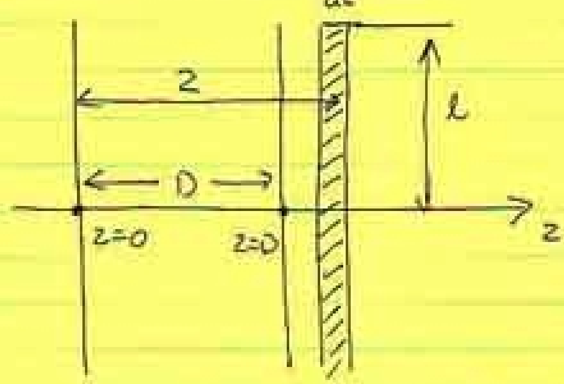
Contact mechanics and adhesion

- Surface and adhesion forces
- Derjaguin approximation
- Indentation forces

Consider the energy per unit area in the interaction of two planar surfaces



Using the earlier derived potential for the interaction between molecule and surface

$$w(D) = -2\pi C\rho \Big/ (n-2)(n-3) D^{n-3}$$

For a thin sheet of molecules with unit area and thickness dz, at a distance z away from another surface (of large area), the interaction energy is:

$$\underbrace{w(z)}_{\text{interaction energy for a sheet of unit area}} = -2\pi C\rho \underbrace{(\rho\, dz)}_{\text{number of molecules in a sheet of unit area and thickness } dz} / (n-2)(n-3) z^{n-3}$$

For two surfaces (integration over all sheets dz):

$$W(D) = -\frac{2\pi C\rho^2}{(n-2)(n-3)} \int_D^\infty \frac{dz}{z^{n-3}} =$$

$$= -\frac{2\pi C\rho^2}{(n-2)(n-3)(n-4) D^{n-4}}.$$

For $n = 6$

$$\boxed{W(D) = -\pi C\rho^2 / 12 D^2}$$

per unit area!

Interaction energy per unit area for two planar surfaces can be used to describe the ~~an~~ interaction force between macroscopic bodies.

Consider the force between sphere and flat surface:

$$F(D) = -\frac{\partial W(D)}{\partial D} = -\frac{4\pi^2 C \rho^2 R}{(n-2)(n-3)(n-4)\,D^{n-4}}$$

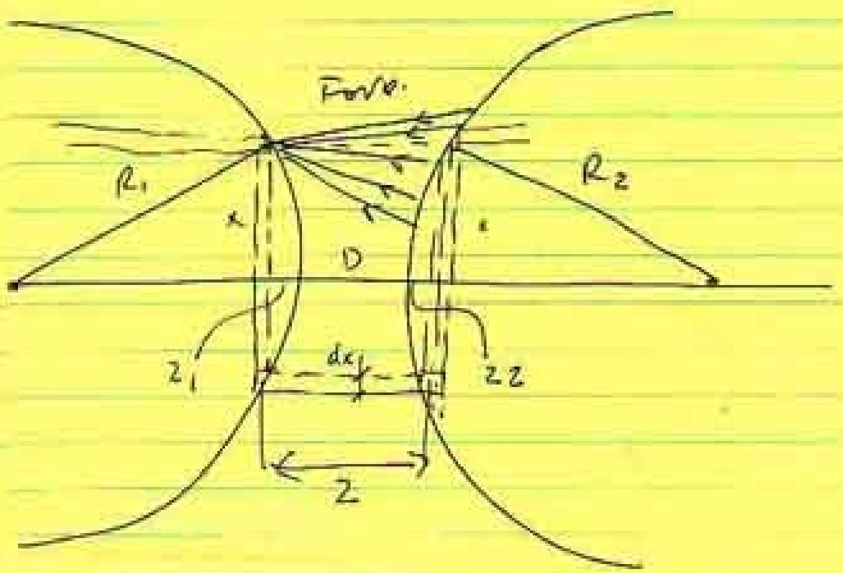
compare with

$$F(D)_{sphere} = \boxed{2\pi R\, \underbrace{W(D)}_{\text{planes per unit area}}}$$

So far, we have dealt only with additive potentials $w(r) = -C/r^n$.

The obtained relationship is however valid for any type of force law.

Consider two large spheres of radii R_1 and R_2, separated by small distance D



If $R_1 \gg D$ and $R_2 \gg D$, the force between the spheres can be obtained by integrating the force between the small circular regions $2\pi x\, dx$ as shown above

$$F(D) = \int_{z=D}^{z=\infty} 2\pi x\, dx\, f(z)$$

$f(z)$: normal force per unit area between two flat surfaces.

using chord theorem

$$x^2 \approx 2R_1 z_1 = 2R_2 z_2$$

$$z = D + z_1 + z_2 = D + \frac{x^2}{2}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

thus:

$$dZ = \left(\frac{1}{R_1} + \frac{1}{R_2}\right) x \, dx$$

and

$$F(D) \approx \int_D^\infty 2\pi \left(\frac{R_1 R_2}{R_1 + R_2}\right) f(z)\, dZ =$$

$$= \boxed{2\pi \left(\frac{R_1 R_2}{R_1 + R_2}\right) \underbrace{W(D)}_{}}$$

Notice: independent of the type of potential we use! → energy per unit area for two surfaces at separation D

Derjaguin approximation.

Valid for any force law, provided that the separation distance D, is much larger than the radii of the spheres.

Some special cases:

- $R_2 >> R_1$

$$F(D) = 2\pi R_1 W(D)$$

the same as

- $R = R_1 = R_2$ (two equal spheres).

$$F(D) = \pi R W(D).$$

- two spheres in contact ($D = \sigma$)

the value of $W(\sigma) \Rightarrow$ can be associated with the surface free energy per unit area of the surface γ

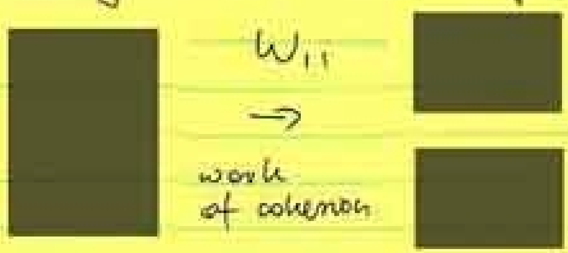
$$W(\sigma) = 2\gamma$$

$$F(\sigma) = F_{ad} = \frac{4\pi \gamma R_1 R_2}{R_1 + R_2}$$

$\uparrow$ adhesion force between two spheres.

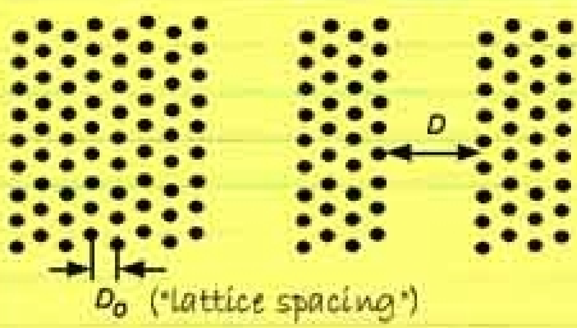
Surface free energy

Consider the cost of separating a body into two pieces



$\gamma_1 = \frac{1}{2} W_{11}$ Surface free energy (per unit area)

We can calculate W_{11} (and thus γ_1) based on our simple pair-wise summation of interaction energies between identical media.



$W = -A/(12\pi D^2)$

Work required to separate two pieces by D:

$$\Delta W = \frac{A}{12\pi}\left(\frac{1}{D_0^2} - \frac{1}{D^2}\right) = \frac{A}{12\pi D_0^2}\left(1 - \frac{D_0^2}{D^2}\right)$$

Separation to infinity $D = \infty$

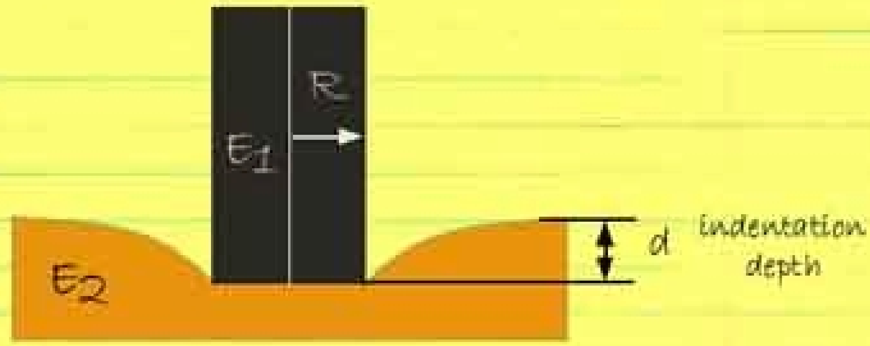
$$\Delta W = W_{11} = \frac{A}{12\pi D_0^2} = 2\gamma$$

$$\boxed{\gamma = \frac{A}{24\pi D_0^2}}$$

Surface free energy as a function of Hamaker constant!

Indentation

Flat cylindrical indenter



E_1, E_2 - Sample and indenter Young's moduli

R - indenter radius.

ν_1, ν_2 - Poisson coefficients

for $E_2 \gg E_1$

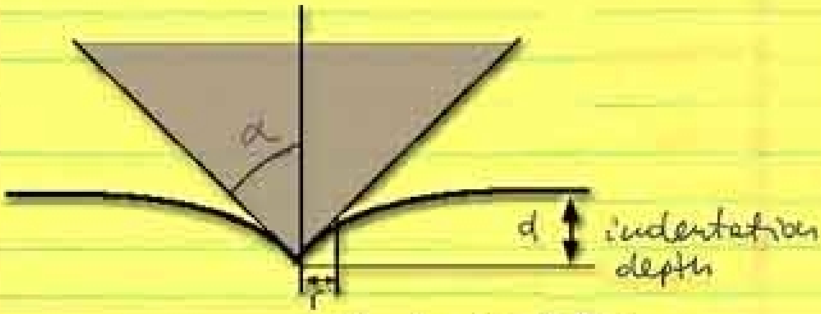
$$F = \frac{2 E_1 R}{1 - \nu_1^2} \, d$$

if E_1 and E_2 are comparable

$$F = \frac{2 \cdot R}{\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}} \, d$$

$$\boxed{d \sim F}$$

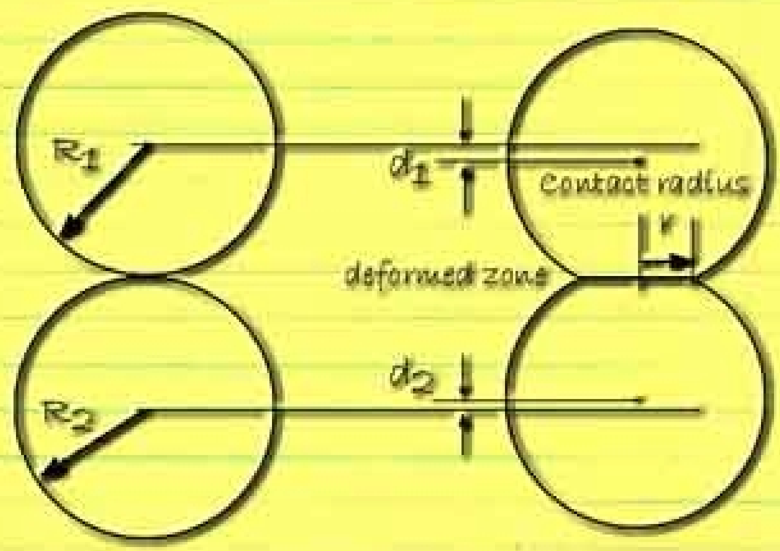
Conical indenter



$$F = \tan\alpha \frac{2E}{\pi(1-\nu^2)} d^2$$

$$\boxed{d \sim F^{1/2}}$$

Sphere - sphere contact (Hertz)



$d = d_1 + d_2$ total deformation

$$K_1 = \frac{1-V_1^2}{E_1} \qquad K_2 = \frac{1-V_2^2}{E_2}$$

$$K_{eff} = K_1 + K_2$$

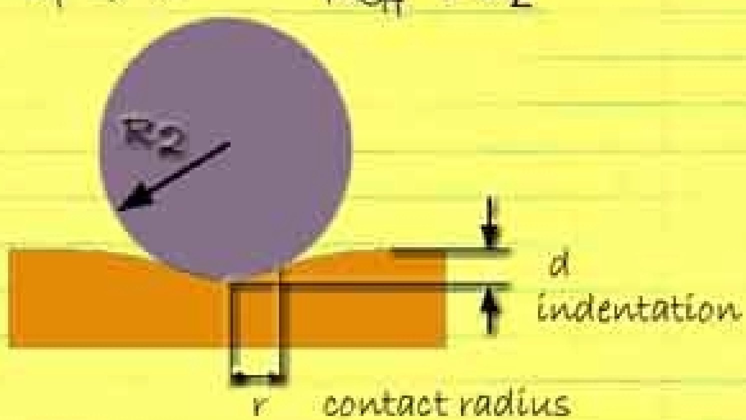
$$R_{eff} = \frac{R_1 R_2}{R_1 + R_2}$$

contact radius:

$$r^3 = \boxed{\frac{3\pi}{4} R_{eff} K_{eff} \cdot F}$$

$$d = \boxed{\frac{r^2}{R_{eff}}}$$

if $R_1 \to \infty$ $\quad R_{eff} = R_2$



indentation vs load in Hertz model:

$$d^3 = \frac{r^6}{R_{eff}^3} = \frac{3\pi}{4} \frac{K_{eff}^2}{R_{eff}} \cdot F^2$$

$$\boxed{d \sim F^{2/3}}$$

distribution of pressure in contact region

$$P(r, x) = \boxed{\frac{2r}{\pi^2 K_{eff} R_{eff}} \sqrt{1-x^2}}$$

where r - contact radius

$x = \frac{\delta}{r}$ normalized distance from the center

δ - distance from the center.

JKR (Johnson-Kendall-Roberts) description of contact: Indentation + Adhesion

$$r^3 = \boxed{\frac{3\pi}{4} R_{eff} K_{eff} \left(F \right.} \underbrace{\left. + q + \sqrt{q^2 + 2qF} \right)}_{\text{Additional load due to adhesion}}$$

$q = 3\pi R_{eff} W_{12}$

$W_{1,2}$ – work of adhesion

If external force $F = 0$

$$r_0 = \left(\frac{R_{eff}}{K_{eff}} 6\pi W_{1,2} \right)^{1/3}$$

Indentation:

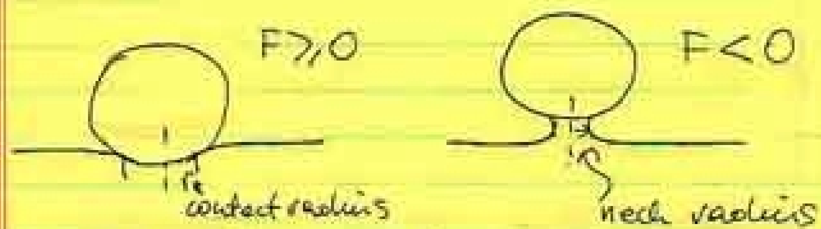
$$d = \boxed{-\frac{r^2}{R_{eff}}} + \frac{2}{3} \frac{r_0^{3/2}}{R_{eff}} r^{1/2}$$

For negative loads $(F < 0)$ contact radius can be still positive, until

$$\sqrt{q^2 + 2qF} > 0.$$

This means that a neck connecting the bodies in contact is formed

Push-in Pull-out



The force at which the neck breaks can be determined from the condition

$$q^2 + 2q F_s = 0$$

$$F_s = \frac{-q}{2} = -\frac{3}{2}\pi R_{eff} W_{12}$$

Neck radius upon breaking

$$r_s^3 = \frac{3\pi}{4} R_{eff} K_{eff} \left(-\frac{q}{2} + q\right) =$$

$$= \frac{3\pi}{4} R_{eff} K_{eff} \frac{q}{2}$$

since $r_0^3 = \frac{3\pi}{4} R_{eff} K_{eff} \cdot 2q$

$$r_s = \frac{r_0}{\sqrt[3]{4}} \approx 0.6\, r_0.$$

Distribution of pressure:

$$P(r,x) = \boxed{\frac{2r}{\pi^2 K_{eff} R_{eff}} \sqrt{1-x^2}} - \frac{\sqrt{\frac{2}{\pi^2} \frac{W_{12}}{K_{eff} \cdot r}}}{\sqrt{1-x^2}}$$