Problem Set #6

This problem set is due at the beginning of your CLASS (not recitation) on Monday, April 30. There are three questions on the homework worth a total of 100 points. The points assigned to each part of each question are indicated in brackets.

Exercise #1. Consider a monopolist whose cost function is:

$$c(y) = 2y,$$

where $y$ is the amount of output produced by the monopolist. The monopolist faces a market demand curve described by equation

$$Q_D(p) = 11 - \frac{p}{2},$$

where $Q_D$ is the quantity demanded when the price of a unit of output is equal to $p$. The monopolist’s goal is to maximize its profits.

(a) [10 pts.] Calculate the monopolist’s optimal level of output. What price does the monopolist charge for its output?

(b) [10 pts.] Calculate consumers’ surplus given the monopolist’s optimal choices for price and output. In addition, compute producer’s surplus (i.e. the monopolist’s profits). What is the total amount of surplus generated by this market?

(c) [10 pts.] Suppose now that the government enacts regulatory legislation requiring the monopolist to behave competitively. That is, the government requires the monopolist to set the price of its output equal to marginal cost. How much output does the monopolist produce? What price does the monopolist charge? Calculate producer’s surplus and consumers’ surplus given the competitive values of price and output. Verify that total surplus is higher than in part (c). Calculate the deadweight loss of monopoly in this market.

Exercise #2. The US pharmaceutical company Pharmacia holds a patent on drug P in the US and an analogous patent in Canada. Its marketing department has identified the following inverse demand curves for drug P by US and Canadian consumers:

$$p_{US} = 1,000 - y_{US}$$
$$p_{CA} = 500 - y_{CA},$$

where $y_{US}$ and $y_{CA}$ denote the quantity demanded by US and Canadian consumers, while $p_{US}$ and $p_{CA}$ denote the price (expressed in US $) at which the drug is sold in the US and Canada respectively.
The cost of producing $y$ units of drug P is given by the cost function

$$c(y) = 100y.$$ 

(a) [10 pts.] Suppose that Pharmacia can sell drug P at different prices in the US and Canada (third degree price discrimination). What price will it charge in the US and which price will it charge in Canada in order to maximize its profits?

(b) [10 pts.] Compute the profits that Pharmacia makes in the US market and in the Canadian market. Also compute the surplus of US consumers and the surplus of Canadian consumers. In a diagram (with $y$ on the x-axis and $p$ on the y-axis) illustrate the US inverse demand curve, the firm’s cost function, the US equilibrium price, the profits that Pharmacia makes in the US, and the surplus of US consumers. Draw a second diagram for Canada.

(c) [15 pts.] Suppose now that US consumers can buy the same drug P in Canada via internet at the Canadian price, without Pharmacia being able to stop these trades (assume also that shipping costs are zero). What price will Pharmacia set for its drug in Canada? [Hint: in this case the monopolist faces only one market with demand given by the sum of the US and Canadian demands. Therefore, the monopolist has to decide only one price at which to sell its drug. Be careful, when summing up the two demand curves, to remember that for $p_{US} > 1,000$, $y_{US} = 0$ and for $p_{CA} > 500$, $y_{CA} = 0$. See the class on aggregation of demand curves.]

(d) [5 pts.] Compute the profits that Pharmacia makes and the surplus that US and Canadian consumers enjoy in (c). What has happened? Why?

Exercise #3. Steel firm A is situated along the banks of the Allegheny river. Further downstream is fishery B. The cost function of A is given by

$$c_A(s, x) = 5s^2 + (1 - x)^2,$$

where $s$ denotes the quantity of steel produced by A in a year and $x$ denotes the quantity of pollutants that A dumps into the Allegheny in a given year. Thus, A can decrease its production costs by dumping more pollutants into the river. Pollutants increase the cost of production for the fishery B, whose cost function is

$$c_B(f, x) = f^2 + 2x,$$

where $f$ denotes the quantity of fish caught in a given year. Notice that the cost of fishing $f$ fish increases with $x$. Suppose that the unit price of fish is $1$ and that the unit price of steel is $10$.

(a) [10 pts.] Compute the profit maximizing quantity of steel $s$ and pollutant $x$ produced by A (assuming A behaves competitively in the output market, i.e., taking the price of steel as given). Also, compute A’s profits.

(b) [10 pts.] Compute the profit maximizing quantity of fish $f$ caught by B (assuming B behaves competitively in the output market, i.e., taking the price of fish as given). Notice that B does not choose $x$. Also, compute B’s profits.

(c) [10 pts.] Suppose now that the two firms A and B merge, creating A&B. The management of A&B now maximizes A&B’s profits by appropriately choosing $x, s,$ and $f$. Find the quantities of steel, fish, and pollutants that the new firm produces. Also, find the profits of A&B.