1. Consider an economy with the following production function:

\[ Y_t = K_t^{1/3} N^{2/3}, \]

where the subscript refers to values in year \( t \). Assume that (i) in year \( t=0 \), government expenditure is \( G_0=100 \); (ii) the labor supply curve is vertical, and \( N \) is fixed always at 1000; (iii) consumption satisfies the equation \( C_t = 0.8 Y_t \) (note that consumption is assumed not to depend on the interest rate).

In class, we have always assumed that the capital stock is fixed. However, we have also seen that increased government expenditure crowds out investment and this may have adverse consequences for the future. This question asks you to explore these consequences.

Capital evolves over time according to the following equation:

\[ K_{t+1} = K_t + I_t - \delta K_t, \]

where \( \delta = 0.1 \) is the capital depreciation rate. This equation says that the capital stock is increased over the previous period’s stock by the amount of investment, but declines each year as a result of the 10% depreciation rate.

a) Prove that if all these initial values remain constant, the capital stock settles down to a constant value of 1,000. (This involves two steps: the first is showing that when the stock of capital is equal to 1000, it will not change from one year to the next. The second is showing that when capital stock is not equal to 1,000, it will move towards that value).

b) Assume now that at time \( t=0 \), the capital stock is given by \( K_0=1000 \) (that is, one can imagine that the values given above have been constant for some time). In year \( t=1 \) the government raises its expenditure in real terms from 100 to 102, and keeps it there for ten years. Then, in year \( t=11 \), the government drops its expenditure back down to 100. Use a spreadsheet to plot the time paths of consumption, output, and investment for each year from \( t=0 \) to 50. What do you conclude about the persistence of the effects of changes in fiscal policy?
2. Consider a production function of the form:

\[ Y = K^\alpha L^{1-\alpha}. \]

If both capital and labor are paid their marginal product, what fraction of real national income is paid as wages to labor, and what fraction is paid to the owners of capital. How much profit is made by the representative firm? Using data from chapter 2 of Froyen, what do you conclude is a reasonable value of \( \alpha \) for the United States?

Assume now that a proportional income tax at the rate \( t \) is paid by workers, so workers receive a net wage of \((1-t)w\). What happens now to the shares of capital and labor in real income? How much does the government get?

3. The classical explanation of the Great Depression is that events occurring in the labor market affecting both the supply of, and demand for, labor, induced a major reduction in hours worked. I have in class expressed skepticism that such a large shift could have occurred. Nevertheless, there were event and policies enacted during this period that adversely affected the labor market. Undertake an internet search to identify some of these, and explain their effect in the labor market. You will find much discussion during your search about matters related to monetary policy; remember that these do not constitute part of the classical explanation of the Great Depression (\textit{Do not try to be exhaustive: a couple of items, and discussion of them is enough; we will use in-class discussion to collate other people’s ideas}).