<u>06-713: Homework 2</u>
Due September 17

<u>Required Problems</u>

1. (a) In class, we derived the so-called normal equations for the least squares fit of a data set to a straight line. But what if we want to fit to the function $y=a+bt+ce^t$, to a collection of *m* experimental data points, $y_i(t_i)$? Show that the least-squares solution of this problem is given by the normal equations and define these equations.
(b) Write down the explicit form of the normal equations for the problem of fitting a collection of experimental data points with a *n*-th order polynomial, $a_0+a_1t+\ldots+a_nt^n$ (you don't have to derive them).
(c) Most mathematical packages include the capability to fit data to polynomials. Generate numerical examples to test whether your favorite package does this by solving a least square problem for linear, quadratic and cubic polynomials (i.e., explicitly solve appropriate least squares problems and compare your results to the package's data fitting results).

2. Consider the two matrices $A=\begin{bmatrix} 1 & 1 & 3 \\ -2 & -3 & 2 \\ 3 & 1 & -2 \end{bmatrix}$ and $B=\begin{bmatrix} 5 & 1 & 2 \\ 0 & 4 & 3 \\ 3 & -2 & -1 \end{bmatrix}$
(a) Calculate the eigenvalues and eigenvectors of A^{-3} without explicitly calculating the matrix A^{-3}. State whether the eigenvectors are linearly independent.
(b) Repeat part (a) by explicitly calculating A^{-3}.
(c) Calculate the eigenvalues and eigenvectors of B^5 without explicitly calculating the matrix B^5. State whether the eigenvectors are linearly independent.
(d) Repeat part (c) by explicitly calculating B^5.

3. Greenberg 10.6.10
4. Greenberg 10.6.13
5. Greenberg 11.2.13
6. Greenberg 11.2.24

Optional Problems

1. This problem illustrates from of the properties that eigenvalues do not have.
   (a) Find two 2 ×2 matrices A and B such that the eigenvalues of AB are not the products of the eigenvalues of A and B.
   (b) Find two 2 ×2 matrices A and B such that the eigenvalues of A+B are not the sums of the individual eigenvalues.
   (c) Can you find matrices for either of the questions above where the given relationship between the eigenvalues does hold?

2. This problem explores properties of transpose matrices.
   (a) If A is a (m × n) matrix and B is a (n × p) matrix, show that $(AB)^T=B^TA^T$. What size matrix is $(AB)^T$?
   (b) If A is a square matrix, show that $(A^{-1})^T=(A^T)^{-1}$.

3. How many numerical operations are required to evaluate the matrix polynomial $c_0I+c_1A + c_2A^2 +\ldots+c_nA^n$ for a square matrix A? What about $c_{-m}A^{-m} + \ldots+c_{-1}A^{-1}+ c_0I+c_1A + c_2A^2 +\ldots+c_nA^n$?

4. Greenberg 10.6.4
5. Greenberg 10.6.7
6. Greenberg 11.2.5 (a)-(f)
7. Greenberg 11.2.7