Hashing (20 Points)

8 (a) Build a hash table using open addressing and linear probing. If the table size is 7 and the hash function is

\[ h(z) = z \mod 7, \]

show what the table looks like after inserting 7, 14, 22, 21, and 9 one after another. (3 Points)

\[
\begin{array}{cccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 \\
7 & 14 & 22 & 21 & 9 & & & \\
\end{array}
\]

(b) Insert the same keys into the table using double hashing to resolve collisions. Let the primary hash function be \( h_1(z) = z \mod 7 \) and the secondary hash function be \( h_2(z) = 1 + (z \mod 5) \). (5 points)

\[
\begin{array}{cccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 \\
7 & 22 & 21 & 9 & 14 & & & \\
\end{array}
\]

(c) In class we discussed the ordered hashing (tough schoolboy) hashing scheme. Assuming that boys later in the alphabet are tougher than boys earlier in the alphabet and that each boy has a preferred seat as well as a jump index (secondary hash), how will this list of boys end up in the following hash table? (3 Points)

\[
\begin{array}{cccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 \\
M & D & Mii & & A & B & & \\
\end{array}
\]

(d) Suppose we are using a very simple open address hashing scheme. The idea is to compute the hash and then try to insert into the table. If there is a collision, we simply check the next available slot. This is the way Michael Main’s example worked in class. Recall the Baby and John Kennedy and Martin Luther King.

Suppose the hash table is of size 100. Suppose to that only three values are in the table. As luck would have it, they are stored at positions 45, 46 and 47. Assuming we have a good hash function, what is the probability that the next value arriving will end up at position 48? (5 Points) \[ \frac{1}{100} \]

(e) Consider again the tough schoolboy hashing of part (c). Describe the important relationship that exists between the secondary hash and the table size? \[ \text{They are relatively} \] (4 Points)