Exercise #1. Anna consumes two goods: milk (measured in gallons) and a composite good (measured in dollars). Let $x_m$ represent the gallons of milk that Anna consumes in a given month and let $x_c$ represent her expenditures on the composite good in a given month. Anna’s preferences over consumption bundles $(x_m, x_c)$ are summarized by the utility function:

$$u(x_m, x_c) = x_m^{0.1} x_c^{0.9}.$$ 

Anna’s monthly income is $400. Let $p_m$ denote the dollar price of a gallon of milk.

(a) [10 pts.] Suppose that $p_m = $1. What is Anna’s optimal consumption bundle? Show your work. Illustrate your answer with a neat and clear diagram showing Anna’s budget line and indifference curves. Label the points at which the budget line intersects the axes and identify the optimal bundle.

(b) [10 pts.] Suppose now that the local grocery store where Anna regularly shops decides to introduce a discount on milk. Specifically, for each gallon of milk that Anna buys, the grocery store reduces its price from $1 per gallon to $0.50 per gallon, up to a maximum number of 50 gallons of milk per month. If Anna buys more than 50 gallons she has to pay the regular price on every gallon beyond the 50-th. In a neat and clear diagram, graph Anna’s budget line. Label the points at which the budget line intersects the axes and determine the coordinates of the kink point.

(c) [15 pts.] Suppose now that the price of milk is again $p_m = $1 (there are no discounts anymore). Due to a shortage of milk, the price of milk increases from $1 to $2. Describe how to compute the extra income that must be given to Anna in order to compensate her for this increase in the price of milk (i.e., the compensating variation) [Here you are not asked to compute this amount. Simply show which steps you would take to compute it.]

Exercise #2. John has the following demand function for beer

$$x_b = m - 2p_b + p_w$$

where $x_b$ denotes the gallons of beer he demands per month, $p_b$ is the dollar price of a gallon of beer, $p_w$ is the dollar price of a bottle of wine, and $m$ denotes John’s income.

(a) [5 pts.] Is beer an ordinary good in this case? Motivate your answer. [Notice: no credit will be given to yes/no type of answers. In order to get credit you need to explain your answer.]
(b) [5 pts.] Is beer a substitute for wine in this case? Motivate your answer. [Notice: no credit will be given to yes/no type of answers. In order to get credit you need to explain your answer.]

(c) [5 pts.] Suppose that the price of a bottle of wine is \( p_w = $10 \), and the price of a gallon of beer is \( p_b = $15 \). What is the relative price of a gallon of beer in terms of bottles of wine?

(d) [10 pts.] Suppose that \( m = $100 \) and that \( p_w = $10 \). Compute the loss in John’s consumer surplus that occurs when the price of a gallon of beer increases from $15 to $20. Support your analysis with a graph representing John’s demand curve and his loss in consumer’s surplus. [Remember that to draw a demand curve you need to place \( p_b \) on the y-axis and \( x_b \) on the x-axis.]

(e) [10 pts.] From point (d) you can see that the loss in consumer’s surplus can be decomposed into two subregions, whose shapes are respectively rectangular and triangular. How can you interpret each of these two subregions?

Exercise #3. Consider the following statements and say whether they are true or false and why. To get credit you should provide a clear justification for your answers.

(a) [10 pts.] If two goods are perfect complements the consumer will be just as well off facing a quantity tax as an income tax.

(b) [5 pts.] If the price of one good increases the demand for that good always decreases.

(c) [5 pts.] The marginal rate of substitution measures the rate at which the market is willing to substitute one good for the other.

(e) [5 pts.] An indifference curve represents the collection of all bundles that a consumer can buy.

(f) [5 pts.] By definition, a lump sum subsidy to a consumer does not affect his/her consumption behavior.