Construct the set of potential contradictions for the derivation below at its current stage. To add a pair to the list, enter a negation from the derivation using the keyboard and the buttons below, indicate the line on which it appears as a positive subformula, and press the "Add" button. When a negation \( \neg \phi \) is added to the list, it will automatically be paired with its immediate subformula \( \phi \) for you.

When you think all the potential contradictions have been added to the list, click the "Done" button to check whether or not you found them all.

1. \( \neg P \lor \neg Q \)          Premise
2. \( \neg P \rightarrow R \)        Premise
3. \( \neg Q \rightarrow \neg S \)  Premise
4. \( \neg ( R \land \neg S ) \)  Premise
5. \( R \rightarrow \neg ( S \lor T ) \) Premise

6. \( \neg ( P \lor T ) \) Assum

\[ \begin{array}{c}
   \vdots \\
   \neg \text{I: ??} \\
   \vdots \\
   n-1. \bot \\
   n. P \lor T
\end{array} \]

\( \neg \text{E: n-1} \)

\( \neg \text{E: n-1} \)
Correct answers:

<table>
<thead>
<tr>
<th>Formula</th>
<th>Line #</th>
</tr>
</thead>
<tbody>
<tr>
<td>¬P</td>
<td>1</td>
</tr>
<tr>
<td>¬Q</td>
<td>1</td>
</tr>
<tr>
<td>¬S</td>
<td>3</td>
</tr>
<tr>
<td>¬( R &amp; ¬S )</td>
<td>4</td>
</tr>
<tr>
<td>¬( S v T )</td>
<td>5</td>
</tr>
<tr>
<td>¬( P v T )</td>
<td>6</td>
</tr>
</tbody>
</table>

Pairs display:

At completion:

```
 P    ¬P
 Q    ¬Q
 S    ¬S
 R & ¬S      ¬( R & ¬S )
 S v T      ¬( S v T )
 P v T      ¬( P v T )
```

Example, partially completed:

```
 P    ¬P
 S    ¬S
 P v T      ¬( P v T )
```

The feedback is designed with the assumption that the number of contradictory pairs is not provided to the student, so the pairs display shouldn't give away the exact number.

If this aspect of the design is problematic, please let me know so that I can rewrite the feedback accordingly.
Feedback:
Formula

- formula is on the list
- expression is not well-formed
- expression is well-formed, but not a negation
- expression is a negation, but not on the list

Now enter the line number.

That expression is not a well-formed formula.

That formula is not a negation. You should only be entering negations that appear as positive subformulae of lines in the derivation.

That formula is a negation, but it doesn't appear as a subformula of any line in the derivation. You should only be entering negations that appear as positive subformulae of lines in the derivation.
That expression is not a well-formed formula.

That formula is not a negation. You should only be entering negations that appear as positive subformulae of lines in the derivation.

That formula is a negation, but it doesn't appear as a subformula of any line in the derivation. You should only be entering negations that appear as positive subformulae of lines in the derivation.

Feedback:

Line Number

formula entered is: \(\neg P\)

That's right.

\(\neg P\) is a subformula of the formula on line 2, but as the antecedent of a conditional, so it isn't a positive subformula.

\(\neg P\) isn't a subformula of the formula on line [# entered]. You should be entering a line on which \(\neg P\) appears as a positive subformula.

formula entered is: \(\neg Q\)

That's right.

\(\neg Q\) is a subformula of the formula on line 2, but as the antecedent of a conditional, so it isn't a positive subformula.

\(\neg Q\) isn't a subformula of the formula on line [# entered]. You should be entering a line on which \(\neg Q\) appears as a positive subformula.
Formula entered is: \( \neg S \)
Feedback:
Line Number

formula entered is:
\( \neg( S \lor T ) \)

That's right.

\( \neg( S \lor T ) \) isn't a subformula of the formula on line [\# entered]. You should be entering a line on which \( \neg( S \lor T ) \) appears as a positive subformula. Don't forget that it is only the antecedent of a conditional that is not positive, the consequent is a positive subformula.

That's right.

formula entered is:
\( \neg( P \lor T ) \)

\( \neg( P \lor T ) \) isn't a subformula of the formula on line [\# entered]. You should be entering a line on which \( \neg( P \lor T ) \) appears as a positive subformula. Don't forget that every formula is a positive subformula of itself, and that the assumption used to open the subderivation should be included in the lines to consider for potential contradictions,
Add

- formula and line number are correct

[Add the negation paired with its immediate subformula to the list and clear the entries for formula and line number.]

Done

- all pairs have been added

That's right!

- not all pairs have been added

There are more potential contradictions than those added to the list.
To find the set of potential contradictions, all you need to do is find the negations that are positive subformulae of lines in the derivation up to and including the assumption made in order to apply an indirect rule.

A positive subformula is any subformula that is not itself a subformula of a negation, nor a subformula of the antecedent of a conditional.

There are negations on every line of the derivation. Work your way through them one at a time. If a negation is not inside the antecedent of a conditional, and not inside the scope of another negation, enter that negation.
If a negation appears as a subformula of more than one line, make sure that the line number you enter is a line on which it appears as a positive subformula.

Remember that subformulae that are within the antecedent of a conditional or within a negation are not positive.

[¬P only]
The occurrence of ¬P on line 2 is not positive, since it occurs within the antecedent of a conditional, but the occurrence on line 1 is positive.

[¬Q only]
The occurrence of ¬Q on line 3 is not positive, since it occurs within the antecedent of a conditional, but the occurrence on line 1 is positive.

[¬S only]
The occurrence of ¬S on line 4 is not positive, since it occurs within the scope of another negation, but the occurrence on line 3 is positive (it is only formulae within the antecedent of a conditional that aren't positive, those within the consequent are positive).
formula entered is:
\[ \neg ( R \land \neg S ) \]
or \[ \neg ( P \lor T ) \]

Don’t forget that every formula is a positive subformula of itself.

If the formula on a given line is a negation, then that entire formula is a positive subformula of a line in the derivation.

[\[ \neg ( R \land \neg S ) \text{ only}\]]
\[ \neg ( R \land \neg S ) \] appears as a subformula of only one line in the derivation, line 4, where it is a positive subformula.

[\[ \neg ( P \lor T ) \text{ only}\]]
\[ \neg ( P \lor T ) \] appears as a subformula of only one line in the derivation, line 6, where it is a positive subformula.

formula entered is:
\[ \neg ( S \lor T ) \]

Don’t forget that subformulae within the consequent of a conditional are positive subformulae, it is only subformulae within the antecedent that are not.

\[ \neg ( S \lor T ) \] appears as a subformula of only one line in the derivation, line 5, where it is a positive subformula.