Screening and Adverse Selection in Frictional Markets

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May 2015

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Introduction

Many markets feature adverse selection and imperfect competition

• Examples: insurance, loans, financial securities

In these markets, contracts used to screen different types

• Examples: differential coverage, loan amounts, trade sizes

A unified theoretical framework is lacking

• Large empirical literature (and some theory)

• But typically restricts contracts and/or assumes perfect competition

But many important questions

• Recent push to make these markets more competitive, transparent

• Is this a good idea?
This Paper

A tractable model of adverse selection, screening and imperfect comp.

1. Complete characterization of the unique equilibrium

2. Explore positive predictions for distribution of contracts

3. Policy experiments: changes in competition, transparency
Sketch of Model: Key Ingredients

- **Adverse Selection**: sellers have private info about quality
  
  - A fraction $\mu_h$ have quality $h$, the rest quality $\ell$

- **Screening**: Buyers offer general menus of non-linear contracts
  
  - Price-quantity pairs: induce sellers to self-select

- **Imperfect Comp**: sellers receive either 1 or 2 offers (à la Burdett-Judd)
  
  - Buyer competing with another with prob $\pi$, otherwise monopsonist.
  
  - Contract offered before buyers know
What We Know (Equilibrium)

- Pool High and Low Quality Sellers (Stiglitz ’77)
- No Trade With High Quality Seller (Stiglitz ’77)
- Only Mixed Strategy Equilibria (Rosenthal-Weiss ’84)
- Least Cost Separating Outcome (Rothschild-Stiglitz ‘76)

(Figure: Graph with axes labeled \(\mu_h\), \(\overline{\mu_h}\), \(\pi\), and \(\overline{\pi}\). The graph illustrates various economic scenarios related to quality and price in a market setting.)
Objective

Obj: Characterize eqm for any degree of adverse selection and imperfect comp.

Financial and Insurance markets typically characterized by imperfect comp.

What are the implications of imperfect comp. for....

• Terms of trade
• Welfare
• Policy
Summary of Findings

Methodology

• New techniques to characterize unique eqm for all $(\mu_h, \pi) \in [0, 1]^2$
• Establish important (and general!) property of all equilibria:
  • *Strictly rank preserving*: offers for $\ell$ and $h$ ranked exactly the same
  • No specialization

Positive Implications

• Equilibrium can be pooling, separating, or mix
• Separation when adverse selection severe, trading frictions mild
• Pooling when adverse selection mild, trading frictions severe

Normative Implications

• Adverse selection severe: *interior* $\pi$ maximizes surplus from trade
• Adverse selection mild: welfare unambiguously decreasing in $\pi$
• Increasing transparency/relaxing info frictions can $\uparrow$ or $\downarrow$ welfare
Related Literature

Empirical

- Chiappori and Salanie (2000); Ivashina (2009); Einav et al. (2010); Einav et al. (2012)

Adverse Selection and Screening

- Rothschild and Stiglitz (1976); Dasgupta and Maskin (1986); Rosenthal and Weiss (1984); Mirrlees (1971); Stiglitz (1977); Maskin and Riley (1984); Guerrieri, Shimer and Wright (2010); Many, many others

Imperfect Competition and Selection

- Search Frictions: Burdett and Judd (1983); Garrett, Gomes, and Maestri (2014)

Environment
Model Environment

Large number of buyers and sellers

- Each Seller endowed with 1 divisible asset
  - Seller values asset at rate $c_i$
  - Two types of sellers $i \in \{l, h\}$ with prob. $\mu_i$

- Buyer values type $i$ asset at rate $v_i$

- If $x$ units sold for transfer $t$, payoffs are
  - Seller: $t + (1 - x)c_i$
  - Buyer: $xv_i - t$

- Assumptions:
  - Gains to trade: $v_i > c_i$
  - Lemons Assumption: $v_l < c_h$
  - Adverse Selection: Only sellers know asset quality
Model Environment

Screening

- Buyers post arbitrary menus of exclusive contracts
- Screening menus intended to induce self-selection

Search frictions

- Each seller receives 1 offer w.p. $1 - \pi$ and both w.p. $\pi$
  - Refer to seller with 1 offer as Captive
  - Refer to seller with 2 offers as non-Captive

Stylized Model of Trade

- best examples: corporate loans market; securitization (maybe)
- other examples: information-based trading; insurance
Strategies

- Each buyer offers arbitrary menu of contracts \( \{(x_n, t_n) \in \mathcal{N}\} \)
- Captive seller’s choice: best \((x_n, t_n)\) from one buyer
- Non-captive seller’s choice: best \((x_n, t_n)\) among both buyers

Revelation Principle

sufficient to consider

- menus with two contracts \( z \equiv \{(x_l, t_l), (x_h, t_h)\} \)

\[(IC_j): t_j + c_j(1 - x_j) \geq t_{-j} + c_j(1 - x_{-j}) \quad j \in \{h, l\}\]

- seller \( j \): chooses contract \( j \) from available the set of menus available
Equilibrium Price Dispersion

• Suppose \( \pi \in (0, 1) \): no symmetric pure strategy equilibrium exists
  • buyers can guarantee positive profits: trade only with captive types
  • in a pure strategy equilibrium: have to share non-captive types
    There is always an incentive to undercut

• Only mixed strategy equilibria possible
  \( \Rightarrow \) equilibrium features price dispersion
  \( \Rightarrow \) equilibrium described by buyers’ distribution over menus
Equilibrium definition

A symmetric equilibrium is a distribution $\Phi(z)$ such that almost all $z$ satisfy,

1. **Incentive compatibility:**

   $$t_j + c_j(1 - x_j) \geq t_{-j} + c_j(1 - x_{-j}) \quad j \in \{h, l\}$$

2. **Seller optimality:**

   $$\chi_i(z, z')$$ maximizes her utility

3. **Buyer optimality:** for each $z \in \text{Supp}(\Phi)$

   $$z \in \arg\max_z \sum_{i \in \{l, h\}} \mu_i(v_i x_i - t_i) \left[ 1 - \pi + \int_{z'} \chi_i(z, z') \Phi(dz') \right]$$ (1)
Characterization

Equilibrium described by non-degenerate distribution in 4 dimensions

Proceed in 4 steps

1. Show that menus can be summarized by a pair of utilities \((u_h, u_l)\)
   - Reduces dimensionality of problem to distribution in 2 dimensions

2. Show there is a 1-1 mapping between \(u_l\) and \(u_h\)
   - Reduces problem to distribution in 1 dimension + a monotonic function

3. Construct Equilibrium

4. Show that constructed equilibrium is unique
A utility representation

Result (Dasgupta and Maskin (1986))

In all menus offered in equilibrium,

- the low types trades everything: \( x_l = 1 \)
- \( IC_l \) binds: \( t_l = t_h + c_l(1 - x_h) \)

Result

Equilibrium menus can be represented by \((u_h, u_l)\) with corresponding allocations

\[
\begin{align*}
  t_l &= u_l \\
  x_h &= 1 - \frac{u_h - u_l}{c_h - c_l} \\
  t_h &= \frac{u_l c_h - u_h c_l}{c_h - c_l}
\end{align*}
\]

Since we must have \(0 \leq x_h \leq 1\),

\[
c_h - c_l \geq u_h - u_l \geq 0
\]
A utility representation

Marginal distributions

\[ F_j(u_j) = \int_{z'} 1 \left[ t'_j + c_j (1 - x'_j) \leq u_j \right] d\Phi(z') \quad j \in \{h, l\} \]

Then, each buyer solves

\[
\Pi(u_h, u_l) = \max_{u_l \geq c_l, \ u_h \geq c_h} \sum_{j \in \{l, h\}} \mu_j \left[ 1 - \pi + \pi F_j(u_j) \right] \Pi_j(u_h, u_l)
\]

s. t. \( c_h - c_l \geq u_h - u_l \geq 0 \)

with \( \Pi_l(u_h, u_l) \equiv v_l x_l - t_l = v_l - u_l \)

\( \Pi_h(u_h, u_l) \equiv v_h x_h - t_h = v_h - u_h \frac{v_h - c_l}{c_h - c_l} + u_l \frac{v_h - c_h}{c_h - c_l} \)

Need to characterize the two linked distributions \( F_l \) and \( F_h \)!
Further Simplifying the Characterization

Result

\( F_l \) and \( F_h \) have connected support and are continuous.

- Except for a knife-edge case (see paper)
- Proof more involved than standard case because of interdependencies

Result

The profit function \( \Pi(u_h, u_l) \) is strictly supermodular.

- Intuition: \( u_l \uparrow \Rightarrow \Pi_h \uparrow \Rightarrow \) stronger incentives to attract high types
- \( \Rightarrow U_h(u_l) \equiv \arg\max_{u_h} \Pi(u_h, u_l) \) is weakly increasing
Strict Rank Preserving

**Theorem**

\( U_h(u_l) \) is a **strictly increasing function**.

**Idea of Proof**

- \( U_h(u_l) \) increasing due to super-modularity of profit function
- \( F_l \) and \( F_h \) have no holes or mass points imply \( U_h \) is strictly increasing and not a correspondence
Strict Rank Preserving

**Theorem**

\[ U_h(u_l) \text{ is a strictly increasing function.} \]

Implications for Characterization

- Rank ordering of equilibrium menus identical across types
- Menus attract same fraction of both types \( F_i(u_l) = F_h(U_h(u_l)) \)
- Greatly simplifies the analysis: only have to find \( F_i(u_l) \) and \( U_h(u_l) \)

Broader Implications

- Buyers do not specialize or attract only a subset of types
- Terms of trade offered to both types are positive correlated

Robust to any number of types

- Relies only on utility representation and ability to show distributions are well behaved
Constructing Equilibria
Equilibria: The two limit cases

Monopsony: $\pi = 0$

- $\mu_h < \bar{\mu}_h \Rightarrow$ Sep. with $x_h = 0$ and $\Pi_l > \Pi_h = 0$
  - No Cross-subsidization
- $\mu_h \geq \bar{\mu}_h \Rightarrow$ Pooling with $x_h = x_l = 1$ and $\Pi_h > 0 > \Pi_l$
  - Cross-subsidization

Bertrand: $\pi = 1$

- $\mu_h < \bar{\mu}_h \Rightarrow$ Sep. with $x_h < 1$, $\Pi_h = \Pi_l = 0$
  - No Cross-subsidization
- $\mu_h < \bar{\mu}_h \Rightarrow$ Sep. with $x_h < 1$, $\Pi = 0$, but $\Pi_h > 0 > \Pi_l$
  - Cross-subsidization

Intuition: Higher $\mu_h \Rightarrow$ Relaxing $IC^l$ more attractive
Types of equilibria in the middle

High $\mu_h$
- $\Pi_h > 0 > \Pi_l$
- All separating, all pooling or a mix

Low $\mu_h$
- $\Pi_l, \Pi_h \geq 0$
- All separating, $U_h(u_l) \neq u_l$
No cross-subsidization: Characterization

Focus on separating equilibrium in no-cross subsidization region

Recall problem of a buyer:

\[
\Pi(u_h, u_l) = \max_{u_l \geq c_l, \ u_h \geq c_h} \sum_{j \in \{l, h\}} \mu_j \left[1 - \pi + \pi F_j(u_j)\right] \Pi_j(u_h, u_l)
\]

\[
\text{s. t.} \quad c_h - c_l \geq u_h - u_l \geq 0
\]

- In separating equilibrium we construct, \( c_h - c_l > u_h > u_l \)
- Sufficient to ensure local deviations unprofitable
No cross-subsidization: Characterization

Marginal benefits vs costs of increasing $u_i$

\[
\begin{align*}
\frac{\pi f_i(u_i) \Pi}{1 - \pi + \pi F_i(u_i)} + \frac{\mu_h}{1 - \mu_h} \frac{\nu_h - c_h}{c_h - c_l} &= \frac{1}{MC} \\
\text{MB of more low types} &+ \text{MB of relaxing } IC_i
\end{align*}
\]

Boundary conditions

\[
F_i(c_l) = 0 \quad F_i(\bar{u}_l) = 1 \quad \rightarrow \quad F_i(u_l)
\]

Equal profit condition

\[
[1 - \pi + \pi F_i(u_l)] \Pi(U_h, u_l) = \bar{\Pi} \quad \rightarrow \quad U_h(u_l)
\]

Pursue similar construction in other regions of parameter space
Equilibrium Regions in the Middle

More Competition implies less pooling
- Gains to cream-skimming increase in $\pi$

Milder Adverse Selection (higher $\mu_h$) implies more pooling
- increased incentives to trade high volume
- increased cost of cream-skimming

Theorem

For every $(\pi, \mu_h)$ there is a unique equilibrium.
EQUILIBRIUM IMPLICATIONS
Positive and Normative Implications

Is improving competition desirable for volume or welfare?

- For high $\mu_h$, monopsony dominates perfect competition
- For low $\mu_h$, perfect competition dominates monopsony
- Will show: for low $\mu_h$, welfare maximized at interior $\pi$

Is increasing transparency desirable?

- Allowing insurers, loan officers, dealers to discriminate on observables?
- Interpret increased transparency as increased spread in $\mu_h$
- Desirability depends on curvature of welfare function with respect to $\mu_h$
- Will show: Concavity/Convexity of welfare function depends on $\pi, \mu_h$
EQUILIBRIUM IMPLICATIONS: COMPETITION
Competition with No Cross-Subsidization

Assume $\mu_h$ in no cross-subsidization region

Equilibrium Distribution and $U_h(u_l)$ for $\pi = 0.2$

Shaded Region indicates support of $F_l$
Competition with No Cross-Subsidization

Assume \( \mu_h \) in no cross-subsidization region

Equilibrium Distribution and \( U_h(u_l) \) for \( \pi = 0.5 \)

Shaded Region indicates support of \( F_l \)

- Increase in \( \pi \) increases \( F_l \) in sense of FOSD
Assume $\mu_h$ in no cross-subsidization region

Equilibrium Distribution and $U_h(u_l)$ for $\pi = 0.9$

Shaded Region indicates support of $F_l$

- Increase in $\pi$ increases $F_l$ in sense of FOSD
- Driven by increased competition for (abundant) low-quality sellers
Competition with No Cross-Subsidization

How is trade volume related to $U_h$?

$$x_h(u_l) = 1 - \frac{U_h(u_l) - u_l}{c_h - c_l}$$

$$x'_h(u_l) > 0 \Leftrightarrow U'_h(u_l) > 1$$
Competition with No Cross-Subsidization

Equilibrium Objects for $\pi = 0.2$
Competition with No Cross-Subsidization

Equilibrium Objects for \( \pi = 0.5 \)

- From low \( \pi \), increase in \( \pi \) increases volume
Competition with No Cross-Subsidization

Equilibrium Objects for $\pi = 0.9$

- From moderate $\pi$, increase in $\pi$ decreases volume
Competition and Welfare

When no cross-subsidization

\[
W(\mu_h, \pi) = (1 - \mu_h)(v_l - c_l) + \mu_h(v_h - c_h) \int x_h(u_l) dF(u_l)
\]

Why is welfare decreasing?

- \(\mu_h \) low implies few high types
- Competition less fierce for high types
- Demand from high types relatively inelastic
- Equal profits \( \Rightarrow \) greater dispersion in prices
- Implies \( U'_h(u_l) > 1 \)

Welfare maximized for interior \( \pi \)

With Cross-Subsidization, welfare (weakly) maximized in monopsony outcome

- Full trade \( \Rightarrow \) all gains to trade exhausted
EQUILIBRIUM IMPLICATIONS:
TRANSPARENCY
Desirability of Transparency

Do the following policies improve welfare?

- Allowing insurance providers to discriminate based on observables
- Introducing credit scores in loan markets
- Requiring OTC market participants to disclose trades

In model, interpret increased transparency as mean-preserving spread of $\mu_h$

- Each seller has individual $\mu'_h$; Buyers know distribution over $\mu'_h$
- Buyers restricted to offering contracts associated with $E[\mu'_h]$
- Under transparency, buyers allowed to offer $\mu_h$-specific menus
- Need to compare $E[W(\mu'_h, \pi)]$ to $W(E[\mu'_h], \pi)$

Is Transparency Desirable? Answer: Depends on $\pi$!

- $W$ is linear when $\pi = 0$ and $\pi = 1$ $\Rightarrow$ no effect on welfare
- $W$ is concave when $\pi$ is high $\Rightarrow$ bad for welfare
Desirability of Transparency: The two limit cases

Monopsony: $\pi = 0$

- $\mu_h < \bar{\mu}_h \Rightarrow x_h = 0$ so that

$$W(\mu_h) = (1 - \mu_h)v_l + \mu_h c_h$$

- $\mu_h > \bar{\mu}_h \Rightarrow x_h = 1$ so that

$$W(\mu_h) = (1 - \mu_h)v_l + \mu_h v_h$$

- Welfare is linear in $\mu_h$

Bertrand: $\pi = 1$

- $\mu_h < \bar{\mu}_h \Rightarrow x_h$ independent of $\mu_h$

- Implies welfare is linear in $\mu_h$

In these cases, welfare is linear in $\mu_h$ so that mean-preserving spread (locally) has no impact on welfare
Desirability of Transparency: The general cases

With cross-subsidization, welfare is concave
\[ \mu_h > \bar{\mu}_h \]
Rightarrow increases in transparency harm welfare

Without cross-subsidization, welfare is concave only for high \( \pi \)
\[ \mu_h < \bar{\mu}_h \]
Rightarrow increases in transparency harm welfare when markets competitive

- With cross-subsidization, welfare is concave
  \Rightarrow increases in transparency harm welfare
- Without cross-subsidization, welfare is concave only for high \( \pi \)
  \Rightarrow increases in transparency harm welfare when markets competitive
Conclusion

Methodological contribution

• Imperfect competition and adverse selection with optimal contracts
• Rich predictions for the distribution of observed trades

Substantive insights

• Depending on parameters, pooling and/or separating menus in equilibrium
• Competition, transparency can be bad for welfare

Work in progress

• Generalize to $N$ types, curved utility
• Non-exclusive trading
No cross-subsidization: Price vs quantity (conditional)

Correlation $< 0$ for suff. high $\pi$

A strategy to infer competitiveness?