The Maturity Structure of Inside Money

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Motivation

- Do Banks transform risk and maturity efficiently?

- We analyze bank balance sheet transformation when banks create liquidity and face aggregate shocks
  - Bank liabilities used to facilitate transactions—“inside money”
  - Bank investments subject to aggregate risk

- Find banks under-provide maturity and risk transformation
  - Bank liabilities too long and too risky relative to social optimum
Our Mechanism

• Claims to bank cash flows serve as inside money
  ○ Partially backed by productive assets with aggregate risk
  ○ Partially backed by bank’s equity

• If inside money is scarce, households liquidity constrained

• Liquidity constraints introduce additional curvature in private and social value functions
  ○ Implies role for banks to provide aggregate liquidity insurance

• Limited bank commitment impedes provision of insurance
  ○ Banks cannot fully commit to transfer equity in low-return states
  ○ Shortening maturity via liquidation relaxes commitment problem

• Pecuniary externality associated with bank liabilities causes too little transformation
Related Literature

- Banks’ Maturity Transformation
  - Diamond and Dybvig (1983) and Calomiris and Kahn (1991)

- Too Much Maturity Transformation
  - Stein (2016), Brunnermeier and Oehmke (2015)

- Too Little Maturity Transformation

- Information Insensitivity of Bank Claims

- Bankers’ Role as Providers of Inside Money
  - Cavalcanti and Wallace (1999), Monnet and Sanches (2012), Gu, Mattesini, Monnet, and Wright (2013)
Plan for Rest of Talk

- Environment
- Constrained Efficient Benchmark
- Equilibrium Risk and Maturity Transformation
- Conclusions
ENVIRONMENT
Key Ingredients

- **Banks:**
  - Issue claims subject to limited commitment
  - Use proceeds and endowments to purchase capital
  - Capital subject to risk and costly liquidation
    - Only source of aggregate risk

- **Households:**
  - Periodically trade in frictional markets
  - Use bank claims to relieve liquidity constraints in frictional markets
Environment: Households

- Adapts standard monetary economy to finite horizon: $t = 0, 1, 2$

- Preferences:
  - Buyers, measure 1:
    $$v(x_0) - y_0 + \sum_{t=1,2} [u(q_t) + v(x_t) - y_t]$$
  - Sellers, measure $n$:
    $$v(x_0) - y_0 + \sum_{t=1,2} [-c(q_t) + v(x_t) - y_t]$$

- No risk over buyer/seller type not critical
  - Insurance mechanism different than in Diamond and Dybvig (1983)

- Endowed with $k^i, i = b, s$ capital goods ($K^H = k^b + nk^s$)
Environment: Markets

- Decentralized Market (DM), trade in \( q_t \)
  - Random, pairwise matching; buyer meets seller with pr. \( \alpha(n) \)
  - Trade requires medium of exchange, subject to bargaining
  - Efficient DM trade: \( u'(q^*) = c'(q^*) \)

- Centralized Market (CM), trade \( x_t \) and assets, produce \( y_t \)
  - Market is competitive
Environment: Banks

- Representative bank; only participates in centralized markets

- Preferences: \( \sum_{t=1,2} c^B_t \)

- Endowed with \( K^B \) capital goods

- Uses endowment and proceeds from any issuances to undertake real investment
• Invest $I$ in $\text{CM}_0$

• Info about returns realized in $\text{DM}_1$:
  - $\omega \in \{\omega_l, \omega_h\}$ w. prob $\gamma(\omega)$
  - At maturity, rate of return $z(\omega)$ with $z(\omega_h) > z(\omega_l)$

• Plan to *liquidate* $L(\omega) \in [0, 1]$ in period $\text{CM}_1$ with $\kappa < 1$

• Realized Output:
  - $\text{CM}_1 : \quad L(\omega)Iz(\omega)\kappa$
  - $\text{CM}_2 : \quad (1 - L(\omega))Iz(\omega)$

• Moral Hazard: abscond with $\xi \leq 1$ per unit of capital after $\text{CM}_1$
  - $\text{CM}_2$ payoff: $(1 - L(\omega))Iz(\omega)\xi$
Environment: Bank Claims

- Banks issue claims with coupon payments \(d_t(\omega) \geq 0\),

\[
D = \{D(\omega_l), D(\omega_h)\} = \{d_1(\omega_l), d_2(\omega_l), d_1(\omega_h), d_2(\omega_h)\}
\]

- Function \(p_0(D)\): price of claim with coupon \(D\)
  - More on \(p_0(D)\) later...

- Households purchase claims in period 0; trade claims in future DM and CMs
  - No “early redemption” at bank

- Notation: \(p_t(D(\omega))\) is ex-coupon claim price in CM\(_t\), state \(\omega\)
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Banks: Pay coupons, $d_2(\omega)$
Bank’s Problem

- Representative bank solves

\[
\max_{I,L,D,c^B \geq 0} \sum_{\omega \in \Omega} \gamma(\omega) \left[ c^B_1(\omega) + c^B_2(\omega) \right]
\]

subject to

\[
\begin{align*}
p^k_0 I & \leq p^k_0 K^B + p_0(D) \\
c^B_1(\omega) + d_1(\omega) & = L(\omega) \kappa I_z(\omega) \\
c^B_2(\omega) + d_2(\omega) & = [1 - L(\omega)] I_z(\omega) \\
c^B_2(\omega) & \geq [1 - L(\omega)] I_z(\omega) \xi \\
\sum_{\omega \in \Omega} \gamma(\omega) \left[ c^B_1(\omega) + c^B_2(\omega) \right] & \geq K^B \sum_{\omega \in \Omega} \gamma(\omega) z(\omega)
\end{align*}
\]
Buyers’ Problem

- Given claims issued by banks, period $CM_{t>0}$ value for a Buyer:

$$W_t^b(a; D(\omega)) = \max_{x,y,a'} v(x) - y + V_{t+1}^b(a' ; D(\omega))$$

subject to

$$x + a' p_t(D(\omega)) \leq y + [p_t(D(\omega)) + d_t(\omega)]a$$

$$V_{t+1}^b(a', D(\omega)) = (1 - \alpha(n)) W_{t+1}^b(a'; D(\omega))$$

$$+ \alpha(n) \int_{a^s} \left\{ u[q_{t+1}(a', a^s; D(\omega))] + W_{t+1}^b(a' - m_{t+1}(a', a^s; D(\omega)); D(\omega)) \right\} d\Psi_{t+1}(a^s)$$

where $q_{t+1}$ and $m_{t+1}$ are terms of decentralized trade
Sellers’ Problem

• Given claims issued by banks, period \( CM_t > 0 \) value for a Seller:

\[
W_t^S(a; D(\omega)) = \max_{x,y,a'} v(x) - y + V_{t+1}^S(a'; D(\omega))
\]

subject to

\[
x + a' p_t(D(\omega)) \leq y + [p_t(D(\omega)) + d_t(\omega)] a
\]

\[
V_{t+1}^S(a', D(\omega)) = (1 - \alpha(n)/n) W_{t+1}^S(a'; D(\omega)) \\
+ \alpha(n)/n \int_{a^b} \left\{-c[q_{t+1}(a^b,a'; D(\omega))]+W_{t+1}^S(a'+m_{t+1}(a^b,a'; D(\omega)); D(\omega))\right\} d\Psi_{t+1}^b(a^b)
\]

where \( q_{t+1} \) and \( m_{t+1} \) are terms of decentralized trade
Households’ Period 0 Problem

- Given claims issued by banks, period $CM_{t=0}$ value type $i = \{s, b\}$

$$W^i_0(D) = \max_{x, y, a'} v(x) - y + \sum_{\omega \in \Omega} \gamma(\omega)V^i_1(a', D(\omega))$$  \hspace{1cm} (1)

subject to:

$$x + a' p_0(D) \leq y + p_0^k k^i$$
Decentralized Terms of Trade

- Assume matched buyers and sellers in decentralized market engage in proportional bargaining

- Implies $q_t(a^b, a^s; D(\omega)), m_t(a^b, a^s; D(\omega))$ determined as solution to

$$
\max_{q_t, m_t} u(q_t) + W_t^b(a^b - m_t; D(\omega)) - W_t^b(a^b; D(\omega))
$$

subject to

$$
u(q_t) + W_t^b(a^b - m_t; D(\omega)) - W_t^b(a^b; D(\omega)) = \frac{\eta}{1 - \eta} [-c(q_t) + W_t^s(a^s + m_t; D(\omega)) - W_t^s(a^s; D(\omega))]$$

$$m_t \leq a^b$$
Definition (Competitive Equilibrium)

A competitive equilibrium consists of an allocation for the bank \((I, L, D, (c^B_t))\), households’ value functions \(\{(W^i_t)_{t=0,1,2}, (V^i_t)_{t=1,2}\}_{i=s,b}\) and policy functions \(\{(x^i_t, y^i_t, a^i_t)_{t=0,1,2}\}_{i=s,b}\), terms of trade, \(\{(q_t, m_t)_{t=1,2}\}\), and prices \(\{p^k_0, p_0(D), (p_t(D(\omega)))_{t=1,2}\}\) such that

- Bank’s allocation solves the bank’s problem
- For \(i \in \{s, b\}\), the policy functions \((x^i_t, y^i_t, a^i_t)\) are optimal and the value functions satisfy the problems described above
- Goods, capital, and claims markets clear
- Decentralized terms of trade satisfy proportional bargaining
Definition *(Market Equilibrium)*

A *market equilibrium* given a claim issue $D$ consists of a collection of history and time dependent functions $(W^i_t, V^i_t, x^i_t, y^i_t, a^i_t, \Psi^i_t)$, terms of trade, $\{(q_t, m_t)_{t=1,2}\}$, and prices $\{(p_t(D(\omega)))_{t=1,2}\}$ such that

- For $i \in \{s, b\}$, the policy functions $(x^i_t, y^i_t, a^i_t)$ are optimal and the value functions satisfy the problems described above
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- For \( i \in \{s, b\} \), the policy functions \( (x^i_t, y^i_t, a^i_t) \) are optimal and the value functions satisfy the problems described above
- Goods, capital, and claims markets clear
- Decentralized terms of trade satisfy proportional bargaining
- Market equilibrium simply takes claim issuance as exogenous
Asset Transformation

- Allocations may feature bank balance sheet transformation
- Will compare structure of coupons to underlying, externally financed investment
Asset Transformation

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- Will compare structure of coupons to underlying, externally financed investment

Risk Transformation

\[ z(\omega_l)K^H < d_1(\omega_l) + d_2(\omega_l) \leq d_1(\omega_h) + d_2(\omega_h) < z(\omega_h)K^H \]
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Maturity Transformation

\[ d_1(\omega) > 0, \text{ or equivalently } L(\omega) > 0 \text{ some } \omega \]
Asset Transformation

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Risk Transformation

\[ z(\omega_l)K^H < d_1(\omega_l) + d_2(\omega_l) \leq d_1(\omega_h) + d_2(\omega_h) < z(\omega_h)K^H \]

Maturity Transformation

\[ d_1(\omega) > 0, \text{ or equivalently } L(\omega) > 0 \text{ some } \omega \]

- Pass-through claim:
  \[ D: d_1(\omega) = 0, \quad d_2(\omega) = z(\omega)K^H \]
Constrained Efficient Liquidation
Constrained Efficient Allocations

Definition (Constrained Efficient Allocation)

A Constrained Efficient Allocation is a solution to the following problem:

$$\max_D W_b^b(D) + nW_s^s(D)$$

subject to there exists \((I, L, c_l^B)\) such that the claim \(D\) satisfies the bank’s constraints and the value functions constitute market equilibrium values.
A Constrained Efficient Allocation is a solution to the following problem:

$$\max_D W^b_0(D) + nW^s_0(D)$$

subject to there exists $(I, L, c^B_t)$ such that the claim $D$ satisfies the bank’s constraints and the value functions constitute market equilibrium values.

- Next, provide partial characterization of market equilibrium
- Determine tradeoffs involved with liquidation
Asset Values and Terms of Trade

- Quasi-linear preferences imply:
  - Deg. end-of-$CM_t$ asset holdings; $CM$-Value functions are linear in assets
  - Buyers each hold 1 unit of bank claims upon entering $DM_t$
  - Decentralized terms of trade function of value of bank claims
Asset Values and Terms of Trade

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  - Buyers each hold 1 unit of bank claims upon entering \(DM_t\)
  - Decentralized terms of trade function of value of bank claims

Terms of Trade

\[
\begin{align*}
  t = 1 \text{ Claim Value: } & p_1(D(\omega)) + d_1(\omega) \\
  t = 2 \text{ Claim Value: } & d_2(\omega)
\end{align*}
\]
Period 1 Asset Prices

\[ p_1(D(\omega)) = d_2(\omega) \left[ 1 + \alpha(n)\eta \frac{u'(q_{2e}^q(D(\omega))) - c'(q_{2e}^q(D(\omega)))}{(1 - \eta)u'(q_{2e}^q(D(\omega))) + \eta c'(q_{2e}^q(D(\omega)))} \right] \]

If liquidity scarce in period 2, period 1 asset price incorporates liquidity premium.

Period 1 price increasing in \( d_2 \) since \( d_2 \) increases asset price directly through increasing dividends.

\( d_2 \) decreases asset price by decreasing liquidity premium (\( q_{eq}^2 \uparrow \)).
Period 1 Asset Prices

\[ p_1(D(\omega)) = d_2(\omega) \left[ 1 + \alpha(n) \eta \frac{u'(q_2^{eq}(D(\omega))) - c'(q_2^{eq}(D(\omega)))}{(1 - \eta)u'(q_2^{eq}(D(\omega))) + \eta c'(q_2^{eq}(D(\omega)))} \right] \]

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- If liquidity scarce in period 2, period 1 asset price incorporates liquidity premium
- Period 1 price increasing in \( d_2 \)
  - \( d_2 \) increases asset price directly through increasing dividends
  - \( d_2 \) decreases asset price by decreasing liquidity premium (\( q_2^{eq} \uparrow \))

Period 0 Asset Prices

\[ p_0(D) = \sum_{\omega} \sum_{t} \gamma(\omega) \left[ 1 + LP_t(\omega; D(\omega)) \right] d_t(\omega) \]

where \( LP_t(\omega; D(\omega)) \) is an analogously defined liquidity premium
Welfare Objective

- Planner’s objective equivalent to

\[ W^P_0(D) = (1 + n)\bar{\delta} + \sum_{\omega} \gamma(\omega) \sum_t d_t(\omega) + \alpha(n) \sum_{\omega} \sum_t \left[ u(q^{eq}_t(D(\omega))) - c(q^{eq}_t(D(\omega))) \right] \]
Welfare Objective

- Planner’s objective equivalent to

\[
W_0^P(D) = (1 + n) \bar{\sigma} + \sum_\omega \gamma(\omega) \sum_t d_t(\omega) + \alpha(n) \sum_\omega \sum_t \left[ u(q_t^{eq}(D(\omega))) - c(q_t^{eq}(D(\omega))) \right]
\]

- Efficient coupons balance:
  - Maximization of expected PDV of cash flows
  - Smoothing of expected inter-temporal liquidity distortions
When NO Transformation is Optimal

- Planner’s objective:

\[ W_0^P(D) = (1 + n) \bar{\vartheta} + \sum_{\omega} \gamma(\omega) \sum_t d_t(\omega) + \alpha(n) \sum_{\omega} \sum_t [u(q_{t}^{eq}(D(\omega))) - c(q_{t}^{eq}(D(\omega)))] \]
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Lemma (No Transformation of Non-Money Like Assets)

Suppose \( \alpha(n) = 0, \kappa < 1 \) and commitment constraint not too severe.

Then the pass-through claim is efficient (\( L = 0, d_2(\omega) = z(\omega) K_H \)).

- Only concern is maximizing coupon PDV
- Commitment constraint not too severe if \( \xi \leq KBK_H + KB \)
  Implies pass-through claim commitment-feasible
  (will maintain this assumption and \( \kappa < 1 \) throughout)
When NO Transformation is Optimal

- Planner’s objective:

\[ W_0^P(D) = (1 + n)\bar{v} + \sum_{\omega} \gamma(\omega) \sum_t d_t(\omega) + \alpha(n) \sum_{\omega} \sum_t [u(q^eq_t(D(\omega))) - c(q^eq_t(D(\omega)))] \]

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- Only concern is maximizing coupon PDV

- Commitment constraint not too severe if

\[
\xi \leq \frac{K^B}{K^H + K^B}
\]

Implies pass-through claim commitment-feasible
(will maintain this assumption and \(\kappa < 1\) throughout)
When NO Transformation is Optimal

- Suppose pass through claim attains $q_{t}^{eq}(D(\omega)) = q^*$

$$W_0^P(D) = (1 + n)\bar{\vartheta} + \sum_{\omega} \gamma(\omega) \sum_{t} d_t(\omega) + \alpha(n) \sum_{\omega} \sum_{t} [u(q_{t}^{eq}(D(\omega))) - c(q_{t}^{eq}(D(\omega)))]$$
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When NO Transformation is Optimal

- Suppose pass through claim attains $q_t^{eq}(D(\omega)) = q^*$

$$W_0^P(D) = (1 + n)\bar{\nu} + \sum_\omega \gamma(\omega) \sum_t d_t(\omega) + \alpha(n) \sum_\omega \sum_t \left[ u(q_t^{eq}(D(\omega))) - c(q_t^{eq}(D(\omega))) \right] q^*$$

Lemma (No Transformation of Valuable Assets)

Suppose $z(\omega_l)K^H \geq d^*$. Then the pass-through claim is efficient $(L = 0, d_2(\omega) = z(\omega)K^H)$. 
When NO Transformation is Optimal

- Suppose pass through claim attains $q^e_t(D(\omega)) = q^*$

$$W^P_0(D) = (1 + n)\bar{v} + \sum_\omega \gamma(\omega) \sum_t d_t(\omega) + \alpha(n) \sum_\omega \sum_t \left[ u(q^e_t(D(\omega))) - c(q^e_t(D(\omega))) \right] \eta^*$$

**Lemma (No Transformation of Valuable Assets)**

Suppose $z(\omega_1)K^H \geq d^*$. Then the pass-through claim is efficient ($L = 0, d_2(\omega) = z(\omega)K^H$).

- Maximizing Coupon PDV consistent with smoothing inter-temporal liquidity distortions

- In excess liquidity states, transformation has no impact on DM trade
When ONLY Risk Transformation is Optimal

- Suppose pass-through claim does not attain $q^*$ (e.g. $z(\omega_l)K^H < d^*$)
When ONLY Risk Transformation is Optimal

- Suppose pass-through claim does not attain $q^*$ (e.g. $z(\omega_l)K^H < d^*$)

Lemma (Only Risk Transformation)

There exists $z < d^*/K^H$ such that if $z(\omega_l) \in [z, d^*/K^H]$ then efficient allocations feature risk transformation and no maturity transformation.
When ONLY Risk Transformation is Optimal

- Suppose pass-through claim does not attain $q^*$ (e.g. $z(\omega_1)K^H < d^*$)

**Lemma (Only Risk Transformation)**

There exists $z < d^*/K^H$ such that if $z(\omega_1) \in [z, d^*/K^H]$ then efficient allocations feature risk transformation and no maturity transformation.

- Easy case: If claim with $d_2(\omega_1) = d^*$, $d_2(\omega_h) \in [d^*, z(\omega_h)K^H]$ with

$$\sum_{\omega} \gamma(\omega)d_2(\omega) = K^H \sum_{\omega} \gamma(\omega)z(\omega)$$

feasible, then must be efficient
  - Maximizes coupon PDV and decentralized trade
When ONLY Risk Transformation is Optimal

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Lemma (Only Risk Transformation)

There exists $z < d^*/K^H$ such that if $z(\omega) \in [z, d^*/K^H]$ then efficient allocations feature risk transformation and no maturity transformation.

- Easy case: If claim with $d_2(\omega_l) = d^*$, $d_2(\omega_h) \in [d^*, z(\omega_h)K^H]$ with

$$\sum \gamma(\omega)d_2(\omega) = K^H \sum \gamma(\omega)z(\omega)$$

feasible, then must be efficient
  - Maximizes coupon PDV and decentralized trade
  - $z(\omega_l)$ not too small ensures bank commitment constraints slack
Constrained Efficient Maturity Transformation

Proposition (Efficient Maturity Transformation)

There exists $\kappa$, $\xi$, and $z < d^*/K^H$ such that if $z(\omega_l) < z$, then efficient allocations feature both risk and maturity transformation ($d_1(\omega_l), L(\omega_l) > 0$).
Proposition (Efficient Maturity Transformation)

There exists $\kappa, \xi$, and $z < d^*/K^H$ such that if $z(\omega_l) < z$, then efficient allocations feature both risk and maturity transformation $(d_1(\omega_l), L(\omega_l) > 0$.

Proof:

- $z(\omega_l)$ low $\Rightarrow$ DM$_t$ trade distorted, commitment constraint binds
Constrained Efficient Maturity Transformation

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There exists $\kappa, \xi$, and $z < d^*/K^H$ such that if $z(\omega_l) < z$, then efficient allocations feature both risk and maturity transformation $(d_1(\omega_l), L(\omega_l)) > 0$.

Proof:

- $z(\omega_l)$ low $\Rightarrow$ DM$_t$ trade distorted, commitment constraint binds

- Liquidation increases DM$_1$ trade $(d_1(\omega_l) \uparrow)$ in liq. scarce state

- Liquidation decreases:
  - DM$_2$ trade $(d_2(\omega_l) \downarrow)$
  - DM$_1$ trade $(d_1(\omega_l) \downarrow$ since $p_1(D(\omega)) \downarrow)$
    - Partially offset by $\uparrow$ in liquidity premium in $p_1(D(\omega))$
  - CM$_2$ coupons $(d_2(\omega_h) \downarrow)$ in liq. excess state
Proposition (Efficient Maturity Transformation)

There exists a region of $\kappa, \xi$ and threshold $z < d^*/K^H$ such that if $z(\omega_l) < z$, then efficient allocations feature both risk and maturity transformation $(d_1(\omega_l), L(\omega_l) > 0$.

Proof:

- Net benefit of liquidation proportional to

$$\kappa \times \left[ 1 + \frac{d \text{DM Utility}_1}{dd_1(\omega_l)} \right] - (1 - \xi) \times \left[ 1 + \frac{d (\text{DM Utility}_1 + \text{DM Utility}_2)}{dd_2(\omega_l)} \right] - \xi$$
Constrained Efficient Maturity Transformation

Proposition (Efficient Maturity Transformation)

There exists a region of $\kappa$, $\xi$ and threshold $z < d^*/K^H$ such that if $z(\omega_l) < z_\ast$, then efficient allocations feature both risk and maturity transformation ($d_1(\omega_l), L(\omega_l) > 0$).

Proof:

- **Net benefit of liquidation proportional to**
  
  $$\kappa \times \left[ 1 + \frac{d \text{DM Utility}_1}{dd_1(\omega_l)} \right] - (1 - \xi) \times \left[ 1 + \frac{d (\text{DM Utility}_1 + \text{DM Utility}_2)}{dd_2(\omega_l)} \right] - \xi$$

- When $\kappa$ and $\xi$ large, exogenous and endogenous costs of liquidation are low
Proposition (Efficient Maturity Transformation)

There exists a region of $\kappa$, $\xi$, and threshold $z < d^*/K^H$ such that if $z(\omega_l) < z$, then efficient allocations feature both risk and maturity transformation $(d_1(\omega_l), L(\omega_l) > 0$.

Proof:

- Net benefit of liquidation proportional to
  
  $\kappa \times \left[ 1 + \frac{d \text{DM Utility}_1}{dd_1(\omega_l)} \right] - (1 - \xi) \times \left[ 1 + \frac{d(\text{DM Utility}_1 + \text{DM Utility}_2)}{dd_2(\omega_l)} \right] - \xi$

- When $\kappa$ and $\xi$ large, exogenous and endogenous costs of liquidation are low

- Exist $\xi < K^B/(K^H + K^B)$ and $\kappa < 1$ so that benefit higher than cost
Constrained Efficient Transformation: Summary

- Risk Transformation as soon as $z(\omega_l) < d^*/K^H$

- Maturity Transformation only when $z(\omega_l)$ sufficiently low

- Maturity Transformation allows for more risk transformation
  - (not shown, but) efficient coupons smoother than best allocations with no liquidation
Necessity of Risk and Limited Commitment

- No maturity transformation in absence of risk
  - Within a given state, shortening maturity necessarily costly
  - Implies in absence of risk, if liquidation has direct costs, efficiency features no liquidation

- No maturity transformation with full commitment
  - Backloading of payments desirable
    - Implied by forward looking asset prices
  - Liquidation only desirable when limited commitment binding
EQUILIBRIUM RISK AND MATURITY TRANSFORMATION
Decentralization of the Liquidation Choice

- Recall in equilibrium, banks issue claims and use proceeds to finance real investment.

- Key equilibrium object: period 0 price of bank claims
  - Must define this price for any possible coupon plan.
Decentralization of the Liquidation Choice

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- Given function $p_0(D)$, we will construct equilibrium.
Decentralization of the Liquidation Choice

- Recall in equilibrium, banks issue claims and use proceeds to finance real investment

- Key equilibrium object: period 0 price of bank claims
  - Must define this price for any possible coupon plan

- Given function $p_0(D)$, we will construct equilibrium

- We show
  - If no maturity transformation is efficient, then equilibrium is efficient
  - If maturity transformation is efficient, then less maturity transformation in equilibrium
    - Equilibrium is (constrained) inefficient
Constructing Claim Prices

- Consider market eq'm when banks issue symmetric claims, $D^*$

- Implies period 0 claim price

\[ p_0(D^*; D^*) = \sum_{\omega} \sum_t \gamma(\omega)[1 + LP_t(\omega; D^*(\omega))]d_t^*(\omega) \]

- Define $\pi_t(\omega; D^*) = \gamma(\omega)[1 + LP_t(\omega; D^*(\omega))]$

- For alternative claim $D$, assume

\[ p_0(D; D^*) = \sum_{\omega} \sum_t \pi_t(\omega; D^*)d_t(\omega) \]
Constructing Claim Prices

- Consider market eq’m when banks issue symmetric claims, $D^*$

- Implies period 0 claim price

\[
p_0(D^*;D^*) = \sum_{\omega} \sum_{t} \gamma(\omega)\left[1 + LP_t(\omega;D^*(\omega))\right]d^*_t(\omega)
\]

- Define $\pi_t(\omega;D^*) = \gamma(\omega)\left[1 + LP_t(\omega;D^*(\omega))\right]$

- For alternative claim $D$, assume

\[
p_0(D;D^*) = \sum_{\omega} \sum_{t} \pi_t(\omega;D^*)d_t(\omega)
\]

- Interpretation:
  - Banks cannot impact aggregate liquidity, liquidity premia
Simplifying Bank’s Problem

- Bank’s problem written only in terms of coupon issuance

\[
\max_{L, D} \left[ K^b + \frac{1}{p_0^k} \sum_\omega \sum_t \pi_t(\omega; D^*) d_t(\omega) \right] \sum_\omega \gamma(\omega) z(\omega) \left[ \kappa L(\omega) + 1 - L(\omega) \right] - \sum_\omega \gamma(\omega) \sum_t d_t(\omega)
\]

subject to

\[
d_1(\omega) \leq \left[ K^b + \frac{1}{p_0^k} \sum_\omega \sum_t \pi_t(\omega; D^*) d_t(\omega) \right] \kappa L(\omega) z(\omega)
\]

\[
d_2(\omega) \leq \left[ K^b + \frac{1}{p_0^k} \sum_\omega \sum_t \pi_t(\omega; D^*) d_t(\omega) \right] (1 - \xi)(1 - L(\omega)) z(\omega)
\]

\[L(\omega), d_t(\omega) \geq 0\]
Simplifying Bank’s Problem

- Bank’s problem written only in terms of coupon issuance

\[
\begin{align*}
\max_{L, D} & \quad \left[ K^b + \frac{1}{p_0^k} \sum_{\omega} \sum_t \pi_t(\omega; D^*) d_t(\omega) \right] \\
& \quad \sum_{\omega} \gamma(\omega) z(\omega) \left[ \kappa L(\omega) + 1 - L(\omega) \right] - \sum_{\omega} \gamma(\omega) \sum_t d_t(\omega) \\
\end{align*}
\]

subject to

\[
\begin{align*}
d_1(\omega) & \leq \left[ K^b + \frac{1}{p_0^k} \sum_{\omega} \sum_t \pi_t(\omega; D^*) d_t(\omega) \right] \kappa L(\omega) z(\omega) \\
d_2(\omega) & \leq \left[ K^b + \frac{1}{p_0^k} \sum_{\omega} \sum_t \pi_t(\omega; D^*) d_t(\omega) \right] (1 - \xi)(1 - L(\omega)) z(\omega) \\
L(\omega), d_t(\omega) & \geq 0
\end{align*}
\]
Equilibrium with Liquidation

- We look for an equilibrium with:
  - No liquidity premium in high state: \( \pi_t(\omega_h; D^*) = \gamma(\omega_h) \)
  - Liquidity premium in low state: \( \pi_t(\omega_l; D^*) > \gamma(\omega_l) \)
Equilibrium with Liquidation

- We look for an equilibrium with:
  - No liquidity premium in high state: \( \pi_t(\omega_h; D^*) = \gamma(\omega_h) \)
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- Bank Optimality Conditions:
Equilibrium with Liquidation

- We look for an equilibrium with:
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- Bank Optimality Conditions:
  - Bank has no consumption in period 1
    - \( \kappa < 1 \Rightarrow \text{bank prefers period 2 consumption} \)
Equilibrium with Liquidation

- We look for an equilibrium with:
  
  ○ No liquidity premium in high state: \( \pi_t(\omega_h; D^*) = \gamma(\omega_h) \)
  
  ○ Liquidity premium in low state: \( \pi_t(\omega_l; D^*) > \gamma(\omega_l) \)

- Bank Optimality Conditions:
  
  ○ Bank has no consumption in period 1
    
    - \( \kappa < 1 \Rightarrow \) bank prefers period 2 consumption

  ○ If \( \pi_t(\omega; D) > \gamma(\omega) \), commitment constraint binds
    
    - Return from issuing \( d_2(\omega) \) larger than cost
    
    - Conjecture (and verify) commitment constraint binds in low state
Equilibrium with Liquidation

- We look for an equilibrium with:
  - No liquidity premium in high state: \( \pi_t(\omega_h; D^*) = \gamma(\omega_h) \)
  - Liquidity premium in low state: \( \pi_t(\omega_l; D^*) > \gamma(\omega_l) \)

- Bank Optimality Conditions:
  - If \( L(\omega_l) > 0 \), then
    \[
    \kappa \pi_1(\omega_l; D^*) - (1 - \xi) \pi_2(\omega_l; D^*) - \xi \gamma(\omega_l) = 0
    \]
Equilibrium with Liquidation

- We look for an equilibrium with:
  
  ◦ No liquidity premium in high state: $\pi_t(\omega_h; D^*) = \gamma(\omega_h)$
  
  ◦ Liquidity premium in low state: $\pi_t(\omega_l; D^*) > \gamma(\omega_l)$

- Bank Optimality Conditions:

  ◦ If $L(\omega_l) > 0$, then

  \[
  \kappa \pi_1(\omega_l; D^*) \quad \text{Inc in} \quad d_1(\omega_l) - (1 - \xi) \pi_2(\omega_l; D^*) \quad \text{Dec in} \quad d_2(\omega_l) - \xi \gamma(\omega_l) \quad \text{Dec in} \quad \text{Bank Exp. Returns} = 0
  \]
Decentralized Equilibrium Liquidation

**Proposition (Inefficient Liquidation)**

If constrained efficient allocation satisfies $L(\omega_l) \in (0, 1)$, then the equilibrium allocation features strictly less maturity transformation (lower $L(\omega_l)$) and is therefore, constrained inefficient.
Proposition (Inefficient Liquidation)

If constrained efficient allocation satisfies $L(\omega_l) \in (0, 1)$, then the equilibrium allocation features strictly less maturity transformation (lower $L(\omega_l)$) and is therefore, constrained inefficient.

- Banks do not internalize impact of own issuance on liquidity premia
- Banks free ride on high implied liquidity premium associated with efficient allocation
  - Issue claims with larger than efficient period 2 coupons
  - Engage in too little liquidation
- Externality associated with “wrong” price of bank liabilities
Decentralized Equilibrium Liquidation

Proposition (Inefficient Liquidation)

If constrained efficient allocation satisfies \( L(\omega_l) \in (0, 1) \), then the equilibrium allocation features strictly less maturity transformation (lower \( L(\omega_l) \)) and is therefore, constrained inefficient.

- Proof:
Decentralized Equilibrium Liquidation

Proposition (Inefficient Liquidation)
If constrained efficient allocation satisfies $L(\omega_l) \in (0, 1)$, then the equilibrium allocation features strictly less maturity transformation (lower $L(\omega_l)$) and is therefore, constrained inefficient.

- Proof:
  - Bank optimality for $L(\omega_l) > 0$ re-written
    \[ \kappa \pi_1(\omega_l; D^*) - (1 - \xi) \pi_2(\omega_l; D^*) - \xi \gamma(\omega_l) \geq 0 \]
Proposition (Inefficient Liquidation)

If constrained efficient allocation satisfies $L(\omega_l) \in (0, 1)$, then the equilibrium allocation features strictly less maturity transformation (lower $L(\omega_l)$) and is therefore, constrained inefficient.

- Proof:
  - Bank optimality for $L(\omega_l) > 0$ re-written
    \[
    \kappa \pi_1(\omega_l; D^*) - (1 - \xi) \pi_2(\omega_l; D^*) - \xi \gamma(\omega_l) \geq 0
    \]
  - Planner optimality for $L(\omega_l) > 0$
    \[
    \kappa \pi_1(\omega_l; D^*) - (1 - \xi) \pi_2(\omega_l; D^*) - \xi \gamma(\omega_l) = -\gamma(\omega_l)(1 - \eta)(1 - \kappa) - \bar{B} \frac{d \pi_2(\omega_l; D^*)}{d q_2^{eq}}
    \]
    with $\bar{B} > 0$
Decentralized Equilibrium Liquidation

Proposition (Inefficient Liquidation)

If constrained efficient allocation satisfies \( L(\omega_l) \in (0, 1) \), then the equilibrium allocation features strictly less maturity transformation (lower \( L(\omega_l) \)) and is therefore, constrained inefficient.

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    \[
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    \kappa \pi_1(\omega_l; D^*) - (1 - \xi)\pi_2(\omega_l; D^*) - \xi \gamma(\omega_l) = \gamma(\omega_l)(1 - \eta)(1 - \kappa) - \bar{B} \frac{d\pi_2(\omega_l; D^*)}{dq_2}
    \]
    with \( \bar{B} > 0 \)
  - MB of liquidation larger for planner
    - Bargaining inefficiency
    - Pecuniary externality
Decentralized Equilibrium Liquidation

- Planner chooses $L(\omega_l) > 0$ when $z(\omega_l) < z(\omega)$. For these $z(\omega_l)$'s, equilibrium $L(\omega_l)$ strictly lower.

- Planner chooses $L(\omega_l) = 0$ when $z(\omega_l) \geq z(\omega)$. For these $z(\omega_l)$'s, equilibrium $L(\omega_l)$ coincides.

- Banks wants less $L(\omega_l)$ than planner; cannot have $L(\omega_l) < 0$.

- Straightforward to show rest of equilibrium also coincides.

- For these $z(\omega_l)$'s, equilibrium is (constrained) efficient.
Decentralized Equilibrium Liquidation

- Planner chooses $L(\omega_l) > 0$ when $z(\omega_l) < z$
  - For these $z(\omega_l)$'s, equilibrium $L(\omega_l)$ strictly lower
Decentralized Equilibrium Liquidation

- Planner chooses $L(\omega_l) > 0$ when $z(\omega_l) < z$  
  - For these $z(\omega_l)$'s, equilibrium $L(\omega_l)$ strictly lower
- Planner chooses $L(\omega_l) = 0$ when $z(\omega_l) \geq z$  
  - For these $z(\omega_l)$'s, equilibrium $L(\omega_l)$ coincides
    - Banks wants less $L(\omega_l)$ then planner; cannot have $L(\omega_l) < 0$
    - Straightforward to show rest of equilibrium also coincides
    - For these $z(\omega_l)$'s, equilibrium is (constrained) efficient
Optimal Policy

• Previous proposition shows
  ◦ Banks undertake less maturity transformation than efficient
  ◦ Resulting claim issues riskier than efficient

• Role for Policy:
  ◦ Requirement that bankers implement minimal $L$ implements constrained efficiency
  ◦ Interpret as liability liquidity floor: bankers issue sufficient short-term claims
  ◦ Inefficiency in contrast with fire-sale externalities
    - Alternatively, short-term asset liquidity floor on claim-holders
  ◦ Policy ensures banks implement efficient level of insurance against aggregate liquidity risk
  ◦ Inefficiency in contrast with fire-sale externalities
    - Wrong pricing of bank assets in event of fire sale
Conclusions

- Developed theory of bank balance sheet transformation arising from liquidity provision and aggregate risk

- Find if assets are risky and yield insufficient liquidity in some states, efficient for banks to transform risk

- If assets sufficiently risky to cause limited commitment constraints to bind, efficient for banks to transform maturity

- When equilibrium features maturity transformation, banks under-provide maturity and risk transformation

- Policy needed to implement efficient transformation
Appendix
Constrained Efficient Maturity Transformation

Formally

- Marginal impact of perturbation proportional to

\[ \gamma(\omega_l) \left\{ U_{1,1l}^{P} \kappa - \left( U_{1,2l}^{P} + U_{2,2l}^{P} \right) (1 - \xi) \right\} - \gamma(\omega_h) \xi \frac{\gamma(\omega_l)}{\gamma(\omega_h)} \left\{ U_{1,2h}^{P} + U_{2,2h}^{P} \right\} \]

\[ = \gamma(\omega_l) \left[ U_{1,1l}^{P} \kappa - \left( U_{1,2l}^{P} + U_{2,2l}^{P} \right) (1 - \xi) - \xi \right] \]

(equality follows from excess liquidity in high state \( U_{1,2h}^{P} + U_{2,2h}^{P} = 1 \))

- As \( z(\omega_l) \to 0 \), term in brackets tends to

\[ \kappa \left[ 1 + \frac{\alpha(n)}{1-\eta} \right] - (1 - \xi) \left[ 1 + \frac{\alpha(n)}{1-\eta} + \frac{\alpha(n)}{1-\eta} \left( 1 + \frac{\alpha(n)\eta}{1-\eta} \right) \right] - \xi, \]