Exercise #1. The supply of private apartments for rent around CMU is given by the following function
\[ Q_S = 600, \]
where \( Q_S \) denotes the quantity of apartments supplied in a given year. The yearly demand for apartments is given by
\[ Q_D = 1,000 - p, \]
where \( Q_D \) denotes the quantity of apartments demanded in a given year and \( p \) denotes the monthly rent for one apartment.

(a) \([5 \text{ pts.}]\) Find the equilibrium price and quantity in this market.

(b) \([10 \text{ pts.}]\) The university enacts a policy that subsidizes rent for students that decide to rent a private apartment around CMU. Specifically, the university will subsidize rent by paying $100 to a student every time he/she shows a receipt of monthly rent payment signed by a landlord. Compute the equilibrium prices and quantity in this case. \([\text{Hint: there will be two prices. One paid by the student out of his/her own pocket, } p_D, \text{ and the other received by the owner of the apartment, } p_S, \text{ with } p_D = p_S - 100.\\]"

(c) \([10 \text{ pts.}]\) On a diagram that has quantity of apartments on the x-axis and the monthly rent on the y-axis, draw the inverse demand and supply curves and mark the equilibrium prices of points (a) and (b).

(d) \([15 \text{ pts.}]\) Compare the equilibrium in (a) with the equilibrium in (b): i) what is the change in consumer’s surplus induced by the policy of the university? ii) What is the change in supplier’s surplus? iii) How much does the university spend to subsidize rents? iv) What is the deadweight loss induced by this policy?

Exercise #2. Consider a firm which produces a single output using two inputs according to the following production function:
\[ y = \min \{2K,L\}, \]
where \( y \) is the firm’s output, \( K \) is machinery (measured in machine-hours) and \( L \) is labor supply (measured in person-hours). Let \( r \) be the cost of one machine-hour and let \( w \) be the wage rate (i.e. the cost of one person-hour).
(a) [10 pts.] Does the firm’s technology exhibit increasing returns to scale? Explain carefully.

(b) [5 pts.] In a neat and clear diagram, draw the isoquant corresponding to output of 10 units. Be sure to label important features in the diagram.

(c) [10 pts.] Suppose that \( r = 16 \) and \( w = 4 \). Determine the firm’s cost function \( c(y) \).

(d) [10 pts.] On a diagram that has costs on the y-axis and output and the x-axis, plot the firm’s cost curve \( c(y) \), average cost curve \( AC(y) \), and marginal cost curve \( MC(y) \).

Exercise #3. Short questions. To get full credit you should justify your answers.

(a) [5 pts.] Show that, in the short run, if the price of the fixed factor is increased, profits will decrease.

(b) [10 pts.] If \( pMP_1 > w_1 \), (where \( p \) is the output price, \( MP_1 \) the marginal product of factor 1, and \( w_1 \) the rental rate of factor 1), should the firm increase or decrease the amount of factor 1 in order to increase profits? Explain carefully.

(c) [10 pts.] If a firm had everywhere increasing returns to scale, what would happen to its profits if prices remained fixed and if it doubled its scale of operations? Show your work.