06-713: Homework 7
Due Monday November 26

1. (a) In class, we discussed a model for a chromatography column. Solve this model using the Freundlich adsorption isotherm, $\Gamma = KC^n$ (with $n > 1$) with the following initial/boundary conditions:
$$C = \overline{C}\exp(-ax) \text{ for } 0 \le x \le L,\ t = 0$$
$$C = 0 \text{ for } x = 0,\ t > 0.$$
(b) Repeat the problem above with the following initial/boundary conditions:
$$C = 0 \text{ for } 0 \le x \le L,\ t = 0$$
$$C = \overline{C} \text{ for } x = 0,\ t > 0.$$
Describe what these two sets of initial/boundary conditions mean physically in terms of operating a chromatography column.

2. Devise a numerical example in three dimensions to demonstrate that the multivariable Newton's method is quadratically convergent but the steepest descent method is linearly convergent.

3. Find all minima of $f(x) = e^{-x}\sin x$ for $0 < x < 10\pi$. Use two different iterative methods, and use the numerical results from your calculations to estimate the order of convergence for the two methods.

Optional Problems

1. Consider a fluid flowing with velocity v through a pipe immersed in a bath of constant temperature T_0. The fluid temperature changes according to a heat balance: $\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial x} = U(t)(T_0 - T)$ for $0 < x <$ L and $t > 0$. Here, $U(t) = U_0\exp(-\alpha t)$ is the overall heat transfer coefficient, which decreases with time due to fouling. Determine the evolution of the fluid temperature assuming that it is initially constant and equal to T_i, while the fluid enters the pipe at $T = T_f$.
2. Traffic flow of cars on a road can be described by $\frac{\partial u}{\partial \tau} + (1-2u)\frac{\partial u}{\partial z} = 0$ for $-\infty < z < +\infty$ and $\tau > 0$. In this equation, u is the dimensionless car concentration, τ is a dimensionless time, and $z = x/L$ is a dimensionless space coordinate where L is a reference length. We will consider the situation where a traffic light located at z = 0 turns green when an infinitely long line of cars is waiting at the light. That is, the initial conditions are $u = 1$ for $z < 0$ and $\tau = 0$ and $u = 0$ for $z > 0$ and $\tau = 0$. Determine the concentration of cars, $u(z,\tau)$. Also calculate how long it takes a car initially located at a distance L from the light (i.e. $z = -1$) to reach the light.
3. Greenberg Problems 17.3.5 and 17.3.6. These problems give several concrete examples of evaluating Fourier series.
4. Greenberg Problem 17.3.9.