

## Physical Chemistry of Colloids and Surfaces – Final Exam reference

### Constants:

$$\begin{aligned}
 k_B &= 1.38 \times 10^{-23} \text{ J/K} & k_B T (298\text{K}) &= 4.11 \times 10^{-21} \text{ J} & g &= 9.80 \text{ m/s}^2 \\
 R &= 8.314 \text{ J/mol K} & N_{av} &= 6.02 \times 10^{23} & \bar{V} &= 22414 \text{ cm}^3/\text{mol (STP)} \\
 e &= 1.60 \times 10^{-19} \text{ C} & \epsilon_o &= 8.85 \times 10^{-12} \text{ C}^2/\text{Jm} & \epsilon_r, \text{ water} &= 78 \\
 1 \text{ Debye} &= 3.34 \times 10^{-30} \text{ Cm}
 \end{aligned}$$

### Equations:

$$\frac{P}{P_o} = \exp\left(\frac{-Mgz}{RT}\right) \quad \frac{c_i}{c_{io}} = \exp\left(\frac{\Phi_i}{k_B T}\right) \quad \Phi_i = \frac{4\pi r^3}{3} (\mathbf{r}_p - \mathbf{r}_f) \cdot \mathbf{z} = sfgz$$

$$D = \frac{kT}{6\pi\eta r} \quad v_{sed} = sg; \quad v_{sed} = s\omega^2 r \quad s_{sph} = \frac{4\pi r^3}{3f} (\mathbf{r}_p - \mathbf{r}_f)$$

$$\begin{aligned}
 s &= \frac{V}{f} (\mathbf{r}_p - \mathbf{r}_f) & j &= -D \left( \frac{dc}{dx} \right) & \frac{\partial c}{\partial t} &= D \left( \frac{\partial^2 c}{\partial x^2} \right) \\
 \ln\left(\frac{c_2}{c_1}\right) &= \frac{sg}{D} (x_2 - x_1) & \ln\left(\frac{c_2}{c_1}\right) &= \frac{s\omega^2}{2D} (r_2^2 - r_1^2) & \langle r^2 \rangle &= 6Dt
 \end{aligned}$$

$$P(n, x) dx = \frac{1}{\sqrt{4\pi Dt}} \exp\left(\frac{-x^2}{4Dt}\right) dx$$

$$\mathbf{g} = \left( \frac{\partial G}{\partial A} \right)_{T,P} \quad P_{in} - P_{out} = \Delta P = \mathbf{g} \left[ \frac{1}{R_1} + \frac{1}{R_2} \right] \quad \frac{2\mathbf{g}}{R} = \Delta rgh$$

$$\ln\left(\frac{p_{vap}}{p_{vb}}\right) = \frac{2\mathbf{g}\bar{V}_L}{rRT} \quad \ln\left(\frac{p_{vap}}{p_{vb}}\right) = \frac{2\mathbf{g}\bar{V}_g}{rRT}$$

$$\ln\left(\frac{T}{T_o}\right) = \frac{-2\bar{V}_L \mathbf{g}}{\Delta H_{vap} r} \quad \Delta H_{vap} \left[ \frac{1}{T_o} - \frac{1}{T} \right] = R \ln \left[ \frac{\frac{2\mathbf{g}}{r} - P_l}{P_l} \right]$$

$$\cos\mathbf{q} = \frac{\mathbf{g}_{sv} - \mathbf{g}_{sl}}{\mathbf{g}_{lv}} \quad S = \mathbf{g}_{sv} - \mathbf{g}_{sl} - \mathbf{g}_{lv} \quad P_{in} - P_{out} = \Delta P = \frac{2\mathbf{g} \cos\mathbf{q}}{R}$$

$$\frac{2\mathbf{g} \cos \mathbf{q}}{R} = \Delta \mathbf{r} g h \quad \frac{dV^*}{dR} = \left( \frac{-dV^*}{d\Delta P} \right) \frac{(\Delta P)^2}{2\mathbf{g} \cos \mathbf{q}} \quad F_{cap} \cong 2\mathbf{p} \mathbf{r} \mathbf{g} L$$

$$\ln \left( \frac{p_{vap}}{p_{vb}} \right) = \frac{-2\bar{V}_L \mathbf{g}_{lv} \cos \mathbf{q}}{rRT} \quad d\mathbf{g} = - \sum_i \Gamma_i d\mathbf{m}_i \quad \Gamma_{surf} = \frac{1}{RT} \left( \frac{-\partial \mathbf{g}}{\partial \ln c_{surf}} \right)_T$$

$$\Gamma_{surf} = \frac{1}{2RT} \left( \frac{-\partial \mathbf{g}}{\partial \ln c_{surf}} \right)_T \quad \Gamma = \frac{\Gamma_{max} c}{K' + c} \quad \Pi_s = RT \Gamma_{max} \ln \left( \frac{K' + c}{K'} \right)$$

$$\Gamma(t) = 2c_0 \sqrt{\frac{Dt}{\mathbf{p}}} \quad \Pi_s(t) = RT \Gamma_{max} \ln \left( 1 + \frac{\Gamma(t)}{\Gamma_{max} - \Gamma(t)} \right)$$

$$\frac{1}{V} \left( \frac{x}{1-x} \right) = \left( \frac{C-1}{CV_m} \right) x + \frac{1}{CV_m} \quad x = \frac{P}{P_{vap}} \quad C = \exp \left( \frac{\mathbf{e} - \mathbf{e}_v}{kT} \right)$$

$$V_m = \frac{A_{sp} \bar{V}}{N_{Av} \mathbf{s}^0} \quad N_s = \frac{v}{a_0 l}$$

$$l = 0.154 + 0.1265n \quad [=] \text{ nm}$$

$$v = (27.4 + 26.9n) * 10^{-3} \quad [=] \text{ nm}^3$$

$$V_{ij, ion-ion}(R) = \frac{q_1 q_2}{4\mathbf{p} \mathbf{e}_o R} \quad V_{ij, ion-dipole}(R) = \frac{q_1 m_1 \cos \mathbf{q}}{4\mathbf{p} \mathbf{e}_o R_{12}^2}$$

$$U_{Keesom}(R) = \frac{-2m_1^2 m_2^2}{3(4\mathbf{p} \mathbf{e}_o) kTR^6}; kT > \frac{m_1 m_2}{4\mathbf{p} \mathbf{e}_o R^3} \quad U_{Debye}(R) = \frac{-m^2 \mathbf{a}}{(4\mathbf{p} \mathbf{e}_o) R_{12}^6}$$

$$V_{ij, London}(R) = - \left( \frac{3}{2} \right) \left( \frac{I_1 I_2}{I_1 + I_2} \right) \left( \frac{\mathbf{a}_1 \mathbf{a}_2}{(4\mathbf{p} \mathbf{e}_o)^2 R_{12}^6} \right) \quad \mathbf{m}_i = z_i \mathbf{e} \mathbf{f}(x) + kT \ln c_i^*(x)$$

$$c_i^*(x) = c_{i\infty}^* \exp \left( \frac{-z_i \mathbf{e} \mathbf{f}(x)}{kT} \right) \quad \frac{d^2 \mathbf{f}}{dx^2} = \frac{-\sum_i z_i \mathbf{e} c_i^*(x)}{\mathbf{e}_r \mathbf{e}_o}$$

$$\frac{d^2 \mathbf{f}}{dx^2} = \frac{-e}{\mathbf{e}_r \mathbf{e}_o} \sum_i z_i c_{i\infty}^* \exp\left(\frac{-z_i e \mathbf{f}(x)}{kT}\right) \quad \frac{d\mathbf{f}}{dx}\Big|_{x \rightarrow \infty} = 0; \quad \frac{d\mathbf{f}}{dx}\Big|_{x=0} = -\frac{\mathbf{s}}{\mathbf{e}_r \mathbf{e}_o}$$

$$\mathbf{f}(x) = \frac{2kT}{ze} \ln \left\{ \frac{1 + \mathbf{G}_o \exp(-\mathbf{k}x)}{1 - \mathbf{G}_o \exp(-\mathbf{k}x)} \right\} \quad \mathbf{G}_o = \tanh\left(\frac{ze \mathbf{f}_o}{4kT}\right) \quad \frac{1}{\mathbf{k}} = \sqrt{\frac{\mathbf{e}_r \mathbf{e}_o kT}{\sum_i (z_i e)^2 c_{i\infty}^*}}$$

$$\frac{1}{\mathbf{k}} = \frac{0.304}{\sqrt{c}} (1:1) \quad \frac{1}{\mathbf{k}} = \frac{0.176}{\sqrt{c}} (2:1, 1:2) \quad \frac{1}{\mathbf{k}} = \frac{0.152}{\sqrt{c}} (2:2)$$

$$\mathbf{s} = \sqrt{8kTc_{\infty}^* \mathbf{e}_r \mathbf{e}_o} \sinh\left(\frac{ze \mathbf{f}_o}{2kT}\right) = \frac{2\mathbf{e}_r \mathbf{e}_o kT}{ze} \sinh\left(\frac{ze \mathbf{f}_o}{2kT}\right)$$

$$\mathbf{f}_0 = \frac{2kT}{ze} \arcsin h\left(\frac{\mathbf{s}ze}{2\mathbf{e}_r \mathbf{e}_o kT}\right) \quad \mathbf{f}(x) = \frac{4kT}{ze} \tanh\left(\frac{ze \mathbf{f}_o}{4kT}\right) \exp(-\mathbf{k}x)$$

$$\sinh(x) = \frac{\exp(x) - \exp(-x)}{2} \quad \cosh(x) = \frac{\exp(x) + \exp(-x)}{2} \quad \tanh(x) = \frac{\sinh(x)}{\cosh(x)}$$

$$\mathbf{P}_{os} \cong kTc^* \quad F_{ww}(h) = 64kTc_{\infty}^* \left( \tanh\left(\frac{ze \mathbf{f}_o}{4kT}\right) \right)^2 \exp(-\mathbf{k}h)$$

$$U(h) = - \int_{\infty}^h F(h') dh' \quad F(h) = - \frac{dU(h)}{dx}$$

$$F_{ss}(h) = 2\mathbf{p} \left( \frac{R_1 R_2}{R_1 + R_2} \right) U_{ww}(h) \quad F_{sw}(h) = 2\mathbf{p} R U_{ww}(h)$$

$$U_{ww}(h) = - \frac{H_{12}}{12\mathbf{p}h^2} \quad U_{sw}(h) = - \frac{H_{12}R}{6h} \quad U_{ss}(h) = - \frac{H_{12}}{6h} \left( \frac{R_1 R_2}{R_1 + R_2} \right)$$

$$H_{12} \cong \sqrt{H_{11} H_{22}} \quad H_{131} \cong H_{313} \cong H_{11} + H_{33} - 2H_{13}$$

$$H_{132} \cong \left( \sqrt{H_{11}} - \sqrt{H_{33}} \right) \left( \sqrt{H_{22}} - \sqrt{H_{33}} \right)$$

$$CCC \cong \frac{10^5 (\mathbf{e}_r \mathbf{e}_o)^3 (kT)^5 \mathbf{G}_o^4}{z^6 e^6 H^2} \quad CCC \cong \frac{400 (\mathbf{e}_r \mathbf{e}_o)^3 kT \mathbf{f}_o^4}{z^2 e^2 H^2}$$

$$W = 2R \int_{2R}^{\infty} \frac{1}{h^2} \exp\left(\frac{U(h)}{kT}\right) dh$$

$$F_{ww}\left(\frac{h}{d_m}\right) = -kT c_{solv, \infty}^* \cos\left(\frac{2ph}{d_m}\right) \exp\left(\frac{-h}{d_m}\right)$$

$$R_g = \sqrt{\frac{Ll_p}{6}} \propto M_w^{0.5} \quad R_g \propto M_w^{0.6}$$

$$R_g \propto M_w^{1/3}$$

$$F_{ww}(h) \cong \frac{kT}{s^3} \left[ \left(\frac{2L}{h}\right)^{9/4} - \left(\frac{h}{2L}\right)^{3/4} \right]; \quad h < 2L$$

$$U_{ww}(h) = C_o \exp\left(-\frac{h}{I_0}\right)$$

$$u = \frac{v}{E} = \frac{q}{f} = \frac{C \mathbf{e}_r \mathbf{e}_0 \mathbf{z}}{\mathbf{m}}$$

$$v(r) = \frac{DP}{4\mathbf{m}} (R^2 - r^2)$$

$$I_{str} = \frac{\mathbf{p} \mathbf{e}_r \mathbf{e}_0 DP R^2 \mathbf{z}}{\mathbf{m}}$$

$$\mathbf{f}_{str} = \frac{\mathbf{e}_r \mathbf{e}_0 \mathbf{z} DP}{\mathbf{m} k_b}$$

$$Q = \frac{\mathbf{e}_r \mathbf{e}_0 I_{eo} \mathbf{z}}{\mathbf{m} k_b}$$

$$Q = \frac{\mathbf{e}_r \mathbf{e}_0 \mathbf{z} p R^2 \mathbf{f}_{eo}}{\mathbf{m}}$$

$$I_b = k_b EA$$

$$k_{eff} = k_b + \frac{2k_s}{R}$$