have any problems with the installation or any other problems with using the program.**

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### ***DrivenHarmonicExplorer (Some Assembly Required)***

In its basic state, PowerEnergyExplorer allows you to generate solutions of the equations of motion of mass ( $M_{tip}$ ) attached to a spring with a spring constant ( $K_{spring}$ ), subjected to damping ( $B_{damp}$ ), driven at a specific frequency with specified drive amplitude based on the following equation:

$$M_{tip} \ddot{z} + B_{damp} \dot{z} + K_{spring} z = F_o (\sin(2\pi F_{oper} time) + Phi) \quad (1)$$

In this homework assignment, you will explore the energy and power flow through both a simple harmonic oscillator and a driven harmonic oscillator. To accomplish this, some simple additions to your model from homework #3 (**DriveHarmOscModel.mdl**) will be required. When you make these changes, save the model as **PowerEnergyModel.mdl**. **(DO NOT SAVE OVER YOUR OLD MODEL.)** Save the model in the folder that you extracted PowerEnergyExplorer into. You should have already set a path to this folder. All necessary parameters are entered through the provided Graphical User Interface (GUI), which also controls the execution of the Simulink model. Make sure that in your model you call the variables as follows:

Mass: Mtip (This is to make it easier to build this as the tip mass in later models)

Spring constant: Kspring

Damping Coefficient: Bdamp

The simulation step (Fixe-step size): SimStep

The total simulation time (i.e. Stop Time)= SimTime

The initial velocity = Zprime (initial condition for the first integrator)

The initial position = Zstart (initial condition for the second integrator)

The position trajectory = Ztraj

The time = time

The drive amplitude = Drive

The drive frequency (operating frequency) = Foper

The constant Phi = Phi

The potential energy = Penergy

The kinetic energy = Kenergy

The damping power = Bpower

The damping energy = Benergy

The drive power = Dpower

The drive energy = Denergy

**If these names are not used, the GUI will not work.**

All other variables will be defined through the GUI.

Steps in construction of your model:

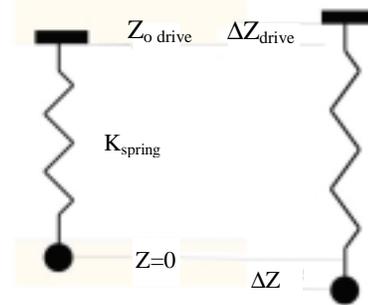
#### **I. Determine and send the potential energy to the workspace.**

Remember from class that the potential energy of a *non-driven* harmonic oscillator is:

$$P_{energy} = \frac{K_{spring} z^2(t)}{2} \quad (2)$$

where  $z(t)$  is the displacement of the mass, which in this case is equal to the deformation of the spring. For a *driven* oscillator we need to modify this expression by noting that the *total* deformation of the spring is equal to (see Figure)

$$\delta z(t) = z(t) - [z_{drive}(t) - z_{o\_drive}] = \Delta z(t) - \Delta z_{drive}(t) \quad (3)$$



So far, your model has been constructed based on drive *force*  $F(t)$  rather than drive *displacement*  $\Delta z_{drive}$ . These two are directly related through

$$F(t) = K_{spring} \Delta z_{drive} \quad (4)$$

Thus, for the driven oscillator the potential energy is given by:

$$P_{energy} = \frac{K_{spring} [\delta z(t)]^2}{2} = \frac{K_{spring}}{2} \left[ z(t) - \frac{F(t)}{K_{spring}} \right]^2 \quad (5)$$

To add this to the model and send it to the workspace, first take the *driving force* signal, divide it by  $K_{spring}$  with the aid of **gain block**, then subtract it from  $z(t)$  using the **sum block**, square the result of subtraction using a **math function block** found in math operations and finally multiply it by  $K_{spring}/2$  using another gain block. (When using the **sum block** to perform subtraction, in the **sum block** properties where it says list of signs, type "+" and "-"). Send this to the workspace using a **To Workspace block** as **Penergy**. Make sure that the save format of the **To Workspace block** is array.

## II. Determine and send the kinetic energy to the workspace.

Remember from class that the kinetic energy of an oscillator is:

$$K_{energy} = \frac{M_{tip} \dot{z}^2(t)}{2} \quad (6)$$

To add this to the model and send it to the workspace, use a **math function block** found in math operations to square the velocity signal ( $\dot{z}$ ) of the model. Then use a **gain block** to multiply this signal by  $M_{tip}/2$ . Send this to the workspace using a **To Workspace block** as **Kenergy**. Make sure that the save format of the **To Workspace block** is array.

## III. Determine and send the damping power to the workspace.

Remember from class that the damping force is:

$$F_{damping} = B_{damp} \dot{z}(t) \quad (7)$$

The infinitesimal work done by the damping force is:

$$dW_{damping} = B_{damp} \dot{z}(t) dz \quad (8)$$

To get the damping power, we divide the infinitesimal work by  $dt$ :

$$\frac{dW_{damping}}{dt} = B_{damp} \dot{z}(t)^2 \quad (9)$$

To add this to the model and send it to the workspace, use a **math function block** found in math operations to square the velocity signal ( $\dot{z}$ ) of the model. Then use a **gain block** to multiply this signal by **Bdamp**. Send this to the workspace using a **To Workspace block** as **Bpower**. Make sure that the save format of the **To Workspace block** is array.

#### IV. Determine and send the drive power to the workspace.

Power delivered by the drive is calculated by observing that the infinitesimal work performed by the drive is equal to the product of elastic force exerted by the spring and infinitesimal drive displacement  $dz_{drive}$  :

$$dW_{drive} = F_{elastic} dz_{drive} = K_{spring} \delta z(t) dz_{drive} = K_{spring} [z(t) - \frac{F(t)}{K_{spring}}] dz_{drive} \quad (10)$$

Consequently, power is equal to:

$$\begin{aligned} \frac{dW_{drive}}{dt} &= K_{spring} [z(t) - \frac{F_{drive}(t)}{K_{spring}}] \frac{dz_{drive}}{dt} = K_{spring} [z(t) - \frac{F_{drive}(t)}{K_{spring}}] \frac{1}{K_{spring}} \frac{dF_{drive}}{dt} = \\ &= [z(t) - \frac{F_{drive}(t)}{K_{spring}}] \frac{dF_{drive}}{dt} \end{aligned} \quad (11)$$

To add this to the model and send it to the workspace, calculate the derivative of the driving force using the **derivative block** found in continuous operations (make sure that you take the derivative before the drive signal is multiplied by Mtip) and multiply it with the aid of the **product block** found in the math operations by the total spring deformation signal

$$\delta z(t) = [z(t) - \frac{F_{drive}(t)}{K_{spring}}]$$

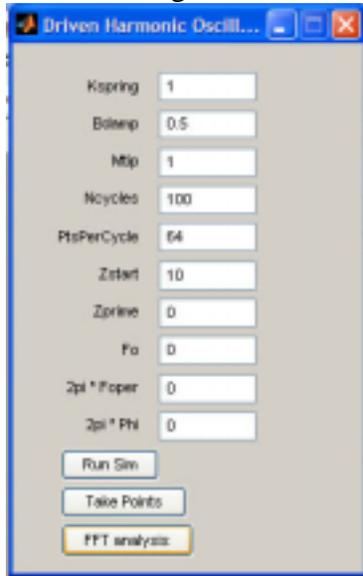
which you have already created earlier when calculating the potential

energy. (Make sure that the **product block** is set for element wise multiplication.) Send this signal to the workspace using a **To Workspace block** as **Dpower**. Make sure that the save format of the **To Workspace block** is array.

Before running the model you need to provide the values of other parameters using the provided GUI. To use the GUI provided, run `Energy_Power` from command line. Unlike before, no SimPrep commands need to be run.

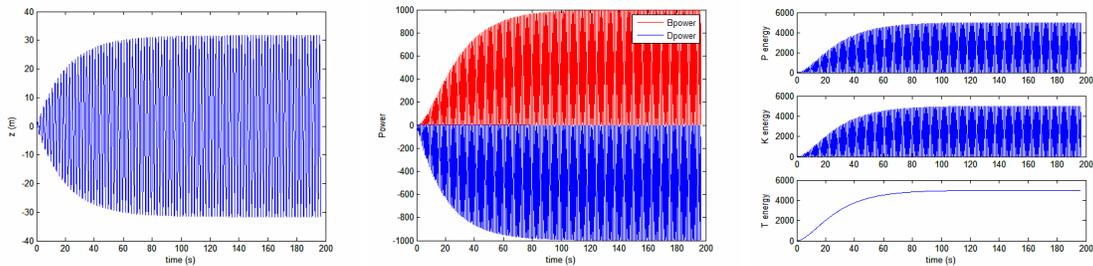
```
>> Energy_Power
```

The following interface should appear.



This GUI is similar to the previous GUI from Homework #3.

If everything is set up, you should be able to run a simulation using the default setting by clicking the **Run Sim** button. The following figures should appear:



## Part 1 Energy flow in a simple Harmonic Oscillator

**1.1** Set **Kspring** to 10, **Bdamp** to 0.1, **Mtip** to 1, **Ncycles** to 100, **PtsPerCycle** to 256, **Zstart** to 1, **Zprime** to 0, **Fo** to 0, **Foper** to 0.5033, and **Phi** to 0.

Run the simulation. What is the relationship between potential and kinetic energy? What happens to total energy? Now set **Bdamp** to 0.1. What happens to the total energy of the system? Convert the damping power to damping energy by integrating **Bpower**. This is easily accomplished by using the `cumsum` function from command line as follows:

```
>> Benergy = (cumsum(Bpower)) * SimStep;
```

Add the damping energy, the potential, and the kinetic energy (potential and kinetic energy are already summed in Tenergy).

```
>> Energy = Benergy + Tenergy;
```

Plot these energies on the same plot.

```
>> figure (9);  
>> plot(time,Benergy, time, Tenergy, time, Energy);  
>> legend ('Benergy', 'Tenergy', 'Energy');
```

What is the relationship between the kinetic and potential energy of the oscillator and the energy dissipated by damping?

## Part 2 Energy flow in a Driven Harmonic Oscillator

**2.1** Set **Kspring** to 10, **Bdamp** to 0.1, **Mtip** to 1, **Ncycles** to 100, **PtsPerCycle** to 256, **Zstart** to 0, **Zprime** to 0, **Fo** to 1, **Foper** to 0.5033, and **Phi** to 0.

Run the sim. Zoom in on the damping power and driving power plot, and comment on the phase of these two power trajectories. Calculate the driving energy and the damping energy using the cumsum function again.

```
>> Benergy = (cumsum(Bpower)) * SimStep;
```

```
>> Denergy = (cumsum(Dpower)) * SimStep;
```

Add these two energies together, and plot the damping energy, drive energy, and the sum of these two energies on the same plot as a function of time. What is the relationship between the damping energy and the drive energy? Notice the offset in the sum of these two energies. Compare this offset with the sum of potential and kinetic energy plotted as Tenergy in the energy plot of the simulation. What is the cause of the offset? Now explore the relationship between the drive energy and the damping energy after the driven harmonic oscillator has reached its steady state. Do this by integrating over the portion of the respective power trajectories that corresponds to the system at steady state.

```
>> Benergy = (cumsum(Bpower(15000:end))) * SimStep;
```

```
>> Denergy = (cumsum(Dpower(15000:end))) * SimStep;
```

Add these together and plot versus time.

```
>> Energy=Benergy+Denergy;
```

```
>>figure (10)

>>plot(time(15000:end),Benergy,time(15000:end),Denergy,time
(15000:end),Energy);

>> legend('Benergy', 'Denergy', 'Energy');
```

Now what is the balance between the driving energy and the damping energy?

**2.2.** Set **Kspring** to 10, **Bdamp** to 0.1, **Mtip** to 1, **Ncycles** to 100, **PtsPerCycle** to 256, **Zstart** to 0, **Zprime** to 0, **Fo** to 1, **Foper** to 0.4000, and **Phi** to 0.

Run the sim. What is the relationship between the drive power and the damping power below resonance? Zoom in on the power trajectories in a region at steady state. What happened to the phase? As in question 2.1, calculate the drive energy and the damping energy. Plot these energies and their sum on the same plot as a function of time. What is the relationship between the damping energy and the drive energy? Compare this plot with the sum of potential and kinetic energy plotted as Tenergy in the energy plot of the simulation. What are the differences between the energy flow when driving the oscillator far below resonance as opposed to driving it at resonance? If the damping energy can not dissipate all of the energy supplied by the drive, where is the extra energy dissipated?

**2.2.** Set **Kspring** to 10, **Bdamp** to 0.1, **Mtip** to 1, **Ncycles** to 100, **PtsPerCycle** to 256, **Zstart** to 0, **Zprime** to 0, **Fo** to 1, **Foper** to 0.6066, and **Phi** to 0.

Run the sim. What is the relationship between the drive power and the damping power above resonance? Zoom in on the power trajectories in a region at steady state. What happened to the phase? As in question 2.1, calculate the drive energy and the damping energy. Plot these energies and their sum on the same plot as a function of time. What is the relationship between the damping energy and the drive energy? Compare this plot with the sum of potential and kinetic energy plotted as Tenergy in the energy plot of the simulation. What are the differences between the energy flow when driving the oscillator far above resonance as opposed to driving it at resonance? If the damping energy can not dissipate all of the energy supplied by the drive, where is the extra energy dissipated?

### Part 3

#### Lennard-Jones potential:

The interaction between two molecules is described by the Lennard-Jones potential:

$$w(r) = \frac{A}{r^{12}} - \frac{B}{r^6} = 4 \epsilon \left[ \left( \frac{\sigma}{r} \right)^{12} - \left( \frac{\sigma}{r} \right)^6 \right]$$

where  $A = 10^{-134} \text{ J m}^{12}$     $B = 10^{-77} \text{ J m}^6$  .

- 3.1. Calculate the distance at which  $w(r) = 0$  .
  
- 3.2. Calculate the minimum value of energy and the distance at which it reaches the minimum.
  
- 3.3. Derive the expression for force as a function of distance  $r$
  
- 3.4. Calculate the maximum value of attractive force.
  
- 3.5. Plot the potential and force as a function of  $r$ .