1. Prove that in a CCC $C$, exponentiation is a functor $C^{op} \times C \to C$. (Hint: Use the bifunctor lemma and reason with the internal $\lambda$-calculus of $C$.)

2. Show that the functions between sets, $\eta_A : A \to PP(A)$,

$$\eta_A(a) = \{ U \subseteq A \mid a \in U \}$$

are the components of a natural transformation $\eta : 1_{\text{Sets}} \to PP^{op}$, where $P : \text{Sets}^{op} \to \text{Sets}$ is the contravariant powerset functor.

3. Recall that a groupoid is a category in which every arrow is an isomorphism. Prove that the category of (small) groupoids is cartesian closed. (Hint: You may use that $\text{Cat}$ is cartesian closed.)

4. ($\ast$) Suppose that $G$ and $H$ are groups, regarded as 1-object categories, and that $S, T : G \to H$ are functors (i.e., group homomorphisms). Prove that a natural transformation $S \to T$ is the same as an element $h \in H$ such that $T(g) = h \cdot S(g) \cdot h^{-1}$ for every $g \in G$. 

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Starred problems are for students enrolled in 80-713