1. Read through section 1.2 of van Dalen.

2. Write down explicit definitions of the functions $f$ and $g$, where
   a. $f$ is defined recursively by $f(0) = 0$, $f(n + 1) = 3 + f(n)$, and
   b. $g$ is defined recursively by $g(0) = 1$, $g(n + 1) = (n + 1)^2 g(n)$. (Hint: use “factorial” notation: $m! = 1 \times 2 \times \ldots \times m$.)

3. Write down an explicit definition of the function $h$, where $h(0) = 0$ and $h(n + 1) = 3 \cdot h(n) + 1$. (Hint: compare to the sequence 1, 3, 9, 27, 81, …)

4. Suppose $g$ is a function from $\mathbb{N}$ to $\mathbb{N}$. Write down a recursive definition of the function $f(n)$, defined by $f(n) = \sum_{i=0}^{n} g(i)$.

5. Do problem 1 on page 30 of the Enderton handout.


7. Suppose, as Section 2.2 of the notes, we are given a set $U$, a subset $B \subseteq U$, and some functions $f_1, \ldots, f_k$. Say a set is inductive if it contains $B$ and is closed under the $f$’s, and let $C^*$ be the intersection of all the inductive subsets of $U$. Show $C^*$ is inductive.

8. Define the set of “babble-strings” inductively, as follows:
   - “ba” is a babble-string
   - if $s$ is a babble-string, so is “ab”$^*$s
   - if $s$ and $t$ are babble-strings, so is $s^*t$

Prove by induction that every babble-string has the same number of $a$’s and $b$’s, and that every babble-string ends with an “a”. Is the set of babble-strings freely generated?