Contagion in the European Sovereign Debt Crisis∗

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Abstract

We use a network model of credit risk to measure market expectations of the potential spillovers from a sovereign default. Specifically, we develop an empirical model based on the recent theoretical literature on contagion in financial networks, which emphasizes a direct, “balance sheet” form of contagion. We estimate the model with data on sovereign credit default swap spreads and the detailed structure of financial linkages among thirteen European sovereigns from 2005 to 2011. Simulating the estimated model, we find that a sovereign default generates only small spillovers to other sovereigns, based on this mechanism.

Keywords: financial networks; sovereign debt crisis; contagion; structural estimation; systemic risk

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1 Introduction

In the recent European sovereign debt crisis, economists, policymakers, and the media have raised concerns over various forms of financial contagion. One common fear is that, given the interconnectedness of financial relationships in Europe, the default of one sovereign would have spillover effects that result in increased borrowing costs for other sovereigns, and perhaps even subsequent defaults. The recent theoretical literature on financial networks has proposed default spillovers as an important source of contagion for European sovereigns (e.g., Elliott, Gohub, and Jackson (2014)). Moreover, European policymakers have invoked these and related concerns to justify sovereign bailouts and interventions. However, the magnitude of the spillovers resulting from a sovereign default, and their effect on the cost of borrowing for other connected sovereigns, remains an open empirical question.

In this paper, we develop and estimate a network model of credit risk to measure market expectations of the potential spillovers from a sovereign default. Specifically, we implement an empirical version of a class of models appearing in recent theoretical papers (e.g., Acemoglu, Ozdaglar, and Tahbaz-Salehi 2015; Elliott, Golub, and Jackson 2014; Glasserman and Young 2015), and estimate it using data on sovereign credit default swap (CDS) spreads and the detailed structure of financial linkages among thirteen European sovereigns from 2005 to 2011.\footnote{This modeling framework originated with the seminal paper by Eisenberg and Noe (2001).} In this framework, the spillovers from a default occur via direct losses to assets (e.g., loans or bonds) held by creditors, in what could be referred to as a “balance sheet” mechanism for contagion. The equilibrium solution for solvency and repayments among the sovereigns in the network expressly accounts for the joint determination of asset values based on this mechanism.

We use data from the Bank for International Settlements (BIS) and IMF to construct an
empirical network of financial linkages among the sovereigns in our sample for each quarter. Additionally, we impute market expectations for the sovereigns’ risk-neutral default probabilities in each quarter using the spreads on their 5-year CDS contracts. Combining these series with data on the sovereigns’ GDP, we estimate the parameters of our network model of spillovers. We then use a series of simulations to quantify the potential spillovers from a sovereign default. Specifically, we consider counterfactuals where we simulate the default of one sovereign and compute the predicted change in the risk-neutral default probabilities of the other sovereigns in the network.

From these simulations, we develop a novel measure of the contagion risk posed by each sovereign in our sample. Our measure has a relationship with centrality measures commonly used in network analysis, but it also has a more direct economic interpretation as the *expected spillover losses per dollar of debt* in the event of a default. This normalizes for a sovereign’s total external debt, so the measure captures differences in contagion risk due to a sovereign’s position in the network. With this measure we show how contagion risk has risen over time, and we arrive at a potentially surprising result for the country with the greatest potential for contagion per unit of debt: Austria. Much of Austria’s debt is held by Italy, a financially vulnerable sovereign with substantial external debt. Thus, while Austria’s default probability is low, the model predicts relatively high spillover losses in the event of an Austrian default.

We then assess how the risk of contagion affected the cost of borrowing for the European sovereigns in our sample. By comparing our estimated model to a counterfactual case in which we rule out the possibility of spillovers from a sovereign default, we are able to measure the effect of this form of contagion risk on borrowing costs for each sovereign at each quarter in our sample. We find that the possibility of contagion resulting from direct losses following a sovereign default had a small effect on sovereign borrowing costs in our sample period. Put
differently, the financial interconnectedness of European sovereign debt holdings per se does not appear to have had an economically significant effect on sovereign costs of borrowing.

Given these results, it is particularly important to consider factors that might bias our estimates toward a finding of no contagion from this channel. To understand this issue, it is useful that our empirical model is directly related to a class of models found in the microeconometric literature on social interactions. The conditions for the identification of endogenous spillover effects are well understood in that literature (e.g., Manski 1993; Blume, Brock, Durlauf, and Ioannides 2011). The basic challenges of simultaneity and the “reflection” problem, which arise in models with interconnected agents, are resolved by our use of data on individual network linkages (Bramouillé, Djebbari, and Fortin 2009).

Another possible source of bias comes from the potential endogeneity of these linkages. Here we follow the theoretical literature on financial contagion, and the empirical work related to this literature (discussed below), by treating the network of financial linkages as exogenous. Hence, any unobserved factors that determine both financial linkages and credit risk would bias our results. We consider this issue in detail and show that any bias is likely to be upward, so it would not affect our overall conclusion that the potential spillovers from the balance sheet mechanism are relatively small.

The basic empirical fact that drives our results is that the differential financial linkages among countries explain relatively little of the differential comovements in sovereign credit risk. Any modeling framework where the transmission of risk is in some way related to the aggregate financial linkages among countries would ultimately yield a small estimate of the spillover effects because of this feature of the data. Other transmission mechanisms exist that do not involve direct financial linkages, such as changes in investor risk preferences or changes
in beliefs regarding the likelihood of an event such as a sovereign default. These mechanisms may deserve further attention in future work, both theoretically and empirically. While our results suggest that credit markets perceived small spillover effects from one channel for contagion, it is possible that other channels might have larger economic effects.

The role of networks in financial contagion has been the subject of a growing literature that largely began with Allen and Gale (2000) and Freixas, Parigi, and Rochet (2000). These papers model a simple interbank market where liquidity shocks arise from consumers. They consider the possible contagion of insolvency if one bank fails, and both models indicate that more connected networks are more resilient against contagion. More recent theoretical work has further examined this result in specific environments. Babus (2009) models the formation of a network of interbank deposits in one region with common liquidity shocks, given pre-existing links with banks in another region that has the opposite liquidity shocks. Acemoglu, Ozdaglar, and Tahbaz-Salehi (2015) and Elliott, Golub, and Jackson (2014) derive several results based on exogenous networks with canonical topologies (e.g., rings, regular graphs, complete graphs), and Glasserman and Young (2015) develop bounds on the probability of contagion under arbitrary topologies.

Despite this rich theoretical literature, there is relatively little empirical work drawing on these models. Both Allen and Gale (2000) and Elliott, Golub, and Jackson (2014) suggest that their frameworks could be applied to a network of countries, but we are the first to do so in an empirically rigorous fashion. The most closely related studies to ours are three recent papers that estimate structural network models of spillovers in interbank markets: Cohen-Cole, Patacchini, and Zenou (2011), Denbee, Julliard, Li, and Yuan (2014); and Bonaldi, Benzoni, Collin-Dufresne, Goldstein, and Helwege (2012) and Kodres and Pritsker (2002).

2 See, for example, Benzoni, Collin-Dufresne, Goldstein, and Helwege (2012) and Kodres and Pritsker (2002).

3 Elliott, Golub, and Jackson (2014) and Glasserman and Young (2015) use relevant empirical data to provide interesting numerical illustrations of their models. However these are not intended to be econometric exercises.
Our modeling framework and empirical approach are broadly similar to theirs. In particular, we follow their use of two key assumptions: that financial linkages established in a previous period are exogenous, and that unobserved shocks are independent over time. This allows each time period to be treated independently for the purpose of estimation, making the application of a structural network model computationally feasible.

Finally, our paper contributes to a broader empirical literature seeking to estimate effects of contagion, both for sovereigns in the recent European crisis and in international financial markets more generally. An important debate in this literature has centered around how to define, measure, and identify contagion. In this paper, we focus on a specific channel for sovereign contagion: direct losses resulting from a default. The structure of our network model, as well as the data used in our estimation, allow us to identify and measure the magnitude of these expected default spillovers, distinct from other channels of contagion and sources of comovement. Thus, we provide estimates for the magnitude of a contagion channel among sovereigns that has been proposed, but yet to be quantified, by the theoretical network literature.

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4 There is also a literature that conducts simulation studies with calibrated models of interbank networks. See Gofman (2014), for example, and Upper (2011) for a survey.

5 See, for example, Forbes and Rigobon (2001), Forbes and Rigobon (2002), Bae, Karolyi, and Stulz (2003), Karolyi and Stulz (2003), and Bekaert, Harvey, and Ng (2005). For surveys of international financial contagion, see Claessens and Forbes (2001) and Karolyi (2003).

6 The existing work on spillovers in the European crisis typically does not specify an exact mechanism and does not consider identification. For example, Arghyrou and Kontonikas (2012) and Beirne and Fratzscher (2013) estimate regression models for sovereign credit risk that include a measure of the average credit risk among other European sovereigns.
2 Theoretical Framework

Our model follows a number of recent papers in the theoretical literature on financial contagion, all of which build on the framework introduced by Eisenberg and Noe (2001).\(^7\) While there are important distinctions in the details of these models and the results they produce, the broad features are as follows. The models in these papers describe a payment equilibrium among a set of financial entities that hold claims on each other and also have outside assets or liabilities that are partly stochastic. Given a network of claims among the entities and realizations of their shocks, the payment equilibrium determines a vector of repayments that clears the system.\(^8\) Default is exogenous and occurs when an entity has insufficient assets to meet all of its obligations in full. Contagion in this framework is therefore understood as defaults or other losses that occur as a consequence of incomplete repayments received from other members of the network. This is an immediate, direct mechanism for the spillovers from a default, which we refer to as the “balance sheet” mechanism of contagion.

Applying this framework to our context, each country is treated as a single, aggregate financial entity, and countries are connected through their aggregate financial claims on each other.\(^9\) Because our focus is on sovereign debt, this aggregate approach relies on the fact that banks hold substantial amounts of foreign sovereign debt, and on the close connection in credit risk between the banks and the central government in each country. Losses in the value of bank holdings of foreign sovereign debt therefore impact the central government (e.g., when there are bank guarantees), which in turn affects the credit risk and cost of


\(^8\)The network of financial linkages is not endogenized in these models.

\(^9\)As noted earlier, Elliott, Golub, and Jackson (2014) specifically suggest that this framework could be applied to countries. When they introduce their model they name “countries, banks, or firms” as examples of the entities in the network (p. 4), and they later provide an empirical illustration of the model using BIS data on the aggregate financial linkages among six European countries.
Two recent papers on the European debt crisis provide a clear theoretical basis for how this would work, along with empirical evidence showing the importance of the relevant factors. Bolton and Jeanne (2011) propose a model of contagion from the sovereign debt of one country to the financial sector and eventually the economic output of another country, based on bank holdings of foreign sovereign debt used as collateral for loans. They document that such foreign debt holdings were indeed substantial in Europe: in many countries, foreign sovereign debt accounted for the majority of sovereign debt holdings at large banks. Acharya, Drechsler, and Schnabl (2014) show how government bailouts would then transfer credit risk from the banks to the sovereign debt of their own country. As they point out, most European nations provided some form of bailout to their banks in the fall of 2008.

Formally, then, the entities in our network are sovereigns \( i = 1, \ldots, N \). They are observed over a number of time periods \( t = 1, \ldots, T \), but each period is treated independently as the payment equilibrium is a fundamentally static solution concept. In each period, sovereigns hold debt claims on each other that were established in a previous period. The face value of country \( i \)'s gross, aggregate claims on country \( j \), payable at date \( t \), is denoted \( l_{ijt} \). These bilateral claims are collected into a matrix \( L_t \), which defines a weighted, directed graph that constitutes the financial network in period \( t \). Sovereigns have additional obligations to unspecified entities outside the network, so that the total debt owed by sovereign \( i \) in period \( t \), denoted \( D_{it} \), is more than just the sum of the claims on \( i \) from the other sovereigns in the

\[ \text{For evidence of the risk transfer, Acharya, Drechsler, and Schnabl (2014) show a strong association between domestic bank CDS rates before the bailouts and sovereign CDS rates after the bailouts. One illustrative example is Ireland, as they note, where the sovereign CDS rate rose from 25 basis points (bps) to 400 bps in six months following the bank guarantee made in September 2008.} \]

\[ \text{This is a limitation, but as we discuss in Appendix A, it does not appear to qualitatively impact our estimate of spillovers from the balance sheet mechanism. Other empirical analyses of spillovers in financial networks similarly apply static models to repeated observations on a single set of players, thereby treating each period independently (Cohen-Cole, Patacchini, and Zenou 2011; Denbee, Julliard, Li, and Yuan 2014; Bonaldi, Hortaçsu, and Kastl 2014).} \]
network (i.e., \( D_{it} \geq \sum_{j \neq i} l_{ijt} \)). A sovereign’s output, \( Y_{it} \in \mathbb{R}^+ \), is stochastic and assumed to evolve exogenously. Finally, sovereigns are exposed to an exogenous financial shock, \( X_{it} \in \mathbb{R} \).

The payment equilibrium determines which countries are solvent in a particular period, given their total debt \( (D_{it}) \), aggregate output \( (Y_{it}) \), financial shocks \( (X_{it}) \), and the equilibrium payments on their established claims \( (l_{ijt}) \). Solvency is denoted with indicators \( s_{it} \). If sovereign \( j \) is solvent in period \( t \) \( (s_{jt} = 1) \), then sovereign \( i \) receives the full value of its claims on \( j \); i.e., \( l_{ijt} \). If, on the other hand, a country defaults, its creditors receive a proportion of their claims \( \delta l_{ijt} \), where \( \delta \in [0, 1) \) is a fixed, exogenous recovery rate. This assumption of a fixed recovery rate is common in the credit risk literature, and the value we choose \( (\delta = 0.4) \) is consistent with historical recovery rates for sovereign defaults.\(^{12}\) Given the fixed recovery rate, the contingent payment that country \( i \) receives for its claims on \( j \) in period \( t \) is thus \( l_{ijt}[\delta + (1 - \delta)s_{jt}] \). The total repayments received in period \( t \) is denoted

\[
R_{it} \equiv \sum_{j \neq i} l_{ijt}[\delta + (1 - \delta)s_{jt}]. \tag{1}
\]

A sovereign is solvent if, with these repayments and the shocks \( Y_{it} \) and \( X_{it} \), it has sufficient assets to pay its debts. Accordingly, the solvency of each sovereign is determined as

\[
s_{it} = 1 \{ R_{it} + Y_{it} + X_{it} > D_{it} \}. \tag{2}
\]

A payment equilibrium can then be characterized by either a vector of repayments \( (R_{it})_{i=1}^{N} \), or a vector of solvency indicators \( (s_{it})_{i=1}^{N} \), that solve the system of equations defined by (2).

Depending on the values of \( Y_{it} \) and \( X_{it} \) across all countries, there may be multiple solutions

\(^{12}\)Pan and Singleton (2008) use the term-structure of CDS spreads and estimate a range of risk-neutral recovery rates depending on the sovereign. They use a rate of 25% in their analysis of sovereign risk premia. Longstaff, Pan, Pedersen, and Singleton (2011) similarly use a rate of 25%, while Ang and Longstaff (2013) assume a recovery rate of 50%. In a sample of historical sovereign debt restructurings, Sturzenegger and Zettelmeyer (2008) estimate a range of recovery rates from 30 - 75%. The discrete losses that occur with a fixed recovery rate can be motivated as a consequence of the renegotiations involved in a sovereign default. See, for example, Yue (2010) and Benjamin and Wright (2009).
to (2). Similar to the models in Rogers and Veraart (2013) and Elliott, Golub, and Jackson (2014), this is a consequence of the discrete loss that occurs with a default. When there are multiple solutions (i.e., multiple equilibria), we follow these papers and select the “best-case” equilibrium in which the fewest countries default.\textsuperscript{13} For example, suppose that given the claims, debts, and shocks among all the countries in the network, there are two solutions for countries $i$ and $j$: either both default ($s_{it} = s_{jt} = 0$) or both are solvent ($s_{it} = s_{jt} = 1$), while all other countries remain solvent. This is possible if $i$ and $j$ are both close to the default threshold and need the repayments from each other in order to remain solvent. In such cases, we always select the equilibrium where marginal countries such as these pay each other back and remain solvent. This would be the result if there were some coordination process, as it is reasonable to presume that all countries would be weakly better off if there were fewer defaults. The best-case solution can be found with a simple iterative procedure: start with repayment amounts as though all countries were solvent; use (2) to determine which countries would, in fact, default; reduce the repayment amounts based on these defaults; use (2) to determine if any additional countries would default; repeat this process until no further countries would default.\textsuperscript{14}

Finally, we think it is useful to describe—informally—how the payment equilibrium could fit into a larger process for the evolution of the financial network over time. This makes clear the assumptions about timing that are involved in our use of the data. It also helps to clarify how biases could arise if our econometric assumptions are violated, such as the exogeneity of financial linkages. (These potential biases are discussed in detail in Appendix A.) Accordingly, for these limited purposes, we can put the payment equilibrium in the

\textsuperscript{13}As in Elliott, Golub, and Jackson (2014) the set of equilibria constitutes a finite lattice, so there is a well-defined maximum with the fewest defaults.

\textsuperscript{14}Eisenberg and Noe (2001), Rogers and Veraart (2013), and Elliott, Golub, and Jackson (2014) use similar algorithms.
context of a process that repeats over time if we suppose that each period unfolds as follows:

0. Countries are endowed with bilateral claims \((l_{ijt})\) and total debts \((D_{it})\), which were established in the previous period.

1. Output \((Y_{it})\) and financial shocks \((X_{it})\) are realized.

2. Solvency \((s_{it})\) is jointly determined in the payment equilibrium for period \(t\).

3. Claims and debts are established for the next period.

4. CDS contracts are traded for credit events in the next period.

To be clear, our model only pertains to the payment equilibrium in step 2. This follows the empirical approaches in Cohen-Cole, Patacchini, and Zenou (2011), Denbee, Julliard, Li, and Yuan (2014), and Bonaldi, Hortaçu, and Kastl (2014), which similarly estimate structural models of spillovers in financial networks. All of these papers apply static equilibrium models to repeated observations on a fixed set of entities. In order to treat each time period independently, any adjustment costs or other dynamic aspects of the decision problems are ignored, and unobserved shocks are assumed to be independent over time. The network of financial linkages can then be considered exogenous if actions in a previous period define the network, as above or in Denbee, Julliard, Li, and Yuan (2014), for example. Any attempt to go beyond this static approach and incorporate the dynamic decision problem in step 3 would confront a substantial challenge of finding equilibrium policy functions for the entities in the network, where the state space involves an \(N \times N\) matrix of financial claims. It would also require a number of additional modeling assumptions. We have chosen instead to follow the above papers in the network literature and estimate a static model of the payment equilibrium, to serve as a starting point to assess the quantitative importance of one proposed mechanism for contagion.
3 Empirical Approach

Our goal is to estimate an empirical version of the solvency condition given in (2), which can then be used to quantify the potential spillovers from a sovereign default that arise from the balance sheet mechanism of contagion. Because defaults are not observed in our sample (2005-2011), and in general are very rare among developed sovereigns, we match equilibrium predictions from the model to observable market beliefs about the probability that each sovereign will be solvent. In particular, we use the observed spreads on sovereign CDS contracts to impute a sovereign’s risk-neutral default probability.

To map the data to our model, we suppose that CDS spreads at the end of period \(t-1\) reflect the market’s assessment of the risk-neutral probability that each sovereign will be solvent in the payment equilibrium in period \(t\). These market expectations should therefore be equal to the expected value of the solvency indicators, \(s_{it}\), conditional on the information available at the end of period \(t-1\) (when the claims payable in period \(t\) have already been established). We use \(p_{it}\) to denote these conditional expectations, taken under the risk-neutral measure \(Q\). Formally, we define these as:

\[
p_{it} \equiv \mathbb{E}^Q \left[ s_{it} | L_t, (D_{jt}, Y_{j,t-1}, X_{j,t-1})_{j=1}^N \right]. \tag{3}
\]

These expectations can be found, given a joint distribution of output \((Y_{jt})_{j=1}^N\) and shocks \((X_{jt})_{j=1}^N\), conditional on their lagged values, by solving for the payment equilibrium (i.e., the vector of solvency indicators, \((s_{jt})_{j=1}^N\)) over this distribution.

To adapt the generalized solvency condition in (2) to work with our data, we need to allow for the fact that the exact amounts of claims and debts due each period, and the available tax revenues for debt payments, are not observed. Our data on bilateral claims \((l)\) and total debts \((D)\) consist of their stocks observed at a quarterly frequency. The measure
of aggregate output ($Y$) is quarterly GDP and the financial shocks ($X$) are unobserved. Accordingly, we introduce parameters that express the proportions of these variables that are relevant, on average, for the payment equilibrium in a single period. In addition we allow the threshold required for solvency to take some value other than zero, which could be positive or negative.\(^{15}\) Thus the empirical version of the solvency condition is specified as

$$s_{it} = \mathbb{1} \{ \gamma R_{it} - \alpha D_{it} + \beta Y_{it} + X_{it} > \pi_i + \pi_t \}. \quad (4)$$

The parameters $\gamma$ and $\alpha$ express the proportions of the observed financial claims that are payable each period, and $\beta$ gives the proportion of aggregate output that is available to the central government for payments on its debt obligations. The solvency threshold for sovereign $i$ in period $t$ is $\pi_i + \pi_t$. This threshold varies across sovereigns to capture differences in relatively fixed obligations such as social pension payments, and varies over time to reflect changes in factors like the availability of capital.

We then need to specify the forecasted distributions of aggregate output ($Y_{it}$) and financial shocks ($X_{it}$) conditional on their values in period $t-1$, so that we can integrate the solutions to (4) over their joint distribution and thereby compute the expectations in (3). For output, we specify the forecasted distribution as a function of its previous level ($Y_{i,t-1}$) and growth rate ($\Delta Y_{i,t-1}$). To capture common macroeconomic trends among the sovereigns in our network, we partition the previous growth rate into a common component and country-specific residuals using a principal components analysis. The common component of the growth rate in country $i$, denoted $\Delta Y_{i,t-1}^c$, is the first principal component (PC) for period $t-1$ multiplied by the loading for country $i$. The residual is $\Delta Y_{i,t-1}^r = \Delta Y_{i,t-1} - \Delta Y_{i,t-1}^c$. As

\(^{15}\)The economic and legal environment of sovereign borrowing is such that there is not a clearly defined default threshold. In the case of a corporate borrower, equityholders would choose to optimally default on their obligations when the value of the equity claim goes to zero. An analogous condition does not exist in the case of a sovereign borrower.
the notation indicates, $\Delta Y_{t-1}^c$ varies across countries because it incorporates the loadings. This allows some countries to be more exposed to the aggregate European economy than others. The mean of the forecast for $Y_{it}$ is then specified as a linear combination of the previous level and these two components of the growth rate: $\beta_1 Y_{i,t-1} + \beta_2 \Delta Y_{i,t-1}^c + \beta_3 \Delta Y_{i,t-1}^r$. The distribution of $Y_{it}$ around this mean is assumed to be normal with variance $\sigma_Y^2$. Thus, the forecasted distribution of aggregate output for sovereign $i$ in period $t$ is

$$Y_{it}|(Y_{i,t-1}, \Delta Y_{i,t-1}^c, \Delta Y_{i,t-1}^r) \sim \mathcal{N}(\beta_1 Y_{i,t-1} + \beta_2 \Delta Y_{i,t-1}^c + \beta_3 \Delta Y_{i,t-1}^r, \sigma_Y^2).$$

These are the market beliefs at the end of period $t-1$.

The shock $X_{it}$ is also specified to have a normal distribution, with mean zero and variance $\sigma_X^2$. The variance is the same across countries, but we normalize all variables in levels to be relative to the size of a country’s economy. This is equivalent to setting the standard deviation of the financial shocks in each country to be proportional to the size of its economy; e.g., $\sigma_{Xi} = \sigma_X Y_{i0}$, where $Y_{i0}$ is some baseline level of aggregate output. Thus, we effectively allow for larger shocks in countries with larger economies.\(^{16}\) Beyond this, the output and financial shocks are assumed to be independent across countries and over time, which follows Cohen-Cole, Patacchini, and Zenou (2011), Denbee, Julliard, Li, and Yuan (2014), and Bonaldi, Hortaçsu, and Kastl (2014).\(^{17}\)

Applying these specifications, the network-wide vector of conditional expectations in (3),

\(^{16}\)This assumption also appears in the theoretical literature we draw from (e.g., Glasserman and Young 2015).

\(^{17}\)Appendix A considers the biases that could arise if these independence assumptions are violated. There we show, among other things, that a positive correlation in the shocks among countries could only result in an upward bias in the estimate of $\gamma$, while our main concern is with a downward bias.
which we refer to as the risk-neutral solvency probabilities, can be expressed as follows:

\[
(p_{it})_{i=1}^{N} = \int 1 \left\{ \gamma R_{it} - \alpha D_{it} + \beta_0 (\beta_1 Y_{i,t-1} + \beta_2 \Delta Y_{i,t-1}^c + \beta_3 \Delta Y_{i,t-1}^r) + X_{it} > \pi_i + \pi_t \right\}_{i=1}^{N} \cdot \prod_{j=1}^{N} \frac{1}{\sigma_Y} \phi \left( \frac{\tilde{Y}_{jt}}{\sigma_Y} \right) \frac{1}{\sigma_X} \phi \left( \frac{X_{jt}}{\sigma_X} \right) d\tilde{Y}_{jt} dX_{jt},
\]

where \( \tilde{Y}_{it} \) is the deviation of \( Y_{it} \) from its conditional mean and \( \phi \) is the standard normal density. The vector of indicator functions \( (1\{...\})_{i=1}^{N} \) in the integral gives the vector of solvency indicators \( ((s_{it})_{i=1}^{N}) \) as a function of the vectors of observables and shocks. The interdependencies across countries arise because the indicators \( s_{jt} \) are embedded in each \( R_{it} \). To simplify this expression, we combine the shocks \( \tilde{Y}_{it} \) and \( X_{it} \) as \( \epsilon_{it} \equiv \tilde{Y}_{it} + X_{it} \) and normalize the parameters so that \( \epsilon_{it} \) has unit variance (as in a standard probit model). Also, because \( \beta_0 \) is not separately identified from \( \beta_1, \beta_2, \) and \( \beta_3 \), we set \( \beta_0 = 1 \). Consequently, the parameters \( \beta_1, \beta_2, \beta_3 \) are interpreted as the combination of the forecast for future output and the relationship between output and solvency. Finally, we use a simple linear trend to capture any changes in the default threshold over time, so that \( \pi_t \) is specified as \( \theta t \).\(^{18}\) This yields the ultimate specification that we take to the data:

\[
(p_{it})_{i=1}^{N} = \int 1 \left\{ \gamma R_{it} - \alpha D_{it} + \beta_1 Y_{i,t-1} + \beta_2 \Delta Y_{i,t-1}^c + \beta_3 \Delta Y_{i,t-1}^r + \epsilon_{it} > \pi_i + \theta t \right\}_{i=1}^{N} \cdot \prod_{i=1}^{N} \phi(\epsilon_{it}) d\epsilon_{it} \quad (5)
\]

The integral is computed via simulation.\(^{19}\) For each vector of pseudo-random draws of \( (\epsilon_{it})_{i=1}^{N} \), we solve the system of equations defined by (4) for the vector of solvency indicators. (When there are multiple solutions we select the best-case equilibrium, as described in Section 2.) The average of these vectors of indicators across all draws provides an approximation of

\(^{18}\)The results in Section 4.2 indicate that a linear time trend fits the data reasonably well and that our conclusions would be robust to more flexible specifications. Having a fixed effect for each time period is problematic because it would greatly increase the parameter space and would raise an incidental parameter problem in our nonlinear model (Neyman and Scott 1948).

\(^{19}\)We use antithetic acceleration to improve the precision of the simulator (Stern 1997).
the vector of solvency probabilities above.

We estimate the parameters in (5) by minimizing the squared error between the empirical, risk-neutral solvency probabilities, derived from the observed CDS spreads, and the predicted solvency probabilities from the above model. The identification of the model is discussed next.

3.1 Identification

To consider identification, our empirical model can be understood within a certain class of models from the microeconometric literature on social and spatial interactions. The class consists of static equilibrium models where best responses are nonlinear functions of the realized actions of other players (i.e., these models are based on simultaneous-move games of complete information, typically with binary actions). In our case it is the solvency outcomes in (4) that are nonlinear functions of the realized solvencies of other countries. Krauth (2006) and Soetevent and Kooreman (2007) are two primary examples that estimate models from this class and provide detailed analyses of identification. Their approaches, like ours, involve making joint predictions for the vector of equilibrium outcomes in order to address the mutual endogeneity of outcomes within a network. Also both employ selection rules when multiple equilibria are present, as do we—in our case, motivated by the theoretical literature (e.g., Rogers and Veraart 2013; Elliott, Golub, and Jackson 2014).

The results in Krauth (2006) show that our model is semi-parametrically identified under our assumption that the shocks ($\epsilon_{it}$) are independent across countries and over time (Section

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20The source of the econometric error between the predicted probabilities and the empirical probabilities is left unspecified. However to provide some intuition for our estimation procedure, we can compare (5) with a generalized linear model. If (5) were a single-equation, non-equilibrium model, it could be treated as a GLM where the econometric error yields a quasi-binomial distribution of the observed solvency probabilities. Maximum likelihood estimation would be accomplished with iteratively reweighted least squares (a Fisher scoring algorithm). Our estimation procedure does not weight the observations as they would be in that approach, but those weights would favor the same observations (i.e., those with relatively low empirical solvency probabilities) that drive our estimates.
The main difference between our model and those in Krauth (2006) and Soetevent and Kooreman (2007) is that in our case the spillovers take place on a weighted, directed graph (i.e., the network of financial linkages), while in theirs the interaction effects are uniform within groups (e.g., school classrooms where all students are equally connected). The variation in exposures introduced by using individual linkages does not affect the identification arguments in these papers. In fact, based on existing results for linear network models, this variation might facilitate identification in circumstances where shocks are correlated across units.\footnote{See Bramoullé, Djebbari, and Fortin (2009) and Lee, Liu, and Lin (2010) for identification results on linear network models with group fixed effects and other forms of correlation. Based on these results, we speculate that a contemporaneous correlation in the shocks across countries would be separately identifiable from the endogenous spillover effect in our model, which is not the case in models with only group-based interactions (see Krauth 2006). Specifically, we believe that if $\epsilon_{it}$ were decomposed into a common and idiosyncratic component, such as $u_t$ and $v_{it}$, the variance of $u$ could be identified separately from the parameters in (5). This follows from similar logic as the identification of nonlinear panel models with random effects. All the variables in (5) exhibit variation across countries at a point in time, including the claims that influence $R_{it}$. Hence the distribution of a common shock should be identifiable. However, to our knowledge, such results on identification with correlated unobservables are not currently available for our class of nonlinear network models. Brock and Durlauf (2007) discuss various conditions to achieve partial identification in nonlinear models with group-based interactions. Lee, Li, and Lin (2014) consider a nonlinear model with network interactions, but it is based on a game of incomplete information.} However we do not explore this possibility; instead, we maintain the assumption of independent shocks across countries, but we carefully consider the bias that would arise if this were violated.

Appendix A provides a detailed analysis of the biases that would arise if the independence assumption or other key assumptions in our empirical model were violated. We consider four potential issues: correlations in the shocks between countries, the endogeneity of financial linkages, endogenous default decisions, and internal amplification mechanisms with different impacts across countries. Our focus is on the bias in the estimate of $\gamma$, the parameter that governs the magnitude of spillovers from a default. In each case we show that the likely bias is upward, so none of these departures from the model would affect our overall conclusion that credit market perceptions of the potential spillovers from the balance sheet mechanism
were relatively small.

4 Data

In this section we discuss the data used to estimate the network model described above. We combine data provided by the Bank for International Settlements (BIS) and International Monetary Fund (IMF) to construct an empirical network of bilateral, aggregate financial linkages among sovereigns. We construct this network for a set of European sovereigns for each quarter over the period from 2005-Q3 to 2011-Q3. This is combined with data on sovereign credit default swaps (CDS) and GDP to estimate the specification in (5). Table 1 lists the thirteen sovereigns included in our sample.

The central banks of BIS member countries collect data on the balance sheet composition of the banks in their jurisdiction. They aggregate these data and report to the BIS the breakdown of banks’ assets according to the country of the issuer of the security. For the BIS member countries, this provides a network, at a quarterly frequency, of the claims held by banks headquartered in one country on entities in another. However, these represent all financial claims, not just sovereign debt. The IMF reports the dollar amount of a sovereign’s debt held by foreign creditors. While this gives the amount of a sovereign’s debt held abroad, it is an aggregated measure that does not provide the nationalities of the various foreign creditors holding a given sovereign’s debt. Thus, to construct our empirical network of sovereign debt claims, we weight the external sovereign debt amounts reported by the IMF according to the shares reported by the BIS. Note that this assumes the foreign sovereign debt holdings of a country’s financial institutions are proportional to their total foreign asset holdings. For a concrete example, suppose the BIS data report that 40% of the total financial claims issued by entities located in country A are held by institutions located in country B and 60% are held by institutions located in country C. Additionally, suppose that the IMF reports that of the debt issued by the government of sovereign A, $50 billion is held by foreign creditors.

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22 We use the BIS data on consolidated claims on an ultimate risk basis. See Appendix B for additional details.
23 Note that this assumes the foreign sovereign debt holdings of a country’s financial institutions are proportional to their total foreign asset holdings. For a concrete example, suppose the BIS data report that 40% of the total financial claims issued by entities located in country A are held by institutions located in country B and 60% are held by institutions located in country C. Additionally, suppose that the IMF reports that of the debt issued by the government of sovereign A, $50 billion is held by foreign creditors.
construction.

Figure 1 gives a visual representation of our constructed network in 2011-Q1 (the underlying amounts are reported in Appendix Table A-1). The arrows represent the total amounts of claims that banks headquartered in one country (the “creditor country”) have on the sovereign debt of another. These amounts are normalized by the size of the economy of the creditor country, using 2004 GDP, to reflect their relative exposures. Darker arrows indicate larger proportional amounts, and aggregate claims worth less than one percent of the creditor country’s 2004 GDP are not shown. Many countries have substantial aggregate claims on each other, so arrows can be bi-directional as for example between Austria (AT) and Italy (IT). The algorithm that creates this visual representation places more strongly connected countries in the center and more weakly connected countries in the periphery.\textsuperscript{24} Thus Germany (DE) and France (FR) are located near the center because they have substantial claims (outward arrows) and debts (inward arrows) with many other countries. We also see that France and Portugal (PT) have large total holdings of sovereign debt from Italy and Greece (GR), respectively, relative to their own 2004 GDP: 28.4\% for France and 12.2\% for Portugal (Appendix Table A-1). These will be relevant for the results seen in Section 5.2. Moreover, the relatively large holdings of Greek debt in Portugal, also a financially vulnerable sovereign, provides a good example where economically important spillovers could potentially arise from the balance sheet mechanism.

We collect CDS spreads from Credit Market Analytics (CMA) for each of the thirteen sovereigns in our sample. All spreads are on five-year CDS contracts, referencing the

\textsuperscript{24}There is not a unique visual representation of the network, however, as it is a projection of an \( N \times N \) matrix into two dimensions. Different algorithms (and different initializations) produce different visual representations. Nevertheless the qualitative features are reasonably stable.
sovereign entity and denominated in US dollars. We transform the CDS spreads to compute implied risk-neutral default probabilities.\textsuperscript{25} Specifically we use the 5-year sovereign CDS spreads and the U.S. Treasury yield curve to compute the time series of annualized solvency probabilities (at a quarterly frequency) for each sovereign, $p_{it}$.\textsuperscript{26} Last, in addition to the financial linkages and CDS spreads, we collect data on countries’ GDP. We use quarterly GDP data that is annualized, seasonally adjusted, and measured in fixed PPP, taken from the OECD’s Quarterly National Accounts database. Quarterly growth rates are decomposed with a principal components analysis, as described in Section 3. In addition, the common component of the growth rate is detrended by subtracting the average quarterly growth rate for each country over the period 1995–2004.

In Table 2 we provide summary statistics for variables used in the estimation of the model. The average risk-neutral solvency probability is 0.987, but many sovereigns have averages above 0.99 with relatively little variation. The lowest average solvency probabilities and highest standard deviations are seen, as would be expected, for Greece, Ireland, and Portugal, followed by Spain and Italy. The total normalized claim amounts ($\sum_{j \neq i} l_{ijt}$) vary greatly across sovereigns. Ireland, the Netherlands, and Belgium hold large amounts of sovereign debt of other European countries (relative to the size of their own economies), while Greece and Finland have comparatively negligible holdings. Most other countries have claims worth between one third and one half of their 2004 GDP. The normalized debt amounts, which are similar to debt-to-GDP ratios except that GDP is held constant, show the expected differences across countries, with an average close to one.

\textsuperscript{25}We follow the credit risk literature in analyzing risk-neutral default probabilities. See, for example, Ang and Longstaff (2013). Risk-neutral default probabilities reflect both the objective default probability and a risk premium. As such, they capture the impact of credit risk on a sovereign’s cost of borrowing, which is our ultimate object of interest.

\textsuperscript{26}Note that this transformation of CDS spreads to risk-neutral solvency or default probabilities assumes a 40% recovery rate and a discount factor derived from the current Treasury yield. See Appendix B for details on how we impute a sovereign’s risk-neutral solvency probability from its CDS spread.
4.1 Assessment of the Constructed Network

Our measure of the financial linkages among countries assumes that the allocation of external sovereign debt to foreign banks is proportional to the allocation of all financial assets from a given country. We use this constructed network rather than more direct measures of claims on sovereign debt because the latter are not consistently available for the countries in our sample. However, to assess the validity of our constructed network, we compare it with other data from the BIS and from the European Banking Authority (EBA), which are available either for a subsample of countries or at particular points in time.

The BIS data on bilateral foreign claims are available by the sector of the counterparty, including the public sector, for six of the countries in our sample starting in 2010-Q4. Separately, the EBA has released data from its bank stress tests, which list exposures to the sovereign debt from each country for a sample of large banks. These banks account for a large portion of the banking system in Europe, so adding across the banks headquartered in one country gives a good estimate of the total claims held by banks in that country on the sovereign debt from each other country. The 2011 EBA stress test used data on these exposures as of December 31, 2010. Accordingly, we can make a comparison between these EBA data and the BIS data on claims on public sector counterparties, against our constructed network, using 2010-Q4. Appendix Table A-2 presents the correlations between these alternative measures and our constructed measure, overall and for each country. The overall correlation with our measure is 0.91 for the BIS public sector debt data and 0.88 for the EBA stress test data, which gives us confidence that our constructed network is reasonably accurate.
4.2 Descriptive Linear Regressions

As a final descriptive exercise, we estimate a series of naïve linear regressions using the variables that appear in our network model. These regressions do not account for the joint determination of credit risk in a payment equilibrium, so the coefficients do not have a causal interpretation. Rather, the purpose of this exercise is to illustrate the variation in the data that identifies our stuctural parameters. Thus, the coefficients should be taken simply as conditional correlations. The main specification is

\[ p_{it} = a_0 + a_1 t + b \sum_{j \neq i} l_{ij} p_{jt} + c D_{it} + d_1 Y_{it} + d_2 \Delta Y^c_{it} + d_3 \Delta Y^r_{it} + u_i + v_{it}, \quad (6) \]

where \( a_0, a_1, b, c, d_1, d_2, \) and \( d_3 \) are coefficients, and \( u_i \) and \( v_{it} \) are country fixed effects and random error terms, respectively. The coefficient \( b \) expresses the conditional correlation between sovereign \( i \)'s solvency probability \( (p_{it}) \) and the weighted average of its debtor's solvency probabilities \( (p_{jt}), \) weighted by the financial linkages \( (l_{ij}) \). This is the same cross-moment that identifies the estimate of \( \gamma \) in our network model, although here the estimate of \( b \) is obviously biased from the simultaneity of \( p_{it} \) and \( p_{jt}, j \neq i. \)

The results of this exercise are shown in Table 3. First we estimate (6) with only the time trend and weighted average of debtor solvency probabilities on the right-hand side. When we add the other variables (column 2), the coefficient on the debtor solvencies drops substantially, from 0.040 to 0.026. To interpret these magnitudes, the latter coefficient says that an increase of 100 basis points (bps) in the weighted average of the solvency probabilities among a country’s debtors is associated with a 2.6 bps increase in its own solvency probability. This is quite a small association, which is at least suggestive that the true spillover effects are not large. Columns 3 and 4 replace the linear time trend \( (a_0 + a_1 t) \) with time period fixed effects \( (a_t 1_t) \), which is reasonable here because the fixed
effects difference out in a linear regression. The coefficients are qualitatively similar to the prior estimates, although the magnitude of the coefficient on the debtor solvency probabilities falls to 0.018 in column 4.\footnote{Acharya, Drechsler, and Schnabl (2014) find quite similar magnitudes for the association between individual bank CDS rates and foreign sovereign CDS rates in Europe, also using BIS data to weight the exposures to each foreign sovereign. In a specification with time and bank fixed effects, for example, they estimate that a 10% increase in the weighted average of foreign sovereign CDS rates is associated with a 0.2% increase in domestic bank CDS rates.} The overall similarity indicates that a linear time trend fits the data reasonably well and should not affect the results qualitatively, although there may be a modest upward bias in the estimate of $\gamma$ in our equilibrium network model (but as noted in Section 3.1 our concern is mainly with downward biases).

When time period fixed effects are included, the financial linkages provide a crucial source of variation to estimate $b$ because the overall correlation in solvency probabilities at a point in time would be absorbed by the fixed effects $a_t$. This is why we say the estimate of $\gamma$ ultimately depends on the extent to which differential comovements in solvency probabilities are explained by differential financial linkages. Thus it is reassuring to see that the cross-moment expressed with $b$, which drives the estimate of $\gamma$, remains largely intact when time fixed effects are used in place of a linear time trend.

5 Empirical Results

5.1 Estimates and Model Fit

We now present the results from our network model, expressed in equation (5). Parameter estimates and the marginal effects of the associated variables are listed in Table 4. The key parameter is $\gamma$, which governs the spillovers among sovereigns. In terms of the model, $\gamma$ is interpreted as the effect of repayments received from other sovereigns, on a sovereign’s own (risk-neutral) solvency probability. The average marginal effect is 0.021, which is similar in
magnitude to the coefficient on debtor solvencies in the above regressions (Table 3, column 2). As expected, however, the estimate from our equilibrium model is smaller than the naïve regression estimate because here we properly account for the joint determination of solvency in the network. The effect of a sovereign’s total foreign debt load is given by \( \alpha \). Its marginal effect implies that an increase in the normalized debt load equal to one standard deviation of this variable (0.32, Table 2) would raise the (risk-neutral) probability of default by 25 bps on average.\(^{28}\) The effects of the GDP variables are as expected.

Figure 2 plots the observed and predicted solvency probabilities to illustrate the dispersion in the data and show the model’s fit. In particular, we plot each sovereign’s “observed” risk-neutral solvency probability, derived from its 5-year CDS spread, against the model’s predicted solvency probability, for each of the 293 quarterly observations in our sample. For most countries the observed and predicted solvency probabilities are both quite close to 1. However the figure shows the notable exceptions to this, mainly for Greece, Ireland, and Portugal, and to a lesser extent for Spain and Italy. The model predictions match their empirical counterparts very well, as seen from the fact that most observations fall close to the 45° line. The correlation between the observed and predicted values is 0.965.

5.2 Simulations and Contagion

Using the estimated version of equation (5), we can simulate the short-run effect of the default of one sovereign on the risk of default of other sovereigns in the network. This provides one assessment of the potential for contagion based on the balance sheet mechanism. We then construct a measure of the expected spillover losses due to this increased probability of default of other sovereigns, which can be used to quantify the systemic risk from each

\(^{28}\)This marginal effect is lower than the analogous coefficient in the linear regressions, but we see a more similar magnitude in a naïve nonlinear regression model. (We estimated a version of (6) as a GLM with a probit link function. These results are not shown but are available on request.)
sovereign. Finally, we simulate the change in solvency probabilities if there were no spillover effects, which provides a simple illustration of the impact of this form of contagion risk on a sovereign’s cost of borrowing.

To simulate the default of a given sovereign \( j \) in period \( t \), we fix the solvency indicator for that country at zero (\( s_{jt} = 0 \)) and recompute the solvency probabilities for all other sovereigns according to the estimated version of (5). The four panels in Figures 3 and 4 plot the increase in the (risk-neutral) default probabilities for selected sovereigns, given a default in Greece, Portugal, Italy, or Spain. These simulations should be considered separately for each period, as there are no cumulative or long-run effects expressed in the model.

There are notable differences in the impact of a default by one of these four sovereigns. Greece poses a somewhat substantial threat to Ireland and Portugal, reducing their solvency probabilities by up to 60 bps with a default (Figure 3-A).\(^{29}\) Portugal also poses a small threat to Ireland (but not reciprocally to Greece), and is more of a threat to Spain in the last four quarters in our sample (Figure 3-B). The asymmetric relationship between Greece and Portugal comes from the fact that Portugal held a relatively large amount of Greek debt at that time, but not vice versa.

Figure 4-A shows that a default in Italy would increase the default probability in France by up to 60 bps. This is substantial relative to the baseline default probability in France, which ranges from 100-200 bps at the end of our sample period. Spain would not have such an impact on any large economy, but a Spanish default would increase the default risk in Portugal by up to 50 bps (Figure 4-B). Both Italy and Spain would also impact Ireland, although the effect from Italy is about three times larger (notice that Ireland is plotted on

\(^{29}\)Official GDP data are not available for Greece in 2011-Q2 or Q3, so there are no simulations for that period. The reduction in the spillover effect on Ireland in the two preceding quarters is due to a large drop in Irish holdings of Greek debt in our constructed network, from 0.102 to 0.012 in the normalized measure.
the right-hand axis in Figure 4-A). Overall, the difference between the effects of a default in Italy and Spain is not surprising, given that Italy’s foreign debt was three times larger than Spain’s in 2010 and 2011.

5.3 Spillovers per Dollar of Debt

To account for the large differences across sovereigns in the amount of their foreign debt, we construct a measure that normalizes for the total amount of external debt of the country with the initial default. This measure gives the expected spillover losses due to any additional defaults, per dollar of foreign debt of the initial country. It is analogous to the Katz-Bonacich centrality measure that other authors have used to quantify the systemic importance of each entity in a financial network (e.g., Cohen-Cole, Patacchini, and Zenou 2011; Denbee, Julliard, Li, and Yuan 2014; Bonaldi, Hortacsu, and Kastl 2014). As with that measure, our measure can be used to analyze systemic risk and identify which sovereigns pose the greatest threat.\footnote{The Katz-Bonacich centrality measure does not apply directly in our case because our model is nonlinear. However, our measure similarly captures all the higher order (i.e., indirect) effects of an initial shock.}

Our measure is defined as follows. Given a default by sovereign $j$ in period $t$, we use the above simulations to calculate the change in solvency probabilities among the other sovereigns in the network. Let $\hat{p}_{it}$ be the original predicted solvency probability for some country $i$ in period $t$ using the estimated model, and let $\tilde{p}_{it}(j)$ be the simulated solvency probability under the counterfactual that country $j$ defaults. These simulated probabilities reflect both the direct effects of the loss of repayments from country $j$, and any indirect effects from the further losses of repayments from other countries ($k$, etc.) that default because they are not repaid either. This includes any higher order sequences of losses because a new payment equilibrium is determined for the scenario in which country $j$ defaults. Then, given the baseline and simulated probabilities, the expected spillover losses from country $i$ due to the
default of country \( j \) is \([\hat{p}_{it} - \tilde{p}_{it}(j)]D_{it}\). We add these losses across all countries, and divide by the total foreign debt of country \( j \) to normalize, which yields our measure:

\[
\lambda_{jt} \equiv \frac{1}{D_{jt}} \sum_{i \neq j} [\hat{p}_{it} - \tilde{p}_{it}(j)]D_{it}.
\]  

(7)

This gives the expected spillover losses per dollar of debt of country \( j \). Our measure can be thought of as expressing the “contagiousness” of a sovereign’s foreign debt. It captures differences in the potential for spillovers that arise from a country’s position in the network of financial linkages—i.e., who its creditors are, and how sensitive those creditors are to losses—rather than from the total amount of its foreign debt.

Figure 5 plots \( \lambda_{jt} \) for the most at-risk sovereigns (Panel A) and for five large European economies (Panel B). The magnitudes of these expected spillovers are not large: for each $1 of debt directly lost in default, the expected losses from additional defaults at other countries are less than 2 cents. The levels and trends are generally similar among all the countries in both panels. Greece and Portugal have relatively more contagious debt, as does Germany, followed by Ireland, France, Italy, and Spain, and last the United Kingdom.\(^{31}\) Figure 6 shows \( \lambda_{jt} \) for smaller European economies (Panel A) and the weighted average among all the sovereigns in our sample (Panel B). Austria’s foreign debt has the highest potential for contagion, with expected spillover losses per dollar of debt that are roughly double those of any other sovereign (note the different scale for this plot). This turns out to be the case because Italy holds a relatively large share of Austria’s debt, and Italy is relatively sensitive to losses on its claims because it has a somewhat higher risk of default. Finally, the weighted average of the spillovers, which uses the total foreign debt amounts \( D_{jt} \) as weights, rose to

\(^{31}\)The UK has lower potential spillover losses than other countries because a relatively large proportion of its debt is held outside Europe (e.g., by the United States). This debt is included in the normalization but is not counted toward spillover losses within the sample of European sovereigns.
almost 0.6 cents per dollar during the recession of 2008-09, then leveled at 0.4 cents until the end of 2010, when it began to rise steadily as the sovereign debt crisis widened in Europe.

A natural concern is whether these expected losses incorporate market beliefs about the likelihood of a bailout for a sovereign at risk of default. Indeed we think it is reasonable to assume that the observed CDS spreads do reflect market beliefs about possible bailouts. Accordingly, the total expected losses should be interpreted as expectations for losses that may occur despite efforts to bail out a sovereign (e.g., as in the case of Greece in March 2012). Similarly, the expected losses due to contagion would incorporate beliefs about further bailouts to prevent additional defaults. These beliefs about bailouts should affect the total expected losses and the expected spillover losses in roughly equal fashion. Our finding that the spillovers represent a very small portion of the total losses would not be impacted unless market beliefs about the likelihood of a bailout are drastically different when countries are at risk due to contagion rather than their own internal factors.\textsuperscript{32}

\section{5.4 Contagion Effects on a Sovereign’s Cost of Borrowing}

Finally, we investigate the extent to which the risk of contagion impacts sovereigns’ cost of borrowing. We simulate default probabilities under a scenario where there are no spillovers from a default and compare these with the baseline default probabilities. Specifically, we set the model so that a sovereign experiences no loss in repayments if one of its debtors defaults and recompute the solvency probabilities using the estimated parameters. To assess the extent to which the estimated spillovers affect a sovereign’s cost of borrowing, we convert these risk-neutral probabilities to CDS spreads under our assumed recovery rate of $\delta = 0.4$.

\textsuperscript{32}Specifically, to have a downward bias on our estimate of spillover losses, the market would need to expect greater effort to bail out countries, such as Ireland and Portugal, that have substantial holdings of foreign sovereign debt, compared with Greece. In that case the solvency probability for Ireland and Portugal would not be as strongly related to the solvency probability of Greece, despite their relatively large holdings of Greek debt. We do not think this scenario is very plausible.
For each sovereign at each date, we compare our model-estimated, implied CDS spread to the spread in a counterfactual case where all spillover effects are shut down. For sovereign $i$ at quarter $t$, these quantities are labeled as $\hat{CDS}_{it}^C$ and $\hat{CDS}_{it}^{NC}$, respectively. We measure the proportion of a sovereign’s borrowing cost or credit spread resulting from potential spillovers as

$$\frac{\hat{CDS}_{it}^C - \hat{CDS}_{it}^{NC}}{\hat{CDS}_{it}^C}.$$ 

In Figure 7 we plot the time series of the cross-sectional average of this measure. As evidenced by the figure, the risk of contagion constitutes a small but growing portion of the average sovereign’s credit spread in our sample. Consistent with our prior results, the risk of contagion has a very small effect on the cost of borrowing for the sovereigns in our sample. In particular, Figure 7 shows that contagion risk accounts for less than 1% of the average sovereign’s credit spread.

In Table 5 we report the estimated contagion risk as a fraction of the total credit spread in 2011-Q1 for each sovereign in our sample. The results again show that contagion has a small effect on sovereign borrowing costs, however, we see significant heterogeneity in this effect. In particular, the differential effect of contagion risk on sovereign borrowing costs is not simply a function of a sovereign’s total credit risk. Rather, the impact of contagion risk depends on a sovereign’s network linkages. Perhaps somewhat surprisingly, we find that contagion risk has the largest proportional effect on the borrowing costs of France, accounting for 4.12% of the sovereign’s total credit spread in 2011-Q1.
6 Conclusion

In this paper, we build upon the recent theoretical literature on financial networks to construct a network model of credit risk among thirteen European sovereigns. Using data on sovereign CDS spreads and the cross-holdings of sovereign debt, we estimate the model and conduct counterfactual experiments to quantify the spillover effects from a direct, “balance sheet” mechanism for contagion.

Our estimates imply that credit markets perceived the potential spillovers from a sovereign default to be small in magnitude. On average, the predicted losses due to contagion account for only one percent of the total expected losses implied by the sovereign CDS spreads in our sample of countries. This suggests that the direct spillover, “balance sheet” channel for contagion did not have an economically significant effect on sovereign borrowing costs over this period. However, there exist other channels through which contagion may operate and these may present more significant effects than the channel considered in this paper. Assessing the quantitative importance of these various contagion mechanisms is essential for understanding the relative costs and benefits to a sovereign bailout. Additionally, our estimation framework and approach, along with the contagion measure that we develop, can be used to quantify expected spillovers in other markets.
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Figure 1: Network Graph, 2011-Q1. The figure displays the network structure of aggregate sovereign debt holdings in the first quarter of 2011. Countries are represented by their two letter abbreviation in Table 1. Arrows represent bank holdings from one country on the sovereign debt of another. Darker, thicker arrows indicate larger amounts in proportion to the creditor country’s GDP in 2004.
Figure 2: Predicted and Observed Solvency Probabilities. The figure plots the predicted and observed risk-neutral quarterly solvency probabilities for each country at each quarter in our sample. Observed solvency probabilities are obtained with a transformation of 5-year CDS contract prices, as described in Section 4 and Appendix B. Predicted solvency probabilities are generated from the estimated network model, specified in equation (5). Country abbreviations are listed in Table 1.
Figure 3: Simulated Default Probabilities: Greece and Portugal. The figure shows the change in the risk-neutral probability of default for selected sovereigns, assuming a default in Greece (Panel A) or Portugal (Panel B), in simulations using the estimated model. See Section 5.2 for details. Country abbreviations are listed in Table 1.
Figure 4: Simulated Default Probabilities: Italy and Spain. The figure shows the change in the risk-neutral probability of default for selected sovereigns, assuming a default in Italy (A) or Spain (B), in simulations using the estimated model. See Section 5.2 for details.
Figure 5: Expected Spillover Losses per Dollar of Debt: At-Risk Sovereigns and Large Economies. The figure plots our measure of expected losses due to increased risk of default at other countries, conditional on a default by the country indicated in each series, per dollar of that country’s foreign debt. See equation (7) for the definition of this measure.
Figure 6: Expected Spillover Losses per Dollar of Debt: Medium or Small Economies, and Weighted Average. The figure plots our measure of expected losses due to increased risk of default at other countries, conditional on a default by the country indicated in each series, per dollar of that country’s foreign debt. See equation (7) for the definition of this measure. Weighted average is among all countries in the sample, weighted by each country’s total foreign debt.
Figure 7: Contagion risk as a fraction of a sovereign’s credit spread. The figure plots the time series of the cross-sectional average of contagion risk as a fraction of the sovereign’s total credit spread. For each sovereign $i$ and quarter $t$, the fraction of borrowing costs from contagion is estimated as

$$\frac{\hat{CDS}_t^C - \hat{CDS}_t^{NC}}{\hat{CDS}_t^C}$$

The figure plots the cross-sectional average of this quantity for each quarter of the sample. See Section 5.4 for more details.
Table 1: **List of Sovereigns**

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</tr>
<tr>
<td>Greece</td>
<td>GR</td>
<td>United Kingdom</td>
<td>GB</td>
</tr>
<tr>
<td>Ireland</td>
<td>IE</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Note: The table lists the names and abbreviation codes for the sovereigns in our sample.*

Table 2: **Summary Statistics for Estimation Variables**

<table>
<thead>
<tr>
<th>Country</th>
<th>Solvency Prob. Mean (SD)</th>
<th>Total Claims Mean (SD)</th>
<th>Total Debt Mean (SD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>All</td>
<td>0.9870 (0.024)</td>
<td>0.42 (0.31)</td>
<td>0.94 (0.32)</td>
</tr>
<tr>
<td>AT</td>
<td>0.9927 (0.008)</td>
<td>0.32 (0.06)</td>
<td>0.93 (0.11)</td>
</tr>
<tr>
<td>BE</td>
<td>0.9912 (0.011)</td>
<td>0.76 (0.22)</td>
<td>1.30 (0.16)</td>
</tr>
<tr>
<td>DE</td>
<td>0.9964 (0.004)</td>
<td>0.30 (0.04)</td>
<td>0.94 (0.13)</td>
</tr>
<tr>
<td>ES</td>
<td>0.9855 (0.017)</td>
<td>0.19 (0.04)</td>
<td>0.58 (0.15)</td>
</tr>
<tr>
<td>FI</td>
<td>0.9942 (0.002)</td>
<td>0.07 (0.01)</td>
<td>0.72 (0.06)</td>
</tr>
<tr>
<td>FR</td>
<td>0.9946 (0.006)</td>
<td>0.48 (0.17)</td>
<td>1.03 (0.18)</td>
</tr>
<tr>
<td>GB</td>
<td>0.9906 (0.005)</td>
<td>0.34 (0.06)</td>
<td>0.79 (0.16)</td>
</tr>
<tr>
<td>GR</td>
<td>0.9675 (0.048)</td>
<td>0.02 (0.01)</td>
<td>1.31 (0.25)</td>
</tr>
<tr>
<td>IE</td>
<td>0.9701 (0.038)</td>
<td>1.04 (0.11)</td>
<td>0.79 (0.38)</td>
</tr>
<tr>
<td>IT</td>
<td>0.9865 (0.015)</td>
<td>0.19 (0.08)</td>
<td>1.44 (0.16)</td>
</tr>
<tr>
<td>NL</td>
<td>0.9955 (0.004)</td>
<td>0.86 (0.08)</td>
<td>0.77 (0.14)</td>
</tr>
<tr>
<td>PT</td>
<td>0.9751 (0.040)</td>
<td>0.20 (0.06)</td>
<td>0.84 (0.18)</td>
</tr>
<tr>
<td>SE</td>
<td>0.9950 (0.004)</td>
<td>0.43 (0.07)</td>
<td>0.61 (0.05)</td>
</tr>
</tbody>
</table>

*Notes: Sample averages and standard deviations of the listed variables are given for the entire panel of countries (“All”) and then separately for each country. Solvency probabilities are risk-neutral probabilities derived from 5-year CDS contracts. Total claims are the sum of bilateral financial claims (\(\sum_{j\neq i}^{} l_{ijt}\)), constructed from BIS and IMF data. Total debt is total foreign debt from the IMF. Claims and debt are normalized by each country’s 2004 GDP. See Section 4 and Appendix B for further details.*
Table 3: Linear Regressions for Solvency Probabilities

<table>
<thead>
<tr>
<th>Variable</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Debtor solvencies ( (L \cdot p) )</td>
<td>0.040*</td>
<td>0.026*</td>
<td>0.029*</td>
<td>0.018*</td>
</tr>
<tr>
<td>Own debt ( (D) )</td>
<td></td>
<td>-0.051*</td>
<td></td>
<td>-0.050*</td>
</tr>
<tr>
<td>GDP level ( (Y) )</td>
<td></td>
<td>0.211*</td>
<td></td>
<td>0.354*</td>
</tr>
<tr>
<td>GDP growth, common ( (\Delta Y_c) )</td>
<td></td>
<td>0.001</td>
<td></td>
<td>0.001</td>
</tr>
<tr>
<td>GDP growth, residual ( (\Delta Y_r) )</td>
<td></td>
<td>0.003*</td>
<td></td>
<td>0.003*</td>
</tr>
<tr>
<td>Time control</td>
<td>( t )</td>
<td>( t )</td>
<td>( t ), 1 ( t )</td>
<td>( t )</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.484</td>
<td>0.640</td>
<td>0.552</td>
<td>0.738</td>
</tr>
<tr>
<td>( N )</td>
<td>293</td>
<td>293</td>
<td>293</td>
<td>293</td>
</tr>
</tbody>
</table>

Notes: The dependent variable is the solvency probability for country \( i \) in period \( t \): \( p_{it} \). Each column is a separate regression. All regressions include country fixed effects. See equation (6) for the complete specification. Standard errors are in parentheses; \* p-value < 0.05.
Table 4: Estimated Parameters and Marginal Effects in the Network Model

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>0.746</td>
<td>0.021</td>
<td>Equilibrium repayments on claims ($R$)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.280</td>
<td>0.008</td>
<td>Total foreign debt ($D$)</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>4.363</td>
<td>0.123</td>
<td>GDP level ($Y$)</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>0.135</td>
<td>0.004</td>
<td>Common component of GDP growth rate ($\Delta Y^c$)</td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>0.042</td>
<td>0.001</td>
<td>Residual component of GDP growth rate ($\Delta Y^r$)</td>
</tr>
</tbody>
</table>

*Notes:* The table shows estimates of the parameters in equation (5) and marginal effects of the associated variables. Note that $\alpha$ enters the model negatively. Marginal effects are computed as the average of the marginal effect for each observation. The equilibrium repayments ($R$), foreign debt ($D$), and GDP level ($Y$) are normalized by the country’s 2004 GDP.

Table 5: Contagion Risk as a Fraction of Sovereign Credit Spreads

<table>
<thead>
<tr>
<th>Country</th>
<th>Contagion Risk as a % of Total Credit Spread</th>
</tr>
</thead>
<tbody>
<tr>
<td>AT</td>
<td>1.71</td>
</tr>
<tr>
<td>BE</td>
<td>2.13</td>
</tr>
<tr>
<td>DE</td>
<td>1.72</td>
</tr>
<tr>
<td>ES</td>
<td>0.57</td>
</tr>
<tr>
<td>FI</td>
<td>0.00</td>
</tr>
<tr>
<td>FR</td>
<td>4.12</td>
</tr>
<tr>
<td>GB</td>
<td>0.59</td>
</tr>
<tr>
<td>GR</td>
<td>0.04</td>
</tr>
<tr>
<td>IE</td>
<td>1.57</td>
</tr>
<tr>
<td>IT</td>
<td>0.64</td>
</tr>
<tr>
<td>NL</td>
<td>2.75</td>
</tr>
<tr>
<td>PT</td>
<td>2.07</td>
</tr>
<tr>
<td>SE</td>
<td>1.13</td>
</tr>
</tbody>
</table>

*Notes:* The table presents results from a counterfactual simulation in which spillover effects are eliminated. Specifically, a sovereign is assumed to experience no loss in repayments if one of its debtors defaults. The reported values are the percentage of a sovereign’s credit spread (measured as a 5-year CDS spread) in 2011-Q1, that can be attributed to contagion spillovers. The reported values are in percentage terms, i.e. 1.5 implies that 1.5% of the sovereign’s 5-year CDS spread is due to contagion risk.
Appendices

A Analysis of Potential Biases

Here we provide a detailed analysis of the biases that could arise if certain key assumptions in our empirical model were violated. We consider four potential issues: correlations in the financial shocks between countries, the endogeneity of financial linkages, endogenous default decisions, and internal amplification mechanisms with different impacts across countries. Our focus is on the bias in the estimate of $\gamma$, the parameter that governs the magnitude of spillovers from a default, and in each case we show that the likely bias is upward.

Correlations in financial shocks. Although we speculate that a correlation in the financial shocks among countries could be identified, we estimate the model under the assumption that they are independent. If, in fact, the shocks were correlated, such as $X_{it}$ and $X_{jt}$, the observed correlation between $p_{it}$ and $p_{jt}$ would reflect this in addition to the true effects of repayments between countries $i$ and $j$ (i.e., $\gamma l_{ijt}[\delta+(1-\delta)s_{jt}]$ and $\gamma l_{jyt}[\delta+(1-\delta)s_{it}]$). Because the repayments are an increasing function of the shocks (via $s_{it}$ and $s_{jt}$), the estimate of $\gamma$ would be biased in the same direction as the correlation in these shocks. Hence, a positive correlation in the financial shocks between countries would result in an upward bias.\(^{33}\)

Endogeneity of financial linkages. In general, a bias would arise if the observed claim amounts $l_{ijt}$ were correlated with the unobservables in period $t$. Because these claims are established at the end of the previous period, this would require that the unobservables, specifically the financial shocks, are correlated over time.\(^{34}\) Then, in some model of sovereign debt holdings (which is beyond the scope of this paper), banks in country $i$ might reduce their holdings of debt from country $j$ if a low value of the shock $X_{j,t-1}$ is observed, because this predicts a low value of $X_{jt}$, and hence a higher probability of default for country $j$ in period $t$. This process would yield a positive correlation between the claims of $i$ on $j$ ($l_{ijt}$) and the current shock for $j$ ($X_{jt}$), which determines the solvency of $j$ ($s_{jt}$) and thereby affects the repayments to $i$ ($R_{it}$). However, our predicted repayments in the model (5) do not account for the lagged shocks (i.e., $X_{j,t-1}$), so there would be an error term for the difference between the correct predictions and our predictions (heuristically, $E[R_{it}|X_{j,t-1},\ldots]-E[R_{it}|\ldots]$). This error term would have a positive correlation with our predicted values of $R_{it}$ because of the

\(^{33}\)If this correlation were uniform among all countries, it might be possible to address with period-specific effects like $\pi_t$ (although this can be problematic in a nonlinear model). If the correlations were different for different country-pairs, as in say $\rho_{ij}$, the bias could depend on the relationship between $\rho_{ij}$ and $l_{ijt}$. It seems likely that the shocks would be more highly correlated between countries with stronger linkages, so that $\rho_{ij}$ and $l_{ijt}$ would be positively related, which would again yield an upward bias in the estimate of $\gamma$.

\(^{34}\)As noted previously, none of the empirical papers that estimate structural models of spillovers in financial networks allow for shocks that are correlated over time.
positive correlation between $l_{ijt}$ and $X_{jt}$ described above. Therefore, this positive correlation would generate an upward bias in the estimate of $\gamma$.

On the other hand, if investors in country $i$ tend to diversify away from home when their own country experiences a negative shock, then there could be a downward bias. In this case, given a low value of the domestic shock $X_{i,t-1}$, banks in country $i$ might increase their holdings of debt from other sovereigns. This would introduce a negative correlation between $l_{ijt}$ and $X_{it}$ that is not accounted for in our model. However, we think this is an unlikely scenario for two reasons. First, if shocks are correlated across countries (as discussed above), then typically there would be no relative advantage to increasing investment abroad. Second, there is evidence that banks tend to increase their “home bias” in times of crisis, even in crises which specifically relate to domestic conditions (Giannetti and Laeven 2012a,b).

**Endogenous default.** If default were endogenous, the decision rule for each country would be a function of the state variables known at the time of the payment equilibrium: the network-wide matrix and vectors $L_t$, $D_t$, $Y_t$, and $X_t$. Conceptually, we could incorporate such a decision rule into the solvency condition with a policy function $\pi_i(L_t, D_t, Y_t, X_t)$ that adjusts the threshold for default up or down from fixed values captured by the parameters $\bar{\pi}_i + \pi_t$ (bars added here for clarity). Then $-\pi_i(L_t, D_t, Y_t, X_t)$ would appear in (5) as an error term, and so the question is how it would be correlated with our predicted repayments based on an exogenous default rule. If we assume that a country does not receive payments on its claims when it defaults and goes into autarky, then the relative value of default should be decreasing in the (true) equilibrium repayments. This suggests that the threshold $\pi_i(L_t, D_t, Y_t, X_t)$ would be decreasing in $R_{it}$ (as derived from the state variables and the other decision rules $\pi_j(\cdot)$), making default less likely when $R_{it}$ is larger. This relationship also holds for our predicted repayments based on an exogenous default rule. Hence there would be a positive correlation between the $R_{it}$ we use and this error term, $-\pi_i(\cdot)$, which would produce an upward bias in the estimate of $\gamma$.

**Amplification mechanisms.** To reflect internal amplification mechanisms with potentially different impacts across countries and over time, we could add parameters that vary the effect of repayments over $i$ and $t$, as in $(\bar{\gamma} + \gamma_{it})R_{it}$ (bar added for clarity). Then $\gamma_{it}R_{it}$ would be an error term in (5), and so the question is whether these deviations would be systematically positive or negative when our predicted $R_{it}$ is relatively higher or lower. If we suppose that banks are more leveraged on average when they have more debt holdings, then the sensitivity to losses ($\gamma_{it}$) should be greater when the total holdings ($\sum_{j \neq i} l_{ijt}$) are larger. Accordingly, there would be a positive correlation between our predicted repayments and the error term $\gamma_{it}R_{it}$, which would generate an upward bias in the estimate of $\bar{\gamma}$.

---

35 This assumes the putative equilibrium strategies would have a single crossing property in $X_{it}$.
B Construction of Variables

Here we describe how we construct the network of financial linkages used in our analysis and how we transform the observed spreads on 5-year CDS contracts into risk-neutral solvency probabilities.

Constructing the Network of Financial Linkages

The BIS reports asset holdings of financial institutions according to country of ultimate counterparty at a quarterly frequency. This measure includes all financial assets, not just sovereign debt, that is held by the financial sector. Specifically, we use the BIS consolidated international banking statistics on an ultimate risk basis. The data provided on an ultimate risk basis are more appropriate for our purposes than the data on an immediate borrower basis. For example, in a 2010 Quarterly Review issued by the BIS (Avdjiev, Upper, and von Kleist (2010)), it is stated:

“The BIS consolidated international banking statistics on an ultimate risk basis are the most appropriate source for measuring the aggregate exposures of a banking system to a given country. Unlike the BIS consolidated international banking statistics on an immediate borrower basis, they are adjusted for net risk transfers. For example, suppose that a Swedish bank extends a loan to a company based in Mexico and the loan is guaranteed by a US bank. On an immediate borrower basis, the loan would be considered a claim of a Swedish bank on Mexico, as the immediate borrower resides in Mexico. On an ultimate risk basis, however, the loan would be regarded as a claim of a Swedish bank on the United States since that is where the ultimate risk resides.”

We define $b_{ijt}$ as the value of sovereign $i$’s financial claims on sovereign $j$ at date $t$ as reported by the BIS. We define an adjusted claims measure as

$$\text{BISwgt}_{ijt} = \frac{b_{ijt}}{\sum_{k=1}^{N} b_{jk} + \sum_{k=N+1}^{#BIS} b_{kj}}$$

Note that the data on financial holdings provided by the BIS include countries outside of the 13 European countries in our sample. As indicated by the second term in the denominator above, we include these data to compute our adjusted claims measure (#BIS is the total number of BIS reporting countries). To construct the network of holdings used in our estimation, we use the adjusted claims to weight each counterparty’s total externally held sovereign debt. The measure of a sovereign $j$’s debt that is externally held, $D_{j,t}^{\text{foreign}}$, comes
from data provided by the IMF. Finally, we compute the measure of sovereign $i$’s claims on sovereign $j$ held at date $t$ as

$$l_{ijt} = \text{BISwgt}_{ijt} \cdot D_{j}^{\text{foreign}} / Y_{i,2004}$$

This includes the normalization for sovereign $i$’s 2004 GDP ($Y_{i,2004}$). These $l_{ijt}$’s are the bilateral claims that are ultimately used in our estimation of the network.

**Imputing Risk-Neutral Solvency Probabilities from CDS Spreads**

We use spreads on 5-year CDS contracts to impute risk-neutral solvency probabilities for the sovereigns in our sample. CDS contracts provide insurance against a credit event of a reference entity, which in our case is a sovereign. The purchaser of protection obtains the right to sell bonds issued by the underlying entity, at their face value, to the seller of the CDS contract. In exchange, the purchaser of the CDS contract makes periodic payments to the seller until the occurrence of a credit event by the reference entity or the maturity of the contract.

Let $P_t$ denote the risk-neutral probability of the underlying sovereign remaining solvent (without a credit event) up to date $t$ (i.e., the cumulative survival probability). The present value of the contingent payments received by the buyer of protection can be expressed as

$$\sum_{t=1}^{T} (1 - \delta)(P_{t-1} - P_t)d_t$$

where $\delta$ is an assumed recovery rate on the sovereign bonds and $d_t$ is the risk-free discount factor. Similarly, the present value of the fixed payments made by the buyer of protection can be expressed as

$$\sum_{t=1}^{T} d_t P_{t-1} S$$

where $S$ is the fixed payment rate (commonly referred to as the CDS spread).

We obtain a time series of CDS spreads for each of the 13 sovereigns in our sample using data from CMA. We only have complete data in our sample for 5-year CDS contracts denominated in US dollars. As such, we assume a constant hazard rate for a sovereign default in order to impute risk-neutral solvency probabilities from the CDS spreads. Under this assumption and letting $(1 - \bar{p})$ denote the per period probability of default, so that $\bar{p}$ is the per period solvency probability, we have $P_t = \bar{p}^t$. By no arbitrage, the present value of
the fixed and contingent payments must be equal, which yields

\[ \sum_{t=1}^{T} (1 - \delta)[\hat{p}^{t-1} - \hat{\rho}^t]d_t = \sum_{t=1}^{T} d_t \hat{p}^{t-1} S \]

Following the literature and estimates from a sample of historical sovereign defaults, we assume a recovery rate of \( \delta = 0.4 \). We compute the discount factor, \( d_t \), using empirical yields on US Treasuries. Given data for the CDS spreads, \( S \), we can impute the risk-neutral quarterly solvency probability, \( \bar{p} \), using the equation above. Note that this can be repeated for each sovereign at each date in our sample, providing the panel of implied, risk-neutral quarterly solvency probabilities, \( p_{it} \), required for our estimation.
Table A-1: Aggregate Bank Holdings of Foreign Sovereign Debt in 2011-Q1

Holdings from Each Sovereign as Percentage of Own Country’s 2004 GDP

<table>
<thead>
<tr>
<th>Bank HQ Country</th>
<th>AT</th>
<th>BE</th>
<th>DE</th>
<th>ES</th>
<th>FI</th>
<th>FR</th>
<th>GB</th>
<th>GR</th>
<th>IE</th>
<th>IT</th>
<th>NL</th>
<th>PT</th>
<th>SE</th>
</tr>
</thead>
<tbody>
<tr>
<td>AT</td>
<td>-</td>
<td>0.6</td>
<td>18.6</td>
<td>1.5</td>
<td>0.2</td>
<td>4.4</td>
<td>1.3</td>
<td>3.0</td>
<td>0.3</td>
<td>11.1</td>
<td>2.6</td>
<td>0.4</td>
<td>0.3</td>
</tr>
<tr>
<td>BE</td>
<td>0.6</td>
<td>-</td>
<td>5.0</td>
<td>3.5</td>
<td>0.1</td>
<td>18.6</td>
<td>2.0</td>
<td>1.6</td>
<td>2.2</td>
<td>9.4</td>
<td>2.9</td>
<td>0.8</td>
<td>0.2</td>
</tr>
<tr>
<td>DE</td>
<td>2.7</td>
<td>0.8</td>
<td>-</td>
<td>3.7</td>
<td>0.3</td>
<td>9.2</td>
<td>3.8</td>
<td>2.4</td>
<td>1.3</td>
<td>8.3</td>
<td>2.7</td>
<td>1.1</td>
<td>0.5</td>
</tr>
<tr>
<td>ES</td>
<td>0.3</td>
<td>0.2</td>
<td>4.4</td>
<td>-</td>
<td>0.1</td>
<td>2.9</td>
<td>6.9</td>
<td>0.3</td>
<td>0.3</td>
<td>3.9</td>
<td>0.7</td>
<td>5.3</td>
<td>0.1</td>
</tr>
<tr>
<td>FI</td>
<td>0.2</td>
<td>0.1</td>
<td>1.4</td>
<td>0.5</td>
<td>-</td>
<td>2.8</td>
<td>0.3</td>
<td>0.0</td>
<td>0.1</td>
<td>0.6</td>
<td>0.6</td>
<td>0.1</td>
<td>0.8</td>
</tr>
<tr>
<td>FR</td>
<td>0.8</td>
<td>7.0</td>
<td>12.6</td>
<td>4.2</td>
<td>0.3</td>
<td>-</td>
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<td>8.0</td>
<td>0.5</td>
<td>28.4</td>
<td>3.0</td>
<td>1.1</td>
<td>0.3</td>
</tr>
<tr>
<td>GB</td>
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<td>0.9</td>
<td>8.5</td>
<td>2.7</td>
<td>0.2</td>
<td>16.8</td>
<td>-</td>
<td>1.9</td>
<td>2.0</td>
<td>4.4</td>
<td>2.7</td>
<td>0.9</td>
<td>0.4</td>
</tr>
<tr>
<td>GR</td>
<td>0.0</td>
<td>0.0</td>
<td>1.6</td>
<td>0.1</td>
<td>0.0</td>
<td>0.7</td>
<td>0.9</td>
<td>-</td>
<td>0.1</td>
<td>0.3</td>
<td>0.6</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>IE</td>
<td>1.6</td>
<td>2.0</td>
<td>33.8</td>
<td>4.9</td>
<td>0.0</td>
<td>11.6</td>
<td>19.0</td>
<td>1.4</td>
<td>-</td>
<td>10.5</td>
<td>1.6</td>
<td>1.1</td>
<td>0.6</td>
</tr>
<tr>
<td>IT</td>
<td>5.1</td>
<td>0.2</td>
<td>14.3</td>
<td>1.0</td>
<td>0.0</td>
<td>2.9</td>
<td>0.5</td>
<td>0.7</td>
<td>0.2</td>
<td>-</td>
<td>0.6</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>NL</td>
<td>1.2</td>
<td>12.0</td>
<td>29.9</td>
<td>7.1</td>
<td>0.4</td>
<td>17.0</td>
<td>3.5</td>
<td>2.4</td>
<td>1.0</td>
<td>11.1</td>
<td>-</td>
<td>0.8</td>
<td>0.4</td>
</tr>
<tr>
<td>PT</td>
<td>0.1</td>
<td>0.1</td>
<td>1.8</td>
<td>6.7</td>
<td>0.0</td>
<td>3.9</td>
<td>0.5</td>
<td>12.2</td>
<td>0.7</td>
<td>1.8</td>
<td>2.4</td>
<td>-</td>
<td>0.0</td>
</tr>
<tr>
<td>SE</td>
<td>0.3</td>
<td>0.6</td>
<td>21.8</td>
<td>0.7</td>
<td>21.2</td>
<td>5.4</td>
<td>4.0</td>
<td>0.1</td>
<td>0.2</td>
<td>0.5</td>
<td>1.4</td>
<td>0.1</td>
<td>-</td>
</tr>
</tbody>
</table>

Notes: The table reports our constructed measure of the aggregate holdings of banks headquartered in each country (listed by row) of sovereign debt from each other country (listed by column), in the first quarter of 2011. The data are taken from the BIS and IMF and transformed as described in Appendix B. Gross nominal values are normalized by the 2004 GDP of the home country, and are listed above as percentages. These are the amounts represented in the network graph in Figure 1.
Table A-2: Comparison with Alternative Measures of Claims on Sovereign Debt in 2010-Q4

<table>
<thead>
<tr>
<th>Bank HQ Country</th>
<th>EBA 2011 Stress Test</th>
<th>BIS Public Sector Claims</th>
</tr>
</thead>
<tbody>
<tr>
<td>All</td>
<td>0.883</td>
<td>0.907</td>
</tr>
<tr>
<td>AT</td>
<td>0.955</td>
<td></td>
</tr>
<tr>
<td>BE</td>
<td>0.450</td>
<td>0.573</td>
</tr>
<tr>
<td>DE</td>
<td>0.832</td>
<td>0.813</td>
</tr>
<tr>
<td>ES</td>
<td>0.816</td>
<td>0.865</td>
</tr>
<tr>
<td>FI</td>
<td>0.677</td>
<td></td>
</tr>
<tr>
<td>FR</td>
<td>0.901</td>
<td>0.947</td>
</tr>
<tr>
<td>GB</td>
<td>0.947</td>
<td>0.848</td>
</tr>
<tr>
<td>GR</td>
<td>0.812</td>
<td></td>
</tr>
<tr>
<td>IE</td>
<td>0.491</td>
<td></td>
</tr>
<tr>
<td>IT</td>
<td>0.964</td>
<td>0.983</td>
</tr>
<tr>
<td>NL</td>
<td>0.964</td>
<td></td>
</tr>
<tr>
<td>PT</td>
<td>0.732</td>
<td></td>
</tr>
<tr>
<td>SE</td>
<td>0.979</td>
<td></td>
</tr>
</tbody>
</table>

Notes: The table shows the correlations between our constructed measure of aggregate bank holdings of sovereign debt from each other country, and two other measures of these holdings available from the BIS and EBA. The comparison is made using data for 2010-Q4 based on the timing of an EBA stress test. The first row shows the correlations across all observations and subsequent rows show the correlations for each country in the holdings of sovereign debt from each other country. See Section 4.1 for further details.