Basket Securities in Segmented Markets∗

Preliminary — Comments welcome.

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This version: September 2014

ABSTRACT
I study the design and welfare implications of basket securities issued in markets with limited investor participation. Profit-maximizing intermediaries exploit investors’ inability to trade freely across different markets and choose which market to specialize in. I show that when there is only one intermediary, the equilibrium may not be constrained efficient. Increasing competition among intermediaries increases the variety of baskets issued, but does not always improve investors’ welfare. Although competition increases the variety of baskets issued, many of these baskets are redundant, in the sense that coordination among intermediaries could improve investors’ risk sharing opportunities. The equilibrium basket structure depends on institutional features of a market such as depth and gains from trade.

Keywords: Basket securities, Segmented markets, Limited investor participation.

JEL classification: G10, G11.

∗I thank Burton Hollifield, Jack Stecher, Artem Neklyudov, Bryan Routledge, Francisco Cisternas, Andrés Bellofato, and Stephen Karolyi for their valuable suggestions on early versions of this paper as well as the seminar participants at the 2014 European Finance Association Meeting (DT), 2014 Eastern Finance Association Meeting, the 2014 Midwest Finance Association Meeting, the 2013 Northern Finance Association Meeting, the 13th Trans-Atlantic Doctoral Conference at LBS, Carnegie Mellon, and Universidad de Chile. All remaining errors are my own. Contact: carlosrc@cmu.edu. Website: http://www.andrew.cmu.edu/user/caramire.
Over the past four decades, there has been a substantial increase of financial innovation and investor demand—both institutional and retail—for securities that pool different assets and whose value is determined as an aggregate of the values of those assets. The large variety of asset-backed securities, such as collateralized debt obligations and mortgage-backed securities, as well as index funds and exchange-traded funds, shows the prevalence of basket securities in modern financial markets.

In this paper, I explore how basket securities develop in an incomplete market setting where profit-maximizing intermediaries are involved in financial innovation. Specifically, I study the design and welfare implications of basket securities issued in markets with limited investor participation. The questions I address are: Which baskets are optimal for profit-maximizing intermediaries, what are the welfare implications associated with introducing such baskets, and how does competition among multiple intermediaries affect equilibrium outcomes? My analysis provides a link between the institutional features of a market, such as depth and gains from trade, and the types of basket securities that emerge in equilibrium.

In perfect capital markets the bundling activity is irrelevant. In reality, however, there are many reasons that investors do not replicate basket securities by themselves. In the literature, asymmetric information, transaction costs, and market incompleteness have been cited as possible explanations for the existence of basket securities, e.g. Subrahmanyan (1991), Gorton and Pennacchi (1993), Allen and Santomero (1997), and DeMarzo (2005). However, the cost of information and transaction costs have continuously decreased during the last thirty years, while the demand for basket securities has grown almost exponentially. In this paper, then, I focus on market incompleteness, in the sense that investors have limited access to capital markets as in Rahi and Zigrand (2009, 2010).

The assumption of limited investor participation also better captures some features of today’s most active basket security markets. For example, in some ABSs markets—such as CDOs and MBSs—many of the characteristics of the underlying assets are public information. More important, the selection of the underlying assets and the posterior tranching of these baskets are chosen by profit-maximizing intermediaries. However, it is not easy for a retail investor to become an

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1If investors are asymmetrically informed, a basket security may reduce uninformed investors’ trading losses because the adverse selection costs associated with baskets are typically lower than those associated with individual securities. In terms of transactions costs, basket securities are desirable because high transaction costs make it expensive for individual investors to replicate diversified portfolios on their own.
intermediaries, because it requires excellent distribution channels and paying setup costs that are typically large. ETFs are another important example of limited investor participation because limited arbitrage trading is at the core of ETFs creation. Only “authorized participants,”—typically large brokers or investment banks—are effectively able to arbitrage price differentials between ETFs and their underlying basket. If an ETF is trading at a premium compared to its underlying basket, only authorized participants can create ETF shares and deliver the underlying basket, whereas other investors cannot participate in the creation-redemption process and need instead to rely on short-long strategies.

The main features of the model are as follows. I consider a two-period economy in which trading occurs at period zero and payoffs are realized at period one. There are two states of nature, two securities—each of them paying in a different state of nature—and two market segments. A continuum of measure one of risk-averse investors is associated with each segment. Each segment is endowed with one security. If trading across segments is free, investors share risk perfectly and the equilibrium allocation is Pareto optimal. If markets are segmented, however, this is not necessarily true, because marginal valuations are typically not equalized at equilibrium. To capture market segmentation, trading across segments is not allowed. However, there is a risk-neutral intermediary (hereafter the issuer) with the ability to trade with both segments and who offers shares of one new security—the basket—in exchange for shares of the two initial securities. A basket consists of a linear combination of the two initial securities. The optimal combination is chosen by the profit-maximizing issuer.

The first question I address is whether a monopoly issuer would simply issue a basket to complete both market segments. The answer is not necessarily. If one asset is insuring against a sufficiently remote risk, the issuer may not find it worthwhile to serve all segments and instead choose to tailor her basket to only one of them. When designing a basket, the issuer seeks to both increase trading volume and decrease downside risk. Provided that the issuer cares only about her intermediation profits, her incentives may not be aligned with those of investors. Thus, the equilibrium is not

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2 In the U.S. major ETFs are more traded than any other security. ETFs' sponsors during the last five years have continually increased the variety of investment objectives and the number of funds offered, given the increment of investors' demand. For more details about the growth of ETFs see Deville (2008), Ferri (2009) and Gastineau (2010).

3 These concerns are consistent with evidence on patterns of innovation in exchange-traded contracts, e.g. Black (1986), and on the creation of new zero coupon bonds in the U.S. Treasury STRIPS program, e.g. Grinblatt and Longstaff (2000).
always constrained efficient.\footnote{Elul (1995) shows that in almost every incomplete market economy with more than one consumption good and with sufficiently many uninsured states of nature, one can introduce a set of assets that might make all agents worse-off. In my setting, however, this argument does not necessarily apply, because there are only two states.}

Since competition among issuers may improve investors’ welfare, I then ask what happens when issuers’ barriers to entry are lowered. If that happens, different basket securities may coexist in equilibrium. Many of them, however, are redundant, in the sense that coordination among issuers may improve investors’ risk-sharing opportunities. The non-cooperative nature of issuers’ competition prevents coordination, though. Nonetheless, there are special cases in which issuers introduce the same basket. If that is the case, competition increases the diversification of the basket relative to the monopolistic case. As the number of issuers increases, issuers find it optimal to provide more diversified baskets. They do so to increase trading volume and hence compensate their losses from the entry of new competitors. Moreover, the basket being offered with duopoly competition may differ from that which a monopolist would offer. More important, the switch may not be welfare-improving, but rather Pareto-incomparable, because some market participants may be better off while others may be worse off after introducing competition.

I then compute a numerical example to provide proxies of the effective segmentation investors may encounter when investing across different markets segments. The numerical example suggests that the segmentation investors face when trading across North-America and Europe is comparable to the segmentation they face when trading across Europe and the Asia-Pacific region.

This paper relates to two strands of the literature: one on optimal security design, and other on the creation of basket securities. Excellent surveys of security design in an incomplete market framework are Allen and Gale (1994) and Duffie and Rahi (1995). So far, the main focus in the literature has been on innovations introduced by agents who do not trade the securities they create. In practice, however, agents involved in financial innovation are often profit-seeking institutions that actively make markets and trade their securities across markets, e.g. Allen and Santomero (1997). Duffie and Jackson (1989) and Ross (1989) are among the first studies to consider this profit-maximizing feature. Duffie and Jackson (1989) study the incentives of exchanges that lead them to offer one contract rather than another. Ross (1989) studies investment banks’ incentives to bundle securities to lower searching costs. Rahi and Zigrand (2009, 2010) study a general equilibrium model similar to mine. In Rahi and Zigrand (2009) investors have limited access to capital markets.
and strategic issuers make profits by exploiting mispricings across markets. The asset structure of the security introduced by issuers is endogenized as the outcome of a security design game.

The second related literature is on the creation of basket securities. This strand of literature has focused mainly on either asymmetric information or transaction costs as the cause of the creation of basket securities. For example, Gorton and Pennacchi (1990, 1993) argue that baskets decrease uninformed investors’ trading losses. Baskets decrease uninformed investors’ “lemons” problem, in the sense that baskets split individual securities cash flows and eliminate the private information informed investors may have about individual securities. Along the same lines, Subrahmanyan (1991) shows that strategic liquidity traders may prefer baskets rather than individual securities. DeMarzo (2005), on the other hand, considers the problem of an issuer who may have superior information about the value of her assets. He provides conditions under which originators sell pools of assets, some of which are purchased by informed intermediaries who then further pool and tranche them. Pooling and trancheing allows intermediaries to leverage their capital more efficiently, enhancing the returns on their private information.

The rest of the paper is organized as follows. Section I describes the model. Section II analyzes the constrained efficient allocation as a benchmark. Section III solves for the market-mediated equilibrium and characterizes its properties. Section IV analyzes the effect of competition among issuers. Section V provides the numerical example. Finally, section VI concludes. All proofs, unless otherwise stated, appear in the Appendix.

I. A Baseline Model

A. The Environment

Consider a two-period economy with one good. Two assets, denoted by $x$ and $y$, are traded at date zero, pay at date one, and are in positive net supply $w_x$ and $w_y$, respectively. Two market segments, henceforth segments, are indexed by $i = \{1, 2\}$. A continuum of risk-averse investors with the same preferences is associated with each market segment. As a consequence, investors in both segments act as price takers. Prior to trading, investors in segment one (hereafter investors one) are endowed with all asset $x$ while investors in segment two (hereafter investors two) are endowed with all asset $y$. Two states of nature, $s \in \{s_1, s_2\}$, occur with probability $\phi$ and $(1 - \phi)$.
respectively, with $\phi \in (0,1)$. Assets’ payoff in units of the good are given in Table I.

<table>
<thead>
<tr>
<th>Asset / State</th>
<th>$s = s_1$</th>
<th>$s = s_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x(s)$</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$y(s)$</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

To capture market segmentation, trading across segments is not allowed. Besides investors, there is one intermediary—the issuer—who can trade with investors in both segments. In exchange for shares of asset $x$ or $y$, the issuer offers shares of one new security, asset $z$, to both segments. Asset $z$ is a basket consisting of $\alpha_x$ shares of asset $x$ and $\alpha_y$ shares of asset $y$. One way of justifying the fact that the issuer offers at most one basket is assuming that the issuer may face large costs if issuing several baskets at the same time. An issuer may need both first-class distribution channels and time to market each basket to see whether there is enough demand for that basket.

I also assume that the basket is competitively priced, in the sense that its price, $p_z = \alpha_x p_x + \alpha_y p_y$, where $p_x$ and $p_y$ are the equilibrium prices of one share of asset $x$ and $y$, respectively. One way of justifying this assumption is assuming prices are the ones at the steady state. In practice, issuers may easily replicate new products introduced by their competitors, by either using reverse engineering or just copying trading strategies which may lead to competitive pricing, e.g Tufano (2003). Another way of justifying this assumption is through Merton (1992)’s metaphor of the “financial innovation spiral.” A new product builds upon the previous one, and each new generation attempts to lower trading costs, be more tax-efficient or be more transparent. The pricing assumption also seems to be consistent with features of some commonly traded basket securities. In some highly traded ETFs, for example, the redemption-creation process ensures that the difference between the net asset value and the market price of the basket is negligible.

For each share of the basket, the issuer charges an exogenous fraction $m$ of the basket payoff, with $m \in (0,1)$. Parameter $m$ aims to proxy the effective segmentation investors encounter when investing across different markets. It may also represent the issuer’s potential gains from offering a basket. If $m$ tends to zero, investors can trade at almost no cost across markets and the equilibrium allocation tends to be Pareto optimal. However, as $m$ departs away from zero, trading across market segments gets costly, but more profitable for issuers, and marginal valuations may not be equalized.
at equilibrium.

B. Agents

B.1. Investors

Investors in both segments have smooth preferences over consumption in both states. Hence, they care about consumption of assets $x$ and $y$. Let $c_i^x$ and $c_i^y$ be investors’ $i$’s consumption of asset $x$ and $y$, respectively. For simplicity, investors in both segments have the same preferences, which are determined by $u(\cdot)$, where $u(\cdot)$ is strictly increasing, strictly concave, twice continuously differentiable, and satisfies Inada conditions.

Investors are not allowed to short-sell the basket. Otherwise, they can complete their segments at no cost and the issuer’s activity is redundant. In this setting, short-selling constraints also represent the dual of non-default constraints on the issuer’s side, as will become clear in the next section.

Let $x_1$ be the number of shares of asset $x$ investors one keep for themselves. Let $z_1$ be the number of basket shares investors one buy. Taking asset prices and the basket structure as given, the optimal portfolio of investors one, $(x_1^*, z_1^*)$, solves

$$
\max_{(x_1, z_1)} U_1(c_1^x, c_1^y) = \phi u(c_1^x) + (1 - \phi) u(c_1^y)
$$

s.t. $p_x x_1 + p_z z_1 \leq p_x w_x$

$$
z_1 \geq 0
$$

where $c_1^x = x_1 + (1 - m) \alpha_x z_1$ and $c_1^y = (1 - m) \alpha_y z_1$, respectively. The problem of investors two is symmetric.\(^5\) If both $\alpha_x$ and $\alpha_y$ are strictly positive, then both investor types hold a strictly positive amount of the basket because preferences satisfy Inada conditions.

\(^5\)Let $y_2$ be the number of shares of asset $y$ investors two keep for themselves. Let $z_2$ be the number of basket shares investors two buy. The optimal portfolio of investors two $(y_2^*, z_2^*)$ solves

$$
\max_{(y_2, z_2)} U_2(c_2^x, c_2^y) = \phi u(c_2^x) + (1 - \phi) u(c_2^y)
$$

s.t. $p_y y_2 + p_z z_2 \leq p_y w_y$

$$
z_2 \geq 0
$$

$$
c_2^x = (1 - m) \alpha_x z_2
$$

$$
c_2^y = y_2 + (1 - m) \alpha_y z_2
$$
B.2. The Issuer

Before date zero, a risk-neutral issuer decides the basket structure to maximize her profits. For simplicity, it is assumed that the issuer does not take active positions in either \( x \) or \( y \), so she sells all basket shares. In other words, she profits only from intermediation.

Provided the spanning role of the basket, the issuer cares only about the ratio \( r_\alpha = \frac{\alpha_y}{\alpha_x} \). To see this, suppose \textit{investors one} buy \( \hat{z}_1 \) shares of the basket if it is composed of \( \alpha_x \) shares of \( x \) and \( \alpha_y \) shares of \( y \). Then, if the basket is composed of \( \beta \alpha_x \) shares of \( x \) and \( \beta \alpha_y \) shares of \( y \), with \( \beta > 0 \), \textit{investors one} rescale their demand —now buying \( \frac{1}{\beta} \hat{z}_1 \) shares— through which they obtain the same payoff. Then, without loss of generality I fix \( \alpha_x = 1 \) and assume \( w_x \geq 1 \).

The issuer solves

\[
\max_{r_\alpha} \quad E[\pi] = m \left[ \phi + (1 - \phi)r_\alpha \right] z_t - c_0 1_{z_t \neq 0} \tag{3}
\]

\[\begin{align*}
st. \quad 0 & \leq r_\alpha \leq w_y \\
E[\pi^*] & \geq 0
\end{align*}\]

where \( z_t \) is the number of basket shares the issuer sells, whereas \( c_0 \) is a fixed cost of issuing a basket—which is always assumed to be sufficiently small so the basket is always issued. \( 1_{z_t \neq 0} \) is one if the issuer creates a basket and zero otherwise. The lower and upper bounds for \( r_\alpha \) keep the issuer away from defaulting at equilibrium. If \( r_\alpha < 0 \) the issuer short-sells \( y \) through the basket. As a consequence, if \( s = s_2 \) an investor who buys the basket needs to deliver \( \alpha_y \) shares of \( y \) in period one. Investors two, however, are the only ones who can do so. Thus, default may arise. The intuition behind the upper bound, \( r_\alpha \leq w_y \), is simpler. The issuer cannot offer more shares of \( y \) than the ones available in the economy.

Since the issuer sells all shares of the basket, non-default restrictions on the issuer side can also be interpreted as non short-selling restrictions on the investors’ side. To see this, suppose \textit{investors one} short-sell shares of \( x \). The issuer then includes more shares of \( x \) in the basket than the ones available in the economy. Thus, default may arise. The same applies for \textit{investors two} with asset \( y \). Finally, \( \pi^* \) is the issuer’s profit evaluated at the optimal basket structure. Thus, the last restriction represents the issuer’s participation constraint.
C. Markets

There are two spot markets at date zero, one per each segment, in which shares of \(x\) and \(y\) are traded in exchange for shares of the basket. Take the basket structure as given. Provided that \(p_z = \alpha_x p_x + \alpha_y p_y\), it follows from adding up excess demand functions

\[
0 = p_x (x_1 - w_x) + p_z z_1 + p_y (y_2 - w_y) + p_z z_2 = p_x (x_1 - w_x) + (\alpha_x p_x + \alpha_y p_y) z_t + p_y (y_2 - w_y) = p_x (x_1 + \alpha_x z_t - w_x) + p_y (y_2 + \alpha_y z_t - w_y)
\]

which represents Walras’ Law. Thus, if one segment clears the other clears as well.

D. Equilibrium

In equilibrium, agents maximize their expected utility at period one subject to their respective trading constraints. Prices are such that market clearing conditions are satisfied. An equilibrium of the market-mediated exchange is then defined as

DEFINITION 1: An equilibrium is an array of prices, \(\{p_z, p_x, p_y\}\), asset demand functions, \(\{x_1, z_1, y_2, z_2\}\), and basket structure, \(r_\alpha\), such that:

(E1) Investor’s maximization: Given prices \(\{p_z, p_x, p_y\}\), investors one’s portfolio \((x_1, z_1)\) solves (1) and investors two’s portfolio \((y_2, z_2)\) solves (2).

(E2) Issuer’s maximization: The basket structure, \(r_\alpha\), solves (3).

(E3) Market clearing: Prices \(\{p_z, p_x, p_y\}\) are such that (4) is satisfied.

II. Constrained Efficient Allocation

This section explores the constrained efficient allocation as a benchmark. Consider a benevolent planner who needs to allocate resources among investors and the issuer. The planner’s problem
can be restated as

\[
\begin{array}{l}
\max_{(c_1^x, c_1^y, c_2^x, c_2^y, c_a^x, c_a^y)} \quad U_1(c_1^x, c_1^y) \\
st. \quad U_2(c_2^x, c_2^y) \geq u_0 \\
E[\pi] \geq \pi_0 \\
c_1^x + c_2^x + c_a^x \leq w_x \\
c_1^y + c_2^y + c_a^y \leq w_y
\end{array}
\]  

(5)

where \(c_a^x\) and \(c_a^y\) denote the issuer’s consumption of \(x\) and \(y\). A feasible allocation is a consumption vector, say \((c_1^x, c_1^y, c_2^x, c_2^y, c_a^x, c_a^y)\), that satisfies \(c_1^x + c_2^x + c_a^x \leq w_x\) and \(c_1^y + c_2^y + c_a^y \leq w_y\). Figure 1 represents the Edgeworth box for problem 5 for different parameter values. Provided that there are three types of agents, a feasible allocation is represented by two points inside the box. For the sake of simplicity, take \(c_2^x\) and \(c_2^y\) as given so \(u_0\) in problem 5 is determined. Doing so defines one of the vertices—\(\{t_0, t_1, t_2\}\)—of the right triangles in figure 1. The hypotenuse of a right triangle represents all feasible allocations a benevolent planner may assign to investors one to provide \(\pi_0\) utils to the issuer as well as \(u_0\) utils to investors two. Take \(u_0\) and \(\pi_0\) as given. A feasible allocation that solves problem 5 needs to satisfy \(U_2(c_2^x, c_2^y) = u_0\) and \(E[\pi(c_a^x, c_a^y)] = \pi_0\). In other words, a constrained efficient allocation is a feasible allocation that gives investors one the highest utility, given \(u_0\) and \(\pi_0\).

DEFINITION 2: Suppose \(c_2^x\) and \(c_2^y\) are fixed. A feasible allocation, say \((c_1^x, c_1^y, c_2^x, c_2^y, c_a^x, c_a^y)\), is said to be constrained efficient among all feasible allocations that share coordinates \(c_2^x\) and \(c_2^y\) and provide \(\pi_0\) utils to the issuer, if there does not exist another feasible allocation in that set, say \((\tilde{c}_1^x, \tilde{c}_1^y, \tilde{c}_2^x, \tilde{c}_2^y, \tilde{c}_a^x, \tilde{c}_a^y)\), such that \(U_1(c_1^x, c_1^y) < U_1(\tilde{c}_1^x, c_1^y)\).

Given \(u_0\) and \(\pi_0\), the set of constrained efficient allocations may be non singleton as figure 1 shows. For example, allocations \((A_1, A_2)\), \((A_1, B_2)\) and \((A_1, C_2)\) in figure 1(a) assign the same utils to each investor type. The same happens with allocations \((A_1, A_2)\) and \((B_1, B_2)\) in figure 1(b). To gain a better intuition, in the following example I solve the previous problem assuming \(u(\cdot) = \log(\cdot)\).

EXAMPLE 1: Consider \(u(c) = \log(c)\) and assume that \(u_0\) and \(\pi_0\) are fixed. In addition, assume that \(\phi w_x + (1 - \phi)w_y \geq \pi_0 + c_0 + e^{u_0}\). The set of constrained efficient allocations corresponds to all
Figure 1. Allocations in the constrained efficient set. Investors one’s quantities are measured with the southwest corner as the origin while investors two’s quantities are measured using the northeast corner. Preferences are also depicted using investors’ indifference curves.

non negative consumption bundles, say \((c_1^x, c_1^y, c_2^x, c_2^y, c_a^x, c_a^y)\), such that

\[
\begin{align*}
c_1^x &= c_1^y = \phi w_x + (1 - \phi) w_y - \pi_0 - c_0 - e^{u_0} \\
c_2^x &= c_2^y = e^{u_0} \\
c_a^x &= \pi_0 + c_0 + (1 - \phi)(w_x - w_y) \\
c_a^y &= \pi_0 + c_0 + \phi(w_y - w_x)
\end{align*}
\]

As a consequence, constrained efficient allocations still provides investors perfect risk sharing, because there is perfect insurance against everything except fluctuations in aggregate wealth. However, both investors’ indifference curves are typically not tangent since the issuer still gets \(\pi_0\) utils.

III. Trading Equilibrium

In what follows I study the equilibrium output of a market-mediated exchange to understand the extent to which the market provides the right instruments so that investors share risk.
A. Investors one’s optimal portfolio

Given \( r_\alpha > 0 \), the budget constraint in (1) can be rewritten in terms of investors one’s consumption as

\[
p_x c_1^x + \frac{1}{(1 - m)} \left( m p_x r_\alpha + p_y \right) c_1^y \leq p_x w_x
\]

(6)

The marginal rate of substitution at the optimal consumption bundle is then

\[
\frac{u'(c_1^x)}{u'(c_1^y)} = \left( \frac{1 - \phi}{\phi} \right) \left( \frac{p_x (1 - m)}{mp_x r_\alpha + p_y} \right)
\]

(7)

Given \( r_\alpha \), investors one face the following trade-off. As they buy more shares of the basket, they increase their position not only in \( y \) but also in \( x \). However, buying shares of \( x \) through the basket is inefficient because they pay a markup \( m \) per share. The basket structure then affects investors one’s marginal valuation of both assets.

Investors one’s optimal portfolio \((x_1^*, z_1^*)\) is then

\[
x_1^* = u^{-1} \left( \frac{\lambda_1 p_x}{\phi} \right) - \frac{1}{r_\alpha} u^{-1} \left( \frac{\lambda_1 \left( m p_x \frac{1}{r_\alpha} + p_y \right)}{(1 - \phi)(1 - m)} \right) = c_1^x - c_1^y \]

(8)

\[
z_1^* = \frac{1}{r_\alpha (1 - m)} u^{-1} \left( \frac{\lambda_1 \left( m p_x \frac{1}{r_\alpha} + p_y \right)}{(1 - \phi)(1 - m)} \right) = \frac{c_1^y}{r_\alpha (1 - m)}
\]

(9)

where \( \lambda_1 \) is the Lagrange multiplier associated with investors one’s budget constrain. As the likelihood of state one increases, the price of \( x \) increases. Then, investors one buy fewer basket shares not only because it is less probable that they need insurance against state two, but also because the basket becomes more expensive, as it also contains shares of asset \( x \).

B. Investors two’s optimal portfolio

Similarly, the budget constraint in problem (2) can be rewritten in terms of investors two’s consumption bundle as

\[
p_y c_2^y + \frac{1}{(1 - m)} \left( m p_y r_\alpha + p_x \right) c_2^x \leq p_y w_y
\]

(10)
Provided the symmetry of the problem, the same intuition applies for investors two with asset y. Thus, the optimal portfolio \((y_2^*, z_2^*)\) is
\[
y_2^* = u'^{-1} \left( \frac{\lambda_2 p_y}{1 - \phi} \right) - r_\alpha u'^{-1} \left( \frac{\lambda_2 (m p_y r_\alpha + p_x)}{\phi (1 - m)} \right) = c_2^y - r_\alpha c_2^y
\]
(11)
\[
z_2^* = \frac{1}{(1 - m)} u'^{-1} \left( \frac{\lambda_2 (m p_y r_\alpha + p_x)}{\phi (1 - m)} \right) = \frac{c_2^x}{1 - m}
\]
(12)
where \(\lambda_2\) is the Lagrange multiplier associated with investors two’s budget constrain.

C. Equilibrium

For simplicity, let \(p_z = 1\). Then, equilibrium prices \(p_x^*\) and \(p_y^*\) are such that the following system of equations is satisfied
\[
x_1^* (p_x^*, p_y^*) + z_t (p_x^*, p_y^*) = w_x
\]
(13)
\[
p_x + r_\alpha p_y = 1
\]
(14)
where equation (13) requires market clearing in segment one, whereas equation (14) requires the basket to be competitively priced. To gain intuition about which basket is likely to emerge at equilibrium, in the following examples I solve for the basket structure that emerges in equilibrium under different investors’ preferences.

EXAMPLE 2: Consider \(u(c) = \log(c)\). The equilibrium basket structure belongs to the set
\[
\left\{ \left[ \frac{\phi}{\phi + m} \right] \frac{w_y}{w_x}, \left[ \frac{1 - \phi + m}{1 - \phi} \right] \frac{w_y}{w_x} \right\}
\]
(15)
REMARK 1: If \(r_\alpha^* = \left[ \frac{\phi}{\phi + m} \right] \frac{w_y}{w_x}\) then investors one hold only shares of the basket after trading. If \(r_\alpha^* = \left[ \frac{1 - \phi + m}{1 - \phi} \right] \frac{w_y}{w_x}\) the same happens to investors two. If there is no aggregate risk, i.e. \(w_x = w_y\), and \(\phi = \frac{1}{2}\), the basket fully replicates the market portfolio.

Example 2 highlights that the issuer does not always complete each segment, because she may get higher profits by tailoring her basket to one segment. If there are not clear gains from customizing the basket—for instance, \(\phi = \frac{1}{2}\) and \(w_x = w_y\) as in example 2—the issuer offers a

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6Then \(p_x\) and \(p_y\) are the exchange rates between the security basket and asset x and y, respectively.
baskets similar to a risk-free bond. The following example shows that such a situation occurs as investors become extremely risk averse.

EXAMPLE 3: Consider \( u(c) = \frac{c^{1-\rho} - 1}{1-\rho} \) and \( \rho > \varsigma \) with \( \varsigma \) sufficiently large. From the issuer’s problem follows \( r^*_\alpha = 1 \).

If investors are sufficiently risk averse then the issuer maximizes her intermediation profits by completing both segments. In that case, the basket replicates the market portfolio, because everyone holds the basket after trading. If there is no aggregate risk, such a security corresponds to a risk-free bond.

The difference between examples 2 and 3 is investors’ preferences. In example 3, investors are extremely risk averse. Thus, the gains from providing risk-sharing services are sufficiently large. In example 2, on the other hand, investors are much less risk averse. Therefore, providing risk-sharing services to both investor types may not be optimal. The issuer tailors her basket because either one of the assets may be insuring against a sufficiently remote risk or investors’ demands are not sufficiently large. As a consequence, providing a basket that replicates the market portfolio may not always be optimal for the issuer. From this intuition follows

PROPOSITION 1: A basket security does not necessarily replicate the market portfolio. It does, however, fully cover the trading in the segment the issuer specializes in. Moreover, as the risk-aversion of investors increases, the basket tends to a security that fully replicates the market portfolio.

It is important to appreciate that examples 2 and 3 also stress the two main forces driving the issuer’s behavior within the model. In one side, the issuer acts as if she cares about downside risk—the odds of receiving intermediation profits at date one—even if she is risk neutral. Thus, the issuer provides baskets that pay in highly probable states, which may lead an issuer to customize her basket to serve just one segment. On the other side, the issuer cares about maximizing trading volume as she takes into account investors’ needs for risk-sharing and their relative wealth. For example, consider that both states are equally probable. If investors one have more endowment than investors two, then the issuer includes more shares of \( x \) than \( y \), because \( x \) is cheaper than \( y \). By doing so, the issuer reduces the cost of the basket, which in turn, increases its trading volume.
These two forces may not always go in different directions. For instance, suppose state one is highly probable, such that investors two are interested in buying shares of the basket. If investors are not sufficiently risk-averse, no basket emerges because investors one are not willing to sell shares of asset \( x \). On the other hand, if investors are sufficiently risk-averse, the issuer offers a basket with more shares of asset \( x \). By doing so, the issuer specializes the basket for investors who need it more and thus increases trading volume as well as increasing the odds of receiving her intermediation profits.

D. Efficiency of trading allocations

Provided that investors are not allowed to trade across segments, markets are incomplete. If markets are incomplete, there is no reason to expect that the trading equilibrium is constrained efficient. To assess the efficiency of trading allocations, I compare those allocations obtained by a market-mediated equilibrium with constrained efficient allocations and explore the conditions under which such allocations are equal.

Table II

<table>
<thead>
<tr>
<th>Consumption Bundles</th>
<th>Planner’s Problem</th>
<th>Market Mediated with log(( \cdot )) investors</th>
<th>Market Mediated with highly risk-averse investors</th>
</tr>
</thead>
<tbody>
<tr>
<td>((c_1^x, c_2^y))</td>
<td>((c, c))</td>
<td>((\phi w_x, \frac{\phi^2}{\phi + \mu} w_y))</td>
<td>((\phi - m)w_0, (\phi - m)w_0))</td>
</tr>
<tr>
<td>((c_3^x, c_4^y))</td>
<td>((e^{u_0}, e^{u_0}))</td>
<td>((1 - \phi - m)w_x, (1 - \phi)w_y))</td>
<td>((1 - \phi)w_0, (1 - \phi)w_0))</td>
</tr>
<tr>
<td>((c_a^x, c_d^y))</td>
<td>((w_x - c - e^{u_0}, w_y - c - e^{u_0}))</td>
<td>((mw_x, \frac{m\phi}{\phi + \mu} w_y))</td>
<td>((mw_0, mw_0))</td>
</tr>
</tbody>
</table>

\[
r_\alpha^* = \frac{\phi}{\phi + \mu} \frac{w_y}{w_x}
\]

Table II shows the allocations attained as the outcome of the social planner’s problem as well as the market-mediated exchange equilibrium. The second column in Table II shows the constrained efficient allocation if investors have log(\( \cdot \)) preferences. The third and fourth columns show allocations that emerge in a market mediated-exchange if investors have log(\( \cdot \)) preferences and if investors
are extremely risk averse. As table II shows, the allocations attained as the outcome of a market-mediated equilibrium tend not to be constrained efficient. Therefore, there are many cases when the price mechanism is perfectible. It then follows

PROPOSITION 2: Basket securities are not always constraint efficient. However, as investors risk aversion increases, the basket security allows investors to achieve constrained efficient allocations.

IV. An extended model with competition among issuers

Provided that the price mechanism is perfectible, introducing competition among issuers may increase investors’ welfare. This section studies the impact of competition on: (a) the structure of basket securities, and (b) investors’ welfare.

For simplicity, consider an economy with two issuers. Issuer one offers basket $z$, which consists of $\alpha_z^x$ shares of $x$ and $\alpha_z^y$ shares of $y$ at $p_z = \alpha_z^x p_x + \alpha_z^y p_y$. On the other hand, issuer two offers basket $q$, which consists of $\alpha_q^x$ shares of $x$ and $\alpha_q^y$ shares of $y$ at $p_q = \alpha_q^x p_x + \alpha_q^y p_y$. Issuer $i$ charges an exogenous fraction of the basket $m_i \in (0, 1)$ per share, with $i = \{1, 2\}$.

Issuers face a strategic environment that can be framed as a two-stage non-cooperative game. In the first stage, issuers choose whether or not to enter each market segment. At the end of the first stage, issuers observe who entered each segment. In the second stage, issuers select the structure of their baskets to maximize their profits. After the design game is played, investors observe the structure of baskets and choose whether or not to trade shares of their assets in exchange for shares of the baskets available in each segment.

Let $z_i$ and $q_i$ be investors’ holdings of baskets $z$ and $q$ respectively, $i = \{1, 2\}$. At equilibrium, all agents maximize their expected utility subject to their trading and budget constraints. Prices are such that market clearing conditions are satisfied.

DEFINITION 3: An equilibrium with duopoly competition among issuers is an array of prices, \{p_z, p_q, p_x, p_y\}, asset demand functions, \{x_1, z_1, q_1, y_2, z_2, q_2\}, and baskets such that

(EC1) Investors’ maximization: Given prices \{p_z, p_q, p_x, p_y\}, investors choose their portfolios to maximize their expected utility subject to their budget and trading constraints.

(EC2) Issuers’ maximization: The structures of the baskets corresponds to the outcome of a Perfect
Nash Equilibrium of the two-stage design game. 

(EC3) Markets clear: Prices are such that all markets—two per each market segment—clear.

For simplicity, consider that issuing-costs are sufficiently small, so both issuers enter the market.

The following example shows how investors’ demand affects the basket structures.

EXAMPLE 4: Assume that both issuers offer a basket and \( u(c) = \log(c) \). In addition, suppose \( \phi \in (\phi_l, \phi_u) \) such that both investor types hold only shares of baskets after trading. Investors’ optimality conditions then imply

\[
\begin{align*}
    z_1 &= \frac{p_x w_x}{p_z + p_q \left( \frac{1 - m_1}{1 - m_2} \frac{\alpha^x - \beta}{\beta - \alpha^y} \right)}, \\
    q_1 &= \frac{(1 - m_1) (\alpha^x - \beta)}{(1 - m_2) (\beta - \alpha^y)} z_1 \tag{16} \\
    z_2 &= \frac{p_y w_y}{p_z + p_q \left( \frac{1 - m_1}{1 - m_2} \frac{\alpha^y - \beta}{\beta - \alpha^y} \right)}, \\
    q_2 &= \frac{(1 - m_1) (\alpha^y - \beta)}{(1 - m_2) (\beta - \alpha^y)} z_2 \tag{17}
\end{align*}
\]

where

\[
\beta = \left( \frac{1 - \phi}{\phi} \right) \left( \frac{\omega \alpha^y - \alpha^x}{1 - \omega} \right) \quad \text{and} \quad \omega = \left( \frac{p_x}{p_q} \right) \left( \frac{1 - m_2}{1 - m_1} \right)
\]

Figures 2 and 3 depict the baskets that emerge from duopoly competition among issuers under conditions of example 4. In both figures, \( \alpha^y_z(\alpha^y_q) \) represents the number of shares of \( y \) in basket \( z \), given basket \( q \) structure, \( \alpha^y_q \). In other words, \( \alpha^y_z(\alpha^y_q) \) corresponds to issuer one’s best response function. Similarly, \( \alpha^y_q(\alpha^y_z) \) represents the number of shares of \( y \) in basket \( q \), given \( z \)’s structure, \( \alpha^y_z \), i.e. issuer two’s best response function. Then, an equilibrium of the design game corresponds to an intersection between both issuers’ best response functions. If the intersection between issuers’ best response functions lies on the 45% line, then both issuers provide the same basket. If that does not happen, an intersection between issuers’ response functions represents two different baskets that coexist at equilibrium.

If issuers compete, figures 2 and 3 show that several baskets may coexist. The increased variety of baskets issued does not always improve investors’ welfare, though. The coexistence of several baskets may be redundant, in the sense that cooperation among issuers may increase investors’ welfare. To see this, suppose that issuers cooperate with each other and perfectly split the market demand so that each basket serves only one market segment. These baskets could allow investors to
achieve constrained efficient allocations. It follows from inspection, however, that such a situation may not be sustained at equilibrium. Suppose issuer one tailors basket $z$ such that investors one strictly prefers basket $z$ over basket $q$. Assume further that issuer two does the same with investors two. Namely, issuer two tailors basket $q$ such that investors two strictly prefers basket $q$ over basket $z$. If trading between issuers is not allowed, then basket $q$ cannot have shares of $x$, because only investors one are endowed with $x$. Moreover, basket $z$ cannot have shares of $y$ since only investors two are endowed with $y$. Provided that investors demand baskets only for risk sharing purposes, none of them will demand such customized baskets. If trading between issuers is allowed, on the other hand, investors may not want to trade shares of their assets in exchange for shares of baskets if markups are sufficiently high, because investors will to need pay a markup to both issuers.

Consider another way of cooperation among issuers. Suppose they decide to issue the same basket and split profits. Provided the symmetry, suppose half of investors one buy basket $z$, whereas half of investors two buy basket $q$. This problem is equivalent to a problem in which there is only one issuer who chooses the basket structure and gets $\hat{m} = \frac{m}{2}$ per share. With log($\cdot$) preferences, the basket structure is either $r_1 = \frac{\phi}{\phi + \hat{m}}$ or $r_2 = \frac{1 - \phi}{1 - \phi + \hat{m}}$, as example 2 shows. As a consequence, as the number of issuers increases, the basket gets closer to a basket that equally weights both assets. Therefore, investors may benefit from cooperation between issuers. However, the non-cooperative character of competition among issuers prevents cooperation, as figures 2 and 3 show. It then follows

PROPOSITION 3: Under duopoly competition, issuers tend to introduce redundant baskets.

I now analyze the effect that competition among issuers has on investors’ welfare. Consider one issuer tailors, but not perfectly, her basket to investors one. Since competition may increase the diversification of the basket, investors one may be worse-off with the entry of new issuers. This is because the structure of the basket is different from the structure that satisfies investors one’s risk-sharing needs—which is introduced under monopoly provision. On the other hand, investors two may be better off because the new basket has relatively more shares of $x$. Provided the symmetry, the same idea applies when one issuer tailors her basket to serve investors two. Therefore, the allocation that emerges with duopoly competition not always Pareto dominates the allocation that emerges from monopoly provision.
Figure 2. Basket securities under duopoly competition. Both the 45% line and issuers’ best response functions are depicted. An equilibrium of the design game played by issuers is represented by an intersection of issuers’ best response functions, $\alpha_z^*(\cdot), \alpha_q^*(\cdot)$. In this case two baskets may coexist in equilibrium.

Shocks to issuers’ technology could also make competition non desirable. Suppose $m_1 = m_2 = \hat{m}$. Assume that both issuers face issuing costs equal to $\hat{c}$ and that either $\hat{m}$ decreases or $\hat{c}$ increases due to shocks to issuers’ technology. As a consequence, both issuers have fewer incentives to enter the market, because issuer $i$ receives only $\gamma_i \hat{m}$ per share, with $\gamma_i < 1$. Without competition, on the other hand, an issuer receives $\hat{m}$ per share. If either $\hat{m}$ is not sufficiently large or $\hat{c}$ is not sufficiently small, then issuers do not provide the basket, so no trading emerges as an equilibrium under duopoly competition. In such a case, investors are better off if just one issuer provides the intermediation service. It then follows

PROPOSITION 4: When issuers compete, the increase in the number of issuers is not always beneficial for all market participants.
Figure 3. As in Figure 3, basket securities under duopoly competition are depicted. Both the 45% line and issuers’ best response functions are shown. In this case though, four baskets may coexist in equilibrium.

V. A Numerical Example

In what follows, I try to pin down parameter $m$ from asset market data in order to compute proxies for the effective segmentation investors face when investing across different markets. The basket structures implied by the model are hard to extrapolate to baskets that we observe in practice, though, given the two states–two assets framework of the model. One way to partially circumvent the problem is to consider baskets that invest in different geographical regions. Then, each region can be interpreted as one market segment. In what follows, I focus on index securities—which are among today’s most actively traded basket securities—to assess the degree of segmentation across different markets.

I consider two commonly traded ETFs: the iShares MSCI EAFE ETF and the iShares MSCI ACWI Index Fund. The iShares MSCI EAFE ETF follows the MSCI EAFE Index, which is one of the most widely quoted indexes, e.g Ferri (2007). This index includes stocks from Europe, Australasia, and the Far East. It is a selection of stocks from 22 developed markets, but excludes
those from the U.S. and Canada.\textsuperscript{7} On the other hand, the iShares MSCI ACWI Index Fund invests mostly in companies in U.S. and Europe.\textsuperscript{8}

I then try to compute a proxy for the market segmentation between the following pairs of regions: U.S.–Europe, and Europe–Asia Pacific. To do so, I proceed as follows:

1) I set

\[
\begin{align*}
    r_{iShares \ MSCI \ EAFE \ ETF} &= \frac{\text{fraction of the fund invested in Asia-Pacific}}{\text{fraction of the fund invested in Europe}} \\
    r_{iShares \ MSCI \ ACWI \ Index \ Fund} &= \frac{\text{fraction of the fund invested in Europe}}{\text{fraction of the fund invested in U.S.}}
\end{align*}
\]

2) I consider that investors have CRRA preferences with a risk aversion coefficient \( \rho = 2.5. \textsuperscript{9} 

3) For a given value of \( m \), I solve for \( r_\alpha \) as a function of investors’ risk aversion. I then select the values for \( m \) that allow me to match the pair (risk aversion, basket structure) I observe in data.

Figure 4 shows the structure of a basket as a function of investors’ risk aversion for two different measures of market segmentation, \( m \in \{0.55, 0.6\} \). In other words, each curve in figure 4 takes parameter \( m \) as given and represents the basket structure as a function of investors’ risk aversion. The two measures of market segmentation depicted in Figure 4 are the ones that match the pair (risk aversion, basket structure) I observe in both the iShares MSCI EAFE ETF and the iShares MSCI ACWI Index Fund. Figure 4 suggests that investors encounter about the same degree of market segmentation when investing between the U.S. and Europe than when investing between Europe and the Asia–Pacific region, because \( m_{iShares \ MSCI \ EAFE \ ETF} \approx m_{iShares \ MSCI \ ACWI \ Index \ Fund} \).

\textsuperscript{7} By June 2013, the fund had a net asset value of 45.41 billion with an expense ratio of 0.34%. Fund composition: Japan 22.39%, United Kingdom 21.06%, Switzerland 9.58%, France 8.85%, Germany 8.57%, Australia 7.91%, Sweden 3.07%, Netherlands 2.93%, Hong Kong 2.86%, Other 12.03%. For more details see: http://us.ishares.com/product info/fund/overview/EFA.htm

\textsuperscript{8} By June, 2013 the fund had a net asset value of 3.05 billion and expense ratio of 0.34%. Holdings by country: United States 46.57%, United Kingdom 8.14%, Japan 7.14%, Canada 4.36%, Switzerland 3.26%, Australia 3.12%, Germany 3.06%, France 3.04%, China 2.09%, Other 18.83%. For more details see: http://us.ishares.com/product info/fund/overview/ACWI.htm

\textsuperscript{9} Vissing-Jorgensen (2002) estimates \( \rho = 2.5 \) using market data with limited investor participation.
VI. Concluding Remarks

I present a general equilibrium model of basket securities in segmented markets and explore the design and welfare implications of the introduction of these securities for different investor types. If there is only one intermediary, I find that the market-mediated equilibrium may not be constrained efficient, because the intermediary not only seeks to maximize trading volume but also seeks to decrease downside risk. I then analyze how competition among intermediaries affects the basket structure and investors’ welfare. I show that competition can generate the coexistence of several redundant baskets that do not necessarily improve investors’ risk-sharing opportunities. I also show that an increase in the number of intermediaries may not be desirable.

To increase the accuracy of the numerical example, it may be useful to explore a richer environment where the states of nature are correlated. In the current model, the two states–two securities framework may enhance the trade-off between trading volume and downside risk. In addition, it may be useful to extend the model to allow more assets and periods to increase our understanding

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**Figure 4.** Basket structure as a function of investors’ risk aversion for different levels of market segmentation. Investors are assumed to have CRRA preferences with a risk aversion coefficient $\rho$. Both states are assumed to be equally probable.
of which assets are typically included in baskets by profit-maximizing intermediaries and how the structure of basket securities evolves over time.

REFERENCES


**APPENDIX**

**A. Proofs**

**Proof Example 1**

The Lagrangian of this problem is then

\[
\mathcal{L} = \phi \log(c_1^p) + (1 - \phi) \log(c_1^f) - \lambda_1(\phi \log(c_2^p) + (1 - \phi) \log(c_2^f) - u_0) \\
- \lambda_2(\phi c_3^p + (1 - \phi)c_3^f - \pi_0 - c_0) - \lambda_3(c_4^p + c_4^f + c_2^f - w_x) - \lambda_4(c_1^p + c_2^p + c_2^f - w_y)
\]
If \( c_i^x \neq 0 \) and \( c_i^y \neq 0 \) then the first order conditions yield

\[
\frac{c_i^y}{c_i^x} = \frac{c_i^y}{c_i^x} = 1
\]

It has to be the case that at the optimum \( \phi \log(c_i^x) + (1 - \phi) \log(c_i^y) = u_0 \) and \( \phi c_i^x + (1 - \phi)c_i^y = \pi_0 + c_0 \). Using these expressions follows

\[
c_1 = \phi w_x + (1 - \phi)w_y - \pi_0 - c_0 - e^{u_0}
\]
\[
c_2 = e^{u_0}
\]

From market clearing follows then

\[
c_i^x = \pi_0 + c_0 + (1 - \phi)(w_x - w_y)
\]
\[
c_i^y = \pi_0 + c_0 - \phi(w_x - w_y)
\]

**Proof Example 2**

Suppose \( u_i(c) = \log(c), \ i = 1, 2 \). Take asset prices \( p_x \) and \( p_y \) and the basket structure as given. The optimal consumption bundles are then,

\[
(c_1^x, c_1^y) = \left( \phi w_x, \left( \frac{(1 - \phi)(1-m)p_x}{p_y + m p_x \left( \frac{1}{r} \right)} \right) w_x \right)
\]
\[
(c_2^x, c_2^y) = \left( \frac{\phi p_y (1-m)}{p_x + mr p_y} \right) w_y, \left( 1 - \phi \right)w_y
\]

and investors’ optimal portfolio bundles are,

\[
(x_1^*, z_1^*) = \left( \left( \phi - \frac{p_x (1-m)(1-\phi)}{mp_x + r p_y} \right) w_x, \left( \frac{1 - \phi}{r} \right) \frac{p_x}{p_y} \right) w_x
\]
\[
(y_2^*, z_2^*) = \left( \left( 1 - \phi \right) \frac{\phi r p_y (1-m)}{p_x + mr p_y} \right) w_y, \left( \frac{\phi p_y}{p_y + m r p_y} \right) w_y
\]

Define \( r_{\text{min}}^p = m \left[ \frac{1}{1-\phi} \right] \left[ \frac{w_x}{w_y} \right] \) and \( r_{\text{max}}^p = \left[ \frac{1}{m} \right] \left[ \frac{1}{1-\phi} \right] \left[ \frac{w_x}{w_y} \right] \). Imposing the market clearing condition for asset \( x \) yields

\[
p_x = \frac{\phi w_y - r_{\alpha}(1-\phi)mw_y}{(1-m)(\phi w_y - r_{\alpha}(1-\phi)w_x)}
\]
\[
p_y = \frac{r_{\alpha}(1-\phi)w_x - \phi mw_y}{r_{\alpha}(1-m)(\phi w_y - r_{\alpha}(1-\phi)w_x)}
\]

For both prices to be well-defined I consider \( m \) small enough and \( r_{\text{min}}^p < r_{\alpha} < r_{\text{max}}^p \). The effective rate of exchange between asset \( x \) and \( y \) is then,

\[
\frac{p_x}{p_y} = \left( \frac{\phi w_y - (1-\phi)mw_y}{(1-\phi)w_x - \phi mw_y} \right)
\]

(1)
Provided prices, $x_1$ and $y_2$ are given by

$$
x_1^* = \frac{r(\phi + m)w_x - \phi w_y}{r(1 + m)} \Rightarrow (x_1^*) \geq 0 \iff r \geq \left[ \frac{\phi}{\phi + m} \right] \frac{w_y}{w_x}
$$

$$
y_2^* = \frac{(1 - \phi + m)w_y - r(1 - \phi)w_x}{1 + m} \Rightarrow (y_2^*) \geq 0 \iff r \leq \left[ \frac{1 - \phi + m}{1 - \phi} \right] \frac{w_y}{w_x}
$$

One needs to impose that both $x_1^*$ and $y_2^*$ be non negative. This is equivalent to impose the non default constrains for the issuer, i.e. $\alpha_x \leq w_x$ and $\alpha_y \leq w_y$ in the general case. Define further $r^q_{\min} = \left[ \frac{\phi}{\phi + m} \right] \frac{w_y}{w_x}$ and $r^q_{\max} = \left[ \frac{1 - \phi + m}{1 - \phi} \right] \frac{w_y}{w_x}$.

Consider $\phi \in (m, 1 - m)$. For prices and quantities to be well-defined I need $r^q_{\min} \leq r_\alpha \leq r^q_{\max}$. From investors’ optimal portfolio and equation (1) follow

$$z_1 = z_1 + z_2$$

$$= \left[ \frac{1}{1 + m} \right] \left[ \frac{\phi w_y}{r_\alpha} + (1 - \phi)w_x \right]$$

which implies,

$$E[\pi] = \left[ \frac{m}{1 + m} \right] \left[ \phi + (1 - \phi)r_\alpha \right] \left[ \frac{\phi w_y}{r_\alpha} + (1 - \phi)w_x \right] - c_0$$

Thus,

$$\frac{\partial E[\pi]}{\partial r_\alpha} = \frac{m(r^2_\alpha(1 - \phi)^2w_x - \phi^2w_y)}{r^2_\alpha(1 + m)}$$

$$\frac{\partial^2 E[\pi]}{\partial r_\alpha^2} = \frac{2m\phi^2w_y}{r^3_\alpha(1 + m)}$$

(2)

(3)

The first derivative is zero at points $r_{1,2}^{\text{threshold}} = \pm \frac{\phi}{\sqrt{-w_y}}$. Note from the second derivative that $r_1^{\text{threshold}}$ is a local minimum and $r_2^{\text{threshold}}$ a local maximum.

Since $r_{1}^{\text{threshold}} < 0 < r_{2}^{\text{threshold}}$ then follows

(Case a) $r_{1}^{\text{threshold}} < r_{\min}^q$ (i.e. $\phi < \sqrt{-\frac{m}{w_y}}$) then $r_\alpha = r_{\max}^q$

(Case b) $r_{1}^{\text{threshold}} > r_{\max}^q$ (i.e. $\phi > \frac{1 - m}{\sqrt{w_y}}$) then $r_\alpha = r_{\min}^q$

(Case c) $r_{\min}^q \leq r_{1}^{\text{threshold}} \leq r_{\max}^q$ (i.e. $\frac{1 - m}{\sqrt{w_y}} < \phi \leq \frac{\sqrt{m}}{1 + \sqrt{w_y}}$) and:

(c1) If $w_x = w_y$

(c1.1) $\phi \in \left( \frac{1}{2}, \sqrt{-\frac{m}{w_y}} \right)$ i.e. $E[\pi (r_{\min}^q)] > E[\pi (r_{\max}^q)]$ - then $r_\alpha = r_{\min}^q$ (empty set)

(c1.2) $\phi \in \left( \frac{1 - m}{\sqrt{w_y}}, \frac{1}{2} \right)$ i.e. $E[\pi (r_{\min}^q)] < E[\pi (r_{\max}^q)]$ - then $r_\alpha = r_{\max}^q$

(c2) If $w_x > w_y$

(c2.1) $\sqrt{\frac{w_y(w_x - w_y)}{w_x}} > 4m + \frac{w_y}{w_x}$ and $\phi > \frac{1}{2} \left( 2 + m - \frac{2(1 + m)w_y}{w_x} \right)$ i.e. $E[\pi (r_{\min}^q)] > E[\pi (r_{\max}^q)]$ - then $r_\alpha = r_{\min}^q$

(c2.2) $\sqrt{\frac{w_y(w_x - w_y)}{w_x}} > 4m + \frac{w_y}{w_x}$ and $\phi \leq \frac{1}{2} \left( 2 + m - \frac{2(1 + m)w_y}{w_x} \right)$ i.e. $E[\pi (r_{\min}^q)] < E[\pi (r_{\max}^q)]$ - then $r_\alpha = r_{\max}^q$
Figure 5. Issuer’s expected profits as a function of the basket security structure $r_\alpha$ when $r_1^{\text{threshold}} \in (r_\alpha^{\text{min}}, r_\alpha^{\text{max}})$

\[
\begin{align*}
\text{(c.2.3)} & \quad \sqrt{\frac{w_y(8w_x + w_y)}{w_y^2}} \leq 4m + \frac{w_x}{w_y} \quad \text{i.e. } E[\pi (r_\alpha^{\text{min}})] > E[\pi (r_\alpha^{\text{max}})] \quad \text{then } r_\alpha^* = r_\alpha^{\text{min}} \\
\text{(c.3)} & \quad \text{If } w_y > w_x \\
\text{(c.3.1)} & \quad \sqrt{\frac{w_x(8w_y + w_x)}{w_x^2}} > 4m + \frac{w_y}{w_x} \quad \text{and } \phi > \frac{1}{2} \quad 2 + m - \frac{2(1+m)w_x}{w_x^2-w_y} + \sqrt{\frac{4w_x w_y + 8m w_x w_y + m^2 (w_x + w_y)^2}{(w_x-w_y)^2}} \quad \text{i.e.} E[\pi (r_\alpha^{\text{min}})] > E[\pi (r_\alpha^{\text{max}})] \quad \text{then } r_\alpha^* = r_\alpha^{\text{min}} \\
\text{(c.3.2)} & \quad \sqrt{\frac{w_x(8w_y + w_x)}{w_x^2}} > 4m + \frac{w_y}{w_x} \quad \text{and } \phi \leq \frac{1}{2} \quad 2 + m - \frac{2(1+m)w_x}{w_x^2-w_y} + \sqrt{\frac{4w_x w_y + 8m w_x w_y + m^2 (w_x + w_y)^2}{(w_x-w_y)^2}} \quad \text{i.e.} E[\pi (r_\alpha^{\text{min}})] < E[\pi (r_\alpha^{\text{max}})] \quad \text{then } r_\alpha^* = r_\alpha^{\text{max}} \\
\text{(c.3.3)} & \quad \sqrt{\frac{w_x(8w_y + w_x)}{w_x^2}} \leq 4m + \frac{w_y}{w_x} \quad \text{i.e.} E[\pi (r_\alpha^{\text{min}})] < E[\pi (r_\alpha^{\text{max}})] \quad \text{then } r_\alpha^* = r_\alpha^{\text{max}}
\end{align*}
\]

The consumption allocations for each agent under the different basket structures are then:

**MA1**  
$r_\alpha^* = r_\alpha^{\text{min}}$

\[
\begin{align*}
(e_1^x, c_1^y) &= \left( \phi w_x, \frac{\phi^2}{\phi + m} w_y \right) \\
(e_2^x, c_2^y) &= \left( (1 - \phi - m) w_x, (1 - \phi) w_y \right) \\
(e_3^x, c_3^y) &= \left( m w_x, \frac{m \phi}{\phi + m} w_y \right)
\end{align*}
\]
MA2) \( r^*_\alpha = r^\alpha_{\text{max}} \)

\[
(c^x_1, c^y_1) = (\phi w_x, (\phi - m)w_y) \\
(c^x_2, c^y_2) = \left( \frac{(1 - \phi)^2}{1 - \phi + m} w_x, (1 - \phi)w_y \right) \\
(c^x_3, c^y_3) = \left( \frac{m(1 - \phi)}{1 - \phi + m} w_x, mw_y \right)
\]

For illustrative purposes consider \( r^* = r^\alpha_{\text{min}} \). Consider further \( c_0 \) small enough such that \( \pi^* > 0 \). Then, asset prices at equilibrium are given by

\[
px = \frac{\phi}{(1 - m)} \\
py = \frac{(1 - \phi - m)(\phi + m)w_x}{w_y \phi (1 - m)}
\]

Then the effective exchange rate between asset \( x \) and \( y \) is,

\[
\frac{px}{py} = \frac{\phi^2 w_y}{(1 - \phi - m)(\phi + m)w_x}
\]

**Proof Example 3**

Taking limit when \( \rho \to \infty \) to the first order conditions of both investors follows

\[
c^x_1 = c^y_1 = c^*_1 \\
c^x_2 = c^y_2 = c^*_2
\]

Using the budget constraint yields

\[
c^*_1 = \frac{px w_x r_\alpha (1 - m)}{px r_\alpha (1 - m) + mp_x + r_\alpha py} \\
c^*_2 = \frac{py w_y (1 - m)}{py (1 - m) + mp_y r_\alpha + px}
\]

From the proof of Proposition 1 follows that when \( \rho \to \infty \) then \( r_\alpha \to 1 \). This result also can be found using the non-default restrictions. With them, the only feasible basket structure that generates non-negative asset holdings is \( r_\alpha \to 1 \).

Consider further there is no aggregate risk. From the optimality conditions follows

\[
\frac{px}{py} = (1 - m) \left( \frac{\phi}{1 - \phi} \right) - m
\]

and \( c^*_1 = \theta (1 - m) p_x w_0 \) and \( c^*_2 = \theta (1 - m) p_y w_0 \), with \( \theta = \frac{1}{px + py} \). Therefore, the consumption bundle of each agent
Proof Example 4

For simplicity consider $\alpha^x = \alpha^y = 1$. Investor one then solves

$$\max_{(x_1, z_1)} U_1(c_1^x, c_1^y) = \phi \log(c_1^x) + (1 - \phi) \log(c_1^y)$$

s.t.

$$p_x x_1 + p_z z_1 + p_q q_1 \leq p_x w_x$$

$$x_1 \geq 0, \ z_1 \geq 0, \ q_1 \geq 0$$

$$c_1^x = x_1 + (1 - m_1) z_1 + (1 - m_2) z_1$$

$$c_1^y = (1 - m_1) \alpha^y_z z_1 + (1 - m_2) \alpha^y_q q_1$$

Since at the optimum both $c_1^x$ and $c_1^y$ are strictly positive, both $z_1$ and $q_1$ cannot be zero at the same time. Therefore, the allocation where $x_1 = w_x$ cannot be an optimum of the problem above.

Since only competition among issuers is of interest at this point I focus on the case when both issuers participate on the market. Thus, an allocation where $x_1 > 0, \ z_1 > 0$ and $q_1 = 0$ is of no interest. The same applies to allocations where $x_1 > 0, \ z_1 = 0$ and $q_1 > 0$. Thus, I restrict the analysis to cases where the optimal allocation satisfies either $x_1 = 0, \ z_1 > 0$ and $q_1 > 0$ or $x_1 > 0, \ z_1 > 0$ and $q_1 > 0$.

For simplicity consider that $\phi \in [\phi_l, \phi_u]$ where $\phi_l$ and $\phi_u$ are both close to $\frac{1}{2}$. Since investors one smooth consumption across states the optimum needs to satisfy $x_1 = 0$. I then can rewrite the above problem as:

$$\max_{(x_1, z_1)} U_1(c_1^x, c_1^y) = \phi \log(c_1^x) + (1 - \phi) \log(c_1^y)$$

s.t.

$$p_x z_1 + p_q q_1 = p_x w_x$$

$$z_1 \geq 0, \ q_1 \geq 0$$

$$c_1^x = (1 - m_1) z_1 + (1 - m_2) z_1$$

$$c_1^y = (1 - m_1) \alpha^y_z z_1 + (1 - m_2) \alpha^y_q q_1$$

Consider that $z_1 > 0$ and $q_1 > 0$. The lagrangian of investor one is then

$$\mathcal{L} = \phi \log[(1 - m_1) z_1 + (1 - m_2) q_1] + (1 - \phi) \log[(1 - m_1) \alpha^y_z z_1 + (1 - m_2) \alpha^y_q q_1]$$

$$- \lambda_1 [p_x x_1 + p_z z_1 + p_q q_1 - p_x w_x]$$

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F.O.C are given by:

\[ z_1 : \frac{\phi}{c_1^1} (1 - m_1) + \frac{1 - \phi}{c_1^1} (1 - m_1) \alpha_y^e - \lambda_1 p_z = 0 \]
\[ q_1 : \frac{\phi}{c_1^1} (1 - m_2) + \frac{1 - \phi}{c_1^1} (1 - m_2) \alpha_y^e - \lambda_1 p_q = 0 \]
\[ \lambda_1 : p_x x_1 + p_z z_1 + p_q q_1 = p_x w_x \]

Manipulating the above equations yields

\[ c_1^y = \beta c_1^x \]

where

\[ \beta = \left( \frac{1 - \phi}{\phi} \right) \left( \frac{\omega \alpha_y^e - \alpha_y^e}{1 - \omega} \right) \]
\[ \omega = \left( \frac{p_x}{p_q} \right) \left( \frac{1 - m_2}{1 - m_1} \right) \]
\[ c_1^x = \frac{p_z w_x}{p_z (1 - m_1) (\alpha_y^e - \alpha_y^e) + p_q (1 - m_2) (\alpha_y^e - \alpha_y^e)} \]

Using the above equalities plus the budget constrain yields

\[ z_1 = \frac{p_z w_x}{p_z + p_q \left( \frac{1 - m_1}{1 - m_2} \right) \left( \frac{\alpha_y^e - \alpha_y^e}{\beta - \alpha_y^e} \right) z_1} \]
\[ q_1 = \frac{(1 - m_1) (\alpha_y^e - \beta)}{(1 - m_2) (\beta - \alpha_y^e) z_1} \]

Provided the symmetry, investor two portfolio is given by

\[ z_2 = \frac{p_z w_y}{p_z + p_q \left( \frac{1 - m_1}{1 - m_2} \right) \left( \frac{\alpha_y^e - \alpha_y^e}{\beta - \alpha_y^e} \right) z_2} \]
\[ q_2 = \frac{(1 - m_1) (\alpha_y^e - \beta)}{(1 - m_2) (\beta - \alpha_y^e) z_2} \]

**Proof Proposition 1**

For simplicity consider\(^{10}\)

\[ u_i(c) = \begin{cases} \frac{1 - \rho}{1 - \rho} : \rho > 0 \text{ and } \rho \neq 1 \\ \log(c) : \rho = 1 \end{cases} \]

\(^{10}\)Even when this assumption may seems restrictive it is not since the main ideas behind the following proof are still valid if one considers other commonly utility functions with linear risk tolerance such as CARA utility, Shifted Logarithmic utility, Shifted Power utility, etc.
\( i = \{1, 2\} \). Take asset prices \( p_x \) and \( p_y \) and the basket structure as given. The optimal consumption bundles are then,

\[
(c^i_1, c^i_2) = \left( \frac{w_x}{1 + \left[ \frac{1}{1-m} \left[ \frac{m}{r_\alpha + \frac{p_y}{p_x}} \right] \left( \frac{1-m}{r_\alpha + \frac{p_y}{p_x}} \right) m \right] \right)^{\frac{1}{\beta}}, \frac{w_x \left[ \frac{1-m}{r_\alpha + \frac{p_y}{p_x}} \right]}{1 + \left[ \frac{1}{1-m} \left[ \frac{m}{r_\alpha + \frac{p_y}{p_x}} \right] \left( \frac{1-m}{r_\alpha + \frac{p_y}{p_x}} \right) m \right] \right)^{\frac{1}{\beta}}},
\]

\[
(c^y_1, c^y_2) = \left( \frac{w_y \left[ \frac{m r_\alpha + \frac{p_y}{p_x}}{1-m} \right]}{1 + \left[ \frac{1}{1-m} \left[ \frac{m}{r_\alpha + \frac{p_y}{p_x}} \right] \left( \frac{1-m}{r_\alpha + \frac{p_y}{p_x}} \right) m \right] \right)^{\frac{1}{\beta}}, \frac{w_y \left[ \frac{m r_\alpha + \frac{p_y}{p_x}}{1-m} \right]}{1 + \left[ \frac{1}{1-m} \left[ \frac{m}{r_\alpha + \frac{p_y}{p_x}} \right] \left( \frac{1-m}{r_\alpha + \frac{p_y}{p_x}} \right) m \right] \right)^{\frac{1}{\beta}}},
\]

and investors’ optimal portfolio bundles are,

\[
(x^*_1, z^*_1) = \left( c^x_1 - c^y_1 r_\alpha (1-m) \right),
\]

\[
(y^*_2, z^*_2) = \left( c^y_2 - r_\alpha c^x_2 \right),
\]

From issuer’s problem follows that in case there are no gains from providing risk-sharing services to one investor over the other, then the issuer provides a basket that replicates the market portfolio to maximize the trading volume on the basket. In such case, it has to be the case that at the basket structure which maximizes issuer’s profit the issuer is indifferent between providing risk-sharing services either to investor one or investor two. In case there are gains from providing risk-sharing services to one investor over the other, the issuer structures her basket such that either \( x^*_1 = 0 \) or \( y^*_2 = 0 \). Then, a basket security does not always replicates the market portfolio since the issuer may prefer to customize her basket to increase trading volume on the basket.

When the issuer customizes its basket to \textit{investors one} such that \( x^*_1 = 0 \) then

\[
\frac{c^y}{c^x} = \left[ \frac{1-m}{r_\alpha + \frac{p_y}{p_x}} \right]^{\frac{1}{\beta}},
\]

On the other hand, when the issuer customizes its basket to \textit{investors two} such that \( y^*_2 = 0 \) then

\[
\frac{c^x}{c^y} = \left[ \frac{m r_\alpha + \frac{p_y}{p_x}}{1-m} \right]^{\frac{1}{\beta}}.
\]

To solve for the basket structure as a function of the parameters of the model I need the market clearing conditions to solve for equilibrium prices. Since a closed-form solution can be obtained only for some values of \( \rho \), I resort to numerical computations to solve for different values of \( \rho \).