Roller Coaster Physics: Energy and Forces

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Outline

- Energy, and how to design with it
- Forces, and how to design with them
- Example Design in No Limits
- Some other miscellaneous calculations
Energy
Energy

- Many types of energy, but only three we care about
  - Potential Energy
  - Kinetic Energy
  - Thermal Energy (friction)
- Allows us to relate **height** and **speed**
- Equations derived from these

\[ KE = \frac{1}{2} mv^2 \quad PE = g \times m \times h = 9.8 \times m \times h \]

- Mass ends up not mattering
- Our equations ignore friction (what does this mean?)
Velocity after Height Change

\[ v_f = \sqrt{v_o^2 + 2 \times 9.8 \times h} \]

- \( v_f \) is the final velocity (in m/s)
- \( v_o \) is the initial velocity (in m/s)
- \( h \) is the change in height (in m) where positive means a decrease in height
Example - Nitro

Nitro at Six Flags Great Adventure, a B&M hyper coaster, has a first drop of 215 feet. We estimate that it is going 7 mph at the top of the hill. We want to calculate the speed at the bottom of the first drop.
Example - Nitro

First, convert units

$$215 \text{ feet} / 3.28 = 65.5 \text{m}$$
$$7 \text{ mph} \times 0.4472 = 3.1 \text{m/s}$$

Plug in the values

$$v_f = \sqrt{(3.1)^2 + 2 \times 9.8 \times 65.5} = 36.0$$

Finally, convert units back

$$36.0 \text{m/s} / 0.4472 = 80.5 \text{mph}$$
Example - Crystal Beach Cyclone

The Crystal Beach Cyclone has a first drop of 96 feet. We estimate that it is going 6 mph at the top of the hill. We want to calculate the speed at the bottom of the first drop.
Example - Crystal Beach Cyclone

First, convert units

\[
\frac{96 \text{ feet}}{3.28} = 29.3 \text{m}
\]
\[
6 \text{ mph} \times 0.4472 = 2.7 \text{m/s}
\]

Plug in the values

\[
v_f = \sqrt{(2.7)^2 + 2 \times 9.8 \times 29.3} = 24.1
\]

Finally, convert units back

\[
\frac{24.1 \text{m/s}}{0.4472} = 53.9 \text{mph}
\]
Height Change Based on Target Velocity

\[ h = \frac{v_f^2 - v_o^2}{2 \times 9.8} \]

- \( h \) is the change in height (in m) where positive means a decrease in height
- \( v_f \) is the final velocity (in m/s)
- \( v_o \) is the initial velocity (in m/s)
Example - Top Thrill Dragster

Top Thrill Dragster at Cedar Point, an Intamin Accelerator coaster, has an initial velocity of 120 mph. After the hill, we want the coaster to be going 30 mph. What should the change in height be?
Example - Top Thrill Dragster

First, convert units

\[ 120 \text{ mph} \times 0.4472 = 53.7\text{ m/s} \]
\[ 30 \text{ mph} \times 0.4472 = 13.4\text{ m/s} \]

Plug in the values

\[ h = \frac{(13.4)^2 - (53.7)^2}{2 \times 9.8} = -138.0 \]

Finally, convert units back

\[ -138.0\text{ m} \times 3.28 = -453\text{ feet} \]
Steel Dragon 2000 at Nagashima Spa Land, a Morgan giga coaster, has a initial velocity of 8 mph. After the hill, we want the coaster to be going 95 mph. What should the change in height be?
Example - Steel Dragon 2000

First, convert units

\[ 8 \text{ mph} \times 0.4472 = 3.6 \text{m/s} \]
\[ 95 \text{ mph} \times 0.4472 = 42.5 \text{m/s} \]

Plug in the values

\[ h = \frac{(42.5)^2 - (3.6)^2}{2 \times 9.8} = 91.5 \]

Finally, convert units back

\[ 91.5 \text{m} \times 3.28 = 300 \text{feet} \]
Forces
Forces

- We say force but really mean acceleration (change in velocity)
- Unit of use is “G-Force”, which is acceleration in terms of Earth’s gravity
- Applies in all curves (turns, hills, loops, etc)
- They make coaster exciting; very important
G-Forces - Directions

- Positive G’s (Eyes Down) - Pushes you down in your seat; bottom of a hill
- Negative G’s (Eyes Up) - Lifts you out of your seat; top of a hill
- Lateral G’s (Eyes Left/Right) - Pushes you to your side; turn with no banking
- Acceleration G’s (Eyes Back) - Pushes you back in your seat; launch
- Deceleration G’s (Eyes Front) - Train slows and you keep going forward; brakes
G-Forces - Safety

- Can't measure safety just by g-force; also need duration
  - 10Gs for a sustained time could easily kill you
  - When you sit down suddenly, you might temporarily feel 10Gs
- Too many positive G’s cause blood to leave head
  - Tunnel vision, grey-out
  - Black out (G-LOC)
  - Brain cells start dying (lack of oxygen)
- Too many negative G’s cause blood to enter head
  - “Red out”
  - Blood vessels in brain burst from pressure; stroke
  - Much worse
- Extreme G testing is actually pretty interesting
G-Force - Safety

Recommended Limits - Positive G’s

Figure 9 Time Duration Limits for +Gz (Eyes Down)

Note:
Design must assure that Patron is properly seated in upright position

+Gz Limits if preceded by 3 sec or more of −Gz

Duration (sec)

Acceleration (G)
G-Force - Safety

Recommended Limits - Negative G’s

Figure 8 Time Duration Limits for -Gz (Eyes Up)

Note:
1) Design must assure that Patron is properly restrained in accordance with Section 6 of this practice.
2) Extended -Gz environments. Rides designed to operate in this extended -Gz range require special restraints, which must be addressed in the Ride Analysis.
G-Force - Safety

Recommended Limits - Lateral G’s

Figure 7 Time Duration Limits for +/-Gy (Eyes Left or Eyes Right)

Note:
For bench seat rides (e.g., without individual patron retention or seat dividers) with sustained lateral accelerations greater than 0.7 g, patron seating order must be from smaller patron to larger patron in the direction of the load.
G-Force - Safety

Recommended Limits - Acceleration G’s

Figure 5 Time Duration Limits for $+G_x$ (Eyes Back)

Note:
1) Must have headrest above 1.5 g unless onset rate is less than 5 g/sec; then 2.0 g is permissible. For no headrest case, max duration of above 1.5 g is 4 seconds.
2) Design and operating procedures must assure patron is in contact with and supported by appropriate backrest and headrest.
G-Force - Safety

Recommended Limits - Deceleration G’s

Figure 6 Time Duration Limits for $-G_x$ (Eyes Front)

Note:
1) Over-The-Shoulder restraint must minimize patron forward motion.
2) Over-The-Shoulder limits may be increased to Prone limits providing the onset rate is less than 15 g/sec and the restraint is appropriately padded.
3) Prone Restraint assumes body is supported by appropriately padded restraint.

Duration (sec)
# G-Forces - Recommended Limits by me

<table>
<thead>
<tr>
<th>G-Force Type</th>
<th>Steel</th>
<th>Wood</th>
</tr>
</thead>
<tbody>
<tr>
<td>Positive</td>
<td>3-4</td>
<td>2-3</td>
</tr>
<tr>
<td>Negative</td>
<td>-1 - -1.5</td>
<td>-1 - -1.5</td>
</tr>
<tr>
<td>Lateral</td>
<td>0</td>
<td>1 - 1.25</td>
</tr>
<tr>
<td>Acceleration</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Deceleration</td>
<td>1.5</td>
<td>1.5</td>
</tr>
</tbody>
</table>
G-Force

Note that not everyone agrees on force terminology.
Force in a Curve

\[ a = \frac{v^2}{r} \quad \text{or} \quad g \cdot 9.8 = \frac{v^2}{r} \]

- \( r \) is the radius of the curve (in m)
- \( v \) is the velocity around the curve (in m/s)
- \( a \) is the acceleration (in m/s\(^2\))
- \( g \) is g-force
Ideal Banking (No Lateral Forces)

\[ b = \tan^{-1}\left(\frac{l}{p}\right) \]

- \( b \) is the banking angle (where 0 is no banking)
- \( l \) is the lateral force
- \( p \) is the positive force
No Limits Design Example
Capacity

\[ c = \left( \frac{3600}{d} \right) \times r \times n \]

- \( c \) is capacity (in rph, or riders per hour)
- \( d \) is duration (in seconds)
- \( r \) is the number of riders per train
- \( n \) is the number of trains