The Effect of Store Financing and Consumer Impatience on Timing of New Products*

Elif Incekara-Hafalir
Adjunct Assistant Professor of Economics
Tepper School of Business, Carnegie Mellon University
5000 Forbes Ave, PA 15213, USA
E-mail: incekara@cmu.edu

January 29, 2013

Abstract

We study the monopolist’s timing of product introductions when the consumer is time inconsistent and has the option to finance through the store. We investigate whether the frequency of product introductions is affected by store financing. We show that this frequency does not change if the consumer is time consistent. If the consumer is time inconsistent, however, the monopolist may offer new products more often. Additionally, time inconsistency of the consumer is not the only factor affecting the frequency; the consumer’s awareness of his time inconsistency also matters.

JEL classification: D21, D42, L11

*I would like to thank Sophie Bade, Susanna Esteban, and Ed Green for their comments and suggestions. I am very indebted to Kalyan Chatterjee for his valuable guidance and advice.
1 Introduction

We study the monopolist’s timing of product introductions when the consumer is time-inconsistent and has the option to finance through the monopolist. As an example, we can consider credit cards that are issued by the stores and can only be used for the purchases made from that store. Chakravorti (2000) reports that sales generally increase with the adoption of store credit cards. This is mainly because the store cards, like other bank credit cards, increase the consumer’s ability to pay with financing. We investigate whether, in addition to sales, the frequency of product offerings is affected by store financing.\(^1\) We show that this frequency may be affected by the consumer’s level of time inconsistency as well as his awareness of it. Specifically, for some parameter values, we demonstrate that it may be optimal for the monopolist to offer new products every period when facing a naive consumer (who is not aware of his time inconsistency), but to offer new products every other period when facing a sophisticated consumer (who is aware of his time inconsistency), even if these consumers have the same discount factors.

In our model, there is a time-consistent monopolist and a time-inconsistent consumer.\(^2\) The monopolist decides whether to offer new products every period or every other period. He pays a fixed cost to create a new product as well as a variable cost of production. In the first period, the monopolist offers the consumer a financing option together with the product. The consumer decides whether to buy the product as well as whether to delay payment by paying interest. The consumer cannot delay payment later than the third period and is not allowed to default.\(^3\) In this environment, the total amount a naive time-inconsistent consumer ends up paying for the product may be higher than he expected at the first period, because of his naivete. Therefore, the monopolist can extract more revenue from a naive time-inconsistent consumer than a sophisticated one, and offering the products every period is more likely to cover the fixed cost. As a result, it might be optimal for the monopolist to offer new products every period for a naive consumer and

\(^1\)There is plenty of anectodal evidence, in different user forums, suggesting that Dell, which is one of the electronic stores offering store financing, rushes the products to the market (Notebook Review Forum 2004).

\(^2\)In the literature, time-inconsistent preferences were first explored by Strotz (1956). There is now a large body of evidence that consumers’ preferences are time inconsistent (e.g. Thaler (1981), Frederick et al. (2002).)

\(^3\)The minimum number of periods to observe the effects of time inconsistency is three.
every other period for a sophisticated consumer.

There are two lines of literature related to this paper. The first one is on the timing of product introductions. In a variety of markets, such as software and personal electronics, consumers commonly complain about rushed products. These complaints partially motivate the literature on the timing of new products. Some researchers investigate the link between the timing of product introductions and competition. Cohen et. al. (1996), Bayus (1997) and Bayus et al. (1997) investigate a firm’s decision on timing and performance level of a product in a competitive environment. Lippman and Mamer (1993) show that the quality of a product decreases, implying a shorter product-development stage, with the number of firms in the market. Other papers look at the timing of durable good upgrades. Ellison and Fudenberg (2000) analyze the reasons for a monopoly supplier of software to offer upgrades more often than the social optimum when the upgrades are backward compatible. Fishman and Rob (2000) investigate a monopolist’s product innovation decision under different conditions. The other line of literature related to our paper is on the interaction of time-consistent and time-inconsistent agents. DellaVigna and Malmendier (2004), Eliaz and Spiegler (2006), Laibson and Yariv (2007) and Heidhues and Köszegi (2010) investigate whether time-consistent agents can extract a higher profit from time-inconsistent consumers by offering exploitative contracts.

In our paper we are interested in the monopolist’s launching time of non-durable products depending on financing options and consumer characteristics, specifically the consumer’s impatience. To the best of our knowledge, this is the first study considering the time-to-launch decision of a time-consistent monopolist who faces time-inconsistent consumers.

2 The Model

There are two agents in the model: a time-consistent profit-maximizing monopolist and a time-inconsistent consumer. The monopolist offers new prod-
ucts at a certain frequency either every period or every other period. The production side of our model is based on Fishman and Rob (2000). The monopolist invests in R&D input, denoted by $x$, to create new products with a strictly increasing, concave production function $v(x)$ that satisfies standard Inada conditions. The cost of R&D input to create a new product is given by a linear function $c(x)$. The monopolist also incurs a fixed cost $F$ every time he introduces a product, and we interpret this as an advertisement cost. Fishman and Rob (2000) interpret $F$ as the cost of harnessing new knowledge to the present product. The monopolist discounts the future exponentially at a rate $\delta$, sells the new products at price $p$, and allows the consumer to delay payments by the third period with a per period interest of $r$.

The consumer makes two main decisions. One is whether to buy the offered product. The other is whether to delay payment for the product. If the consumer decides to buy a new product today, he will buy the same quality subsequent products every time they are offered later. In other words, our problem is stationary. We use quasi-hyperbolic discounting to model this time-inconsistent consumer, following Phelps and Pollak (1968) and Laibson (1997). A naive quasi-hyperbolic consumer (a "naive consumer" from here on) is not aware of his time inconsistency. Specifically, he knows that his future discounting today is $\{1, \beta \delta, \beta \delta^2, \beta \delta^3, \ldots\}$, and believes that from tomorrow onward it will be $\{1, \delta, \delta^2, \delta^3, \ldots\}$, although in reality it will be $\{1, \beta \delta, \beta \delta^2, \beta \delta^3, \ldots\}$. A sophisticated quasi-hyperbolic consumer (a "sophisticated consumer" from here on) is aware of his time inconsistency. He knows that his future discounting today is $\{1, \beta \delta, \beta \delta^2, \beta \delta^3, \ldots\}$, and he correctly anticipates that it will be $\{1, \beta \delta, \beta \delta^2, \beta \delta^3, \ldots\}$ from tomorrow onward.

We analyze this model first by assuming that the interest rate is endogenous and then with competitive financial markets implying a fixed interest rate. In each of these cases, we study an exponential consumer as a benchmark case. Then we study a naive and a sophisticated hyperbolic consumer, and we compare them. We focus on a pure strategy subgame perfect equilibrium.

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7 This is not a crucial assumption; It is made just to simplify calculations. We suspect that our results will hold with convex cost functions as well. In fact, we have an example in which the results hold for a strictly convex cost function.

8 From here on, we will use "time-inconsistency" and "impatience" interchangeably to refer to the present-biased preferences modeled with $\beta - \delta$ discounting.
3 Analysis with Endogenous Interest Rate

In this section, we assume that the monopolist chooses the optimal interest rate for delayed payments and we then determine the frequency of new product offerings for each type of consumer. Since we want to analyze the effect of impatience (especially when the hyperbolic consumer’s behavior deviates from the exponential consumer’s behavior) we will assume that $\delta(1 + r) \geq 1$ and $\beta\delta(1 + r) < 1$, so that the exponential consumer will not delay payment of the product although the hyperbolic consumer may delay.\footnote{The other cases do not create any interesting discrepancy between the exponential and hyperbolic consumers’ choice of delay in payments.}

The benefit of offering new products every period is that every period earns revenue for the monopolist. On the other hand, the benefits of offering new products every other period is twofold. First, there is a smaller per unit cost, as the monopolist can optimally spread the cost over two periods. Second, the monopolist pays the fixed cost only every other period. Depending on the production-side parameters, $\delta$ and $F$, the monopolist offers new products every period or every other period. The higher $\delta$ promotes every-period introductions although the higher $F$ promotes every-other-period introductions. This is because the higher values of $\delta$ decreases the advantage of spreading the cost over two periods, and the higher fixed cost makes it more costly to produce every period. Additionally, if the consumer is hyperbolic, the discount factor $\beta$ also affects the frequency of the product introductions. The lower values of $\beta$ increase the consumer’s willingness to pay for a product, as the delayed payments will be discounted more. In this paper, we restrict the $\delta$ parameter to high enough values so that the monopolist will offer new products every period to an exponential consumer only for some lower values of $F$ and every other period for the higher values of $F$. The reason for this is that we want to investigate how the consumer’s impatience (captured by $\beta$) expedites the frequency of product introductions.

In the following lemma, we find the optimal price and the monopolist’s revenue for each type of consumer. Afterward, we determine the payoffs in case of every-period and every-other-period introductions. Then, in the subsequent propositions, we show that there are some parameter values for which the monopolist offers new products every other period for an exponential consumer but offers them every period for hyperbolic consumers.
Lemma 1 Let $\delta(1 + r) \geq 1$ and $\beta \delta(1 + r) < 1$. The optimal price the monopolist charges and the revenue for each type of consumer are:

- $p = v(x)$ and $R = v(x)$ if the consumer is exponential
- $p = v(x) \frac{1}{\beta(1+r)}$ and $R = v(x) \frac{(1+r)}{\beta}$ if the consumer is naive hyperbolic
- $p = v(x)$ and $R = v(x)$ if the consumer is mildly impatient sophisticated hyperbolic with $\beta \geq \beta$
- $p = v(x) \frac{1}{\beta^{2}(1+r)^{2}}$ and $R = v(x) \frac{1}{\beta}$ if the consumer is heavily impatient sophisticated hyperbolic with $\beta < \beta$.

Proof. See the Appendix.

The monopolist’s payoffs if he offers new products every period and every other period for each consumer type $i = \{\text{exponential, naive, mildly impatient sophisticated, heavily impatient sophisticated}\}$ are:

- $\text{payoff }_{i,1} = \frac{1}{1-\delta} [v(x^*) A_i - c(x^*) - F]$
- $\text{payoff }_{i,2} = \frac{1}{1-\delta} [v(y^*) A_i - \delta c(y^*) - F]$

such that $A_i$ is 1 if the consumer is exponential, $\frac{(1+r)}{\beta}$ if the consumer is naive hyperbolic, 1 if the consumer is mildly impatient sophisticated hyperbolic ($\beta \geq \beta$), and $\frac{1}{\beta}$ if the consumer is heavily impatient sophisticated hyperbolic ($\beta < \beta$).

Recall that lower values of fixed cost increase the advantage of producing every period. Next, we show that there exists a cutoff fixed-cost value such that the monopolist offers new products every period if the fixed cost is lower than the cutoff and every other period if the fixed cost is higher than the cutoff.

Proposition 1 There exists a cutoff $F^*_{\text{exp}}(\delta)$ such that the monopolist offers new products to exponential consumers every period if $F \leq F^*_{\text{exp}}(\delta)$ and offers them every other period if $F > F^*_{\text{exp}}(\delta)$ for high enough values of $\delta$.

Proof. See the Appendix.

As we explained before, the $\beta$ discount factor as well as $\delta$ and $F$ affects the frequency of the offers. In the following propositions, we demonstrate how the cutoff $F^*$ changes with $\beta$ for naive and sophisticated hyperbolic consumers.
Proposition 2 There exists an \( F^*_{\text{naive}}(\delta, \beta) \) such that the monopolist offers new products every period for a naive hyperbolic consumer if \( F \leq F^*_{\text{naive}}(\delta, \beta) \) and every other period if \( F > F^*_{\text{naive}}(\delta, \beta) \). Moreover, \( \frac{\partial F^*_{\text{naive}}(\delta, \beta)}{\partial \beta} < 0 \) and \( F^*_{\text{exp}}(\delta) < F^*_{\text{naive}}(\delta, \beta) \).

Proof. See the Appendix. ■

This proposition explains that the monopolist offers a new product more frequently to a naive hyperbolic consumer than to an exponential consumer for some parameter values (e.g. \( F^*_{\text{exp}}(\delta) < F < F^*_{\text{naive}}(\delta, \beta) \)). This is because a naive hyperbolic consumer ends up paying more than he was originally willing to pay, which increases the monopolist’s revenue from each transaction. Consequently, introducing a new product is more likely to cover the fixed cost.

Proposition 3 There exists an \( F^*_{\text{sophis-mild}}(\delta, \beta) \) such that the monopolist offers new products every period for the mildly impatient sophisticated consumer (with \( \beta \geq \beta \)) if \( F \leq F^*_{\text{sophis-mild}}(\delta, \beta) \) and every other period if \( F > F^*_{\text{sophis-mild}}(\delta, \beta) \). Moreover, \( F^*_{\text{exp}}(\delta) = F^*_{\text{sophis-mild}}(\delta, \beta) \).

Proof. See the Appendix. ■

This proposition says that the frequency of product innovations for the mildly impatient sophisticated consumer is the same as the frequency for the exponential consumer. This is intuitive, because the mildly impatient sophisticated consumer does not delay his payment and does not pay interest, and consequently, the monopolist can extract the same payoff from him as from the exponential consumer.

Proposition 4 There exists an \( F^*_{\text{sophis-heavy}}(\delta, \beta) \) such that the monopolist offers new products every period for the heavily impatient sophisticated consumer (with \( \beta < \beta \)) if \( F \leq F^*_{\text{sophis-heavy}}(\delta, \beta) \) and every other period if \( F > F^*_{\text{sophis-heavy}}(\delta, \beta) \). Moreover, \( F^*_{\text{exp}}(\delta) < F^*_{\text{sophis-heavy}}(\delta, \beta) < F^*_{\text{naive}}(\delta, \beta) \) and \( \frac{\partial F^*_{\text{sophis-heavy}}(\delta, \beta)}{\partial \beta} < 0 \).

Proof. See the Appendix. ■

This proposition says that the monopolist offers a new product more frequently to the heavily impatient sophisticated consumer than to the exponential consumer for some parameter values (e.g. \( F^*_{\text{exp}}(\delta) < F < F^*_{\text{sophis-heavy}}(\delta, \beta) \)). This is because an arbitrage opportunity emerges as the monopolist is more
patient than the hyperbolic consumer (the hyperbolic consumer discounts the payment with an additional factor of $\beta$). Note that this effect exists for the naive hyperbolic consumer as well. As an additional factor, however, the naive hyperbolic consumer pays more than he was originally willing to pay. This is why the relation between the cutoff fixed cost values turns out to be $F^*_{\text{naive}}(\delta, \beta) > F^*_{\text{sophis-heavy}}(\delta, \beta) > F^*_{\text{exp}}(\delta)$. This finding is interesting because it tells us that the frequency of new product introductions not only depends on the level of the consumer’s impatience but also on the consumer’s awareness of his impatience.

Figure 1 illustrates the frequency of the monopolist’s product introductions depending on parameter values $\beta$ and $F$ for high-enough $\delta$ and the type of the consumer. In the diagram, lines a, b, and c represent $F^*_{\text{naive}}(\delta, \beta)$ (or $F^*_{\text{sophis–mild}}(\delta, \beta)$), $F^*_{\text{sophis–heavy}}(\delta, \beta)$, and $F^*_{\text{exp}}(\delta)$ respectively.\footnote{The figure is for expositonal purposes and does not reflect the exact shape of the lines b and c other than decreasing and not crossing each other.} The monopolist offers the new products every other period on the right-hand side of the lines and every period on the left-hand side of the lines for the corresponding types of consumers. If the parameter values are on the left-hand side of line a, the monopolist introduces products every period for all types of consumers, as the fixed cost is very low. If the parameter values are on the right-hand side of line c, the monopolist offers new products every other period for all types of consumers, since the fixed cost is too high. If the parameter values are between line a and line b (for $\beta < \frac{1}{\delta^2(1+r)^2}$), the monopolist offers new products every period for naive hyperbolic consumers, although he offers them every other period for exponential consumers, because of the arbitrage opportunity created by different discount rates for monopolist and consumer. If the parameter values are between line b and line c (for $\beta < \frac{1}{\delta^2(1+r)^2}$), the monopolist offers new products every period for naive hyperbolic consumers, although he offers them every other period for heavily impatient sophisticated consumers and exponential consumers because of the naive consumer’s overpayment for the product.

4 Exogenous Interest Rate

In this section we analyze the case in which there is a competitive financial market, implying a fixed interest rate of $\frac{1-\delta}{\delta}$. Since the interest rate affects
the monopolist’s problem only if the consumer is naive hyperbolic, the monopolist’s maximization problem for the other cases is the same. If \( r = \frac{1-\delta}{\delta} \), then \( A = \frac{1}{\beta} \) for the naive consumer. Since \( \frac{\partial A}{\partial \beta} < 0 \), Proposition 2 and Proposition 4 hold for this case as well. As a consequence, we get the same result as in the previous section.

5 Welfare Analysis

We determine that the monopolist may offer new products every period although offering them every other period might be socially better. We evaluate the consumer’s welfare with long-term preferences with \( \beta = 1 \) as in O’Donoghue and Rabin (1999) and Della Vigna and Malmendier (2004).

**Proposition 5** If the consumer is naive, the monopolist offers new products every period even though offering them every other period is socially better for some values of \( F \) and \( \beta \).

**Proof.** See the Appendix.

**Proposition 6** If the consumer is heavily impatient sophisticated hyperbolic, the monopolist offers new products every period even though offering them every other period is socially better for some values of \( F \) and \( \beta \).

**Proof.** See the Appendix.

6 Conclusion and Discussion

In this paper, we demonstrate that the monopolist may introduce products more frequently if the consumer is impatient and if there is a store financing option. In our model, we allow the consumer to delay payment for the product by paying interest, which creates an opportunity for the monopolist to benefit from the consumer’s impatience. The benefit for the monopolist emerges in two ways. One is the increased revenue coming solely from the difference between the discount factors for the monopolist and the consumer. The other is because of the naive consumer’s unrecognized overpayment for the product. The increased revenue because of one or both of these factors makes frequent product introduction more likely to cover the fixed cost.
One can also analyze the frequency of product introductions if the only financing option available for the consumer is through a bank. In that case, the only factor that increases the revenue for the monopolist is the difference between the discount factors for the monopolist and the consumer, since the naive consumer’s unrecognized overpayment will benefit the bank rather than the store. In that case, the frequency of product introductions would be the same for naive and sophisticated hyperbolic consumers, and may be higher than the frequency of product introductions for exponential consumers. Relatedly, the timing of product introductions may be more frequent if a naive consumer finances through the store rather than through a bank.


### Appendix

#### Proof of Lemma 1.

The consumer buys the new product only if the utility he receives is greater than or equal to his total payment. The monopolist gets the entire surplus. Since \(1 \geq \delta(1 + \rho)\) and \(\beta \delta(1 + \rho) < 1\), we find the optimal price and revenue as follows:

- **Exponential consumer:**
  \[ p = v(x) \Rightarrow R = v(x) \]

- **Naive hyperbolic consumer:**
  \[ p\delta(1 + \rho) = v(x) \Rightarrow p = v(x) \frac{1}{\beta \delta(1 + \rho)} \Rightarrow R = v(x) \frac{(1 + \rho)}{\beta} \]

- **Sophisticated hyperbolic consumer with \(\beta \geq \beta^*\)**
  \[ p = v(x) \Rightarrow p = v(x) \text{ and } R = v(x) \]

- **Sophisticated hyperbolic consumer with \(\beta < \beta^*\)**
  \[ \beta \delta^2(1 + \rho)^2 p = v(x) \Rightarrow p = v(x) \frac{1}{\beta} \text{ and } R = v(x) \frac{1}{\beta} \]

#### Proof of Proposition 1.

The difference of the payoffs if they are offered every period and every other period:

\[
\text{payoff}_2 - \text{payoff}_1 = \frac{1}{1 - \delta^2} \left[ \left( Av(y^*) - \delta c(y^*) \right) - \left( (1 + \delta) (Av(x^*) - c(x^*)) + \delta F \right) \right]
\]

where \(A = 1\). The limit of \(y^*(\delta)\) is equal to the limit of \(x^*(\delta)\) as \(\delta\) goes to 1. At this limit, \([-Av(y^*) - \delta c(y^*)] (1 + \delta) (Av(x^*) - c(x^*))\] is equal to \([Av(x^*) + c(x^*)]\) since \(y^*(\delta)\) and \(x^*(\delta)\) are continuous in \(\delta\). We know that \([-Av(x^*) + c(x^*)]\) is negative as the value function, \(v(.)\), is concave and \(v'(x^*) = \frac{c'(x^*)}{A}\). As a result, there exists a cutoff \(F^*_\text{exp}(\delta) > 0\):

\[
F^*_\text{exp}(\delta) = -\frac{1}{\delta} \left[ (Av(y^*) - \delta c(y^*)) - (1 + \delta) (Av(x^*) - c(x^*)) \right]
\]

such that \(\text{payoff}_2 - \text{payoff}_1 \leq 0\) if \(F \leq F^*_\text{exp}(\delta)\) and \(\text{payoff}_2 - \text{payoff}_1 > 0\) if \(F > F^*_\text{exp}(\delta)\) for high enough values of \(\delta\).

#### Proof of Proposition 2.

Recall that the monopolist’s payoff functions increase with \(A\), which is equal to \(\frac{(1 + \rho)}{\beta}\) for a naive hyperbolic consumer.
Therefore, the monopolist will choose \( r = \frac{1-\beta \delta}{\beta \delta} \) as a profit-maximizing interest rate.

\[
\text{payoff}_2 - \text{payoff}_1 = \frac{1}{1 - \delta^2} \left[ - (1 + \delta) \left( \text{Av}(y^*) - c(y^*) \right) + \delta F \right] \tag{2}
\]

where \( A = \frac{1}{\beta \delta} \).

Similar to the previous proof, there exists a cutoff \( F^*_{\text{naive}}(\delta, \beta) > 0 \):

\[
F^*_{\text{naive}}(\delta, \beta) = -\frac{1}{\delta} \left[ \left( \text{Av}(y^*) - \delta c(y^*) \right) - (1 + \delta) \left( \text{Av}(x^*) - c(x^*) \right) \right]
\]

such that \( \text{payoff}_2 - \text{payoff}_1 \leq 0 \) if \( F \leq F^*_{\text{naive}}(\delta, \beta) \) and \( \text{payoff}_2 - \text{payoff}_1 > 0 \) if \( F > F^*_{\text{naive}}(\delta, \beta) \) for high enough values of \( \delta \).

Now, we claim that \( F^*_{\text{naive}}(\delta, \beta) > F^*_{\exp}(\delta) \) for \( \delta \) close to 1. Suppose not:

\[
- \frac{1}{\delta} \left[ \left( \frac{1}{\beta^2 \delta} v(y^{**}) - \delta c(y^{**}) \right) - (1 + \delta) \left( \frac{1}{\beta^2 \delta} v(x^{**}) - c(x^{**}) \right) \right] \leq -\frac{1}{\delta} \left[ \left( \text{Av}(y^*) - \delta c(y^*) \right) - (1 + \delta) \left( \text{Av}(x^*) - c(x^*) \right) \right]
\]

where \(^*\) and \(^{**}\) denote the optimal values if the consumer is exponential and naive, respectively. The previous inequality is true if and only if

\[
\left[ \left( \frac{1}{\beta^2 \delta} v(y^{**}) - \delta c(y^{**}) \right) - (1 + \delta) \left( \frac{1}{\beta^2 \delta} v(x^{**}) - c(x^{**}) \right) \right] \geq \left[ (v(y^*) - \delta c(y^*)) - (1 + \delta) (v(x^*) - c(x^*)) \right].
\]

This is true if and only if

\[
\left( \frac{1}{\beta^2 \delta} v(x^{**}) - c(x^{**}) \right) \leq (v(x^{**}) - c(x^{**})).
\]

which results in \( 1 \leq \beta^2 \delta \), which is a contradiction. Lastly, we claim that \( \frac{\partial F^*_{\text{naive}}(\delta, \beta)}{\partial \beta} < 0 \). By the envelope theorem:

\[
\frac{\partial (\text{payoff}_2 - \text{payoff}_1)}{\partial \beta} = \frac{1}{1 - \delta^2} \left[ v(y^*) - (1 + \delta) v(x^*) \right] \frac{\partial A}{\partial \beta} \tag{3}
\]
then the limit of $y^*(\delta)$ is equal to the limit of $x^*(\delta)$ as $\delta$ goes to 1. At this limit, $[v(y^*) - (1 + \delta)v(x^*)]$ is strictly negative and equal to $-v(x^*)$ since $y^*(\delta)$ and $x^*(\delta)$ are continuous in $\delta$. This means that $\text{payoff}_2 - \text{payoff}_1$ is increasing with $\beta$ for high enough values of $\delta$, and consequently $\frac{\partial F^*_{\text{naive}}(\delta, \beta)}{\partial \beta} < 0$. Moreover, we have $F^*_\text{exp}(\delta) < F^*_{\text{naive}}(\delta, \beta) < F^*_{\text{naive}}(\delta, \beta)$ for $\delta$ close to 1. ■

**Proof of Proposition 3.** If the consumer is sophisticated hyperbolic with $\beta \geq \overline{\beta}$;

$$\text{payoff}_2 - \text{payoff}_1 = \frac{1}{1 - \delta^2} \left[ \begin{array}{c} A(v(y^*) - (1 + \delta)v(x^*)) \\ -\delta c(y^*) + (1 + \delta)c(x^*) + \delta F' \end{array} \right]$$

(4)

where $A = 1$. Since this problem is the same as the problem with the exponential consumer, the cutoff fixed cost values are the same, $F^*_{\text{sophis-mild}}(\delta, \beta) = F^*_\text{exp}(\delta)$. ■

**Proof of Proposition 4.** If the consumer is sophisticated hyperbolic with $\beta < \overline{\beta}$;

$$\text{payoff}_2 - \text{payoff}_1 = \frac{1}{1 - \delta^2} \left[ \begin{array}{c} A(v(y^*) - (1 + \delta)v(x^*)) \\ -\delta c(y^*) + (1 + \delta)c(x^*) + \delta F' \end{array} \right]$$

(5)

where $A = \frac{1}{\beta}$.

Similar to the first proof, there exists a cutoff $F^*_{\text{sophis-low}}(\delta, \beta) > 0$:

$$F^*_{\text{sophis-heavy}}(\delta, \beta) = \frac{1}{\delta} [(Av(y^*) - \delta c(y^*)) - (1 + \delta) (Av(x^*) - c(x^*))]$$

such that $\text{payoff}_2 - \text{payoff}_1 \leq 0$ if $F \leq F^*_{\text{sophis-heavy}}(\delta, \beta)$ and $\text{payoff}_2 - \text{payoff}_1 > 0$ if $F > F^*_{\text{sophis-heavy}}(\delta, \beta)$ for high enough values of $\delta$. Now, we claim that $F^*_{\text{naive}}(\delta, \beta) > F^*_{\text{sophis-heavy}}(\delta, \beta) > F^*_\text{exp}(\delta)$ for $\delta$ close to 1 and $\frac{\partial F^*_{\text{sophis-heavy}}(\delta, \beta)}{\partial \beta} < 0$. The proofs are similar to the proofs in the second proposition. ■

**Proof of Proposition 5.** The welfare of a naive hyperbolic consumer every time he buys a new product is:

$$v(x^*) - p$$

(6)

such that $p$ is the total price he pays. This is the welfare evaluated with long-term preferences. The welfare of the monopolist is $p - (c(x^*) + F)$ if he
offers new products every period and \( p - (\delta c(y^*) + F) \) if he offers them every other period.

The social welfare with every-period introductions is:

\[
W_1 = \frac{1}{1 - \delta} [v(x^*) - (c(x^*) + F)]
\]  

(7)

The social welfare with every-other-period introductions is:

\[
W_2 = \frac{1}{1 - \delta^2} [v(y^*) - (\delta c(y^*) + F)]
\]  

(8)

The difference is:

\[
W_2 - W_1 = \frac{1}{1 - \delta^2} [v(y^*) - \delta c(y^*) - (1 + \delta)v ((x^*) - c(x*)) - \delta F]
\]  

(9)

From the proofs of 1 and 2, the monopolist will offer new products every period, although it is socially better to offer them every other period for \( F_{\text{exp}}(\delta) < F < F_{\text{naive}}(\delta, \beta) \) if the consumer is naive hyperbolic.

**Proof of Proposition 6.** Similar to the previous proof,

\[
W_2 - W_1 = \frac{1}{1 - \delta^2} [v(y^*) - \delta c(y^*) - (1 + \delta)v ((x^*) - c(x*)) - \delta F]
\]  

(10)

From the proofs of 1 and 4, the monopolist will introduce new products every period, although it is socially better to offer them every other period for \( F_{\text{exp}}(\delta) < F < F_{\text{sophis-heavy}}(\delta, \beta) \) if the consumer is heavily impatient sophisticated hyperbolic.
8 References


Figure 1. Product Introductions depending on parameter values