Choice Continued
Interpretation

- At Z:
  \[ \frac{p_1}{p_2} < -MRS(x_1, x_2) \]

- At Y:
  \[ \frac{p_1}{p_2} > -MRS(x_1, x_2) \]
Consumer Demand

- Consumer’s **demanded bundle**: optimal choice of goods 1 and 2 for given prices and income.
- Consumer’s **demand functions**:
  
  \[ x_1 = x_1(p_1, p_2, m) \]
  
  \[ x_2 = x_2(p_1, p_2, m) \]
Cobb-Douglas

- Demand function for good 1:
  \[ x_1 = c \frac{m}{p_1} \]

- Demand function for good 2:
  \[ x_2 = (1 - c) \frac{m}{p_2} \]
Perfect Substitutes

Demand function for good 1:

\[ x_1 = \frac{m}{p_1} \quad \text{if} \quad p_1 < p_2 \]

\[ x_1 = 0 \quad \text{if} \quad p_1 > p_2 \]

\[ x_1 = (0, \frac{m}{p_1}) \quad \text{if} \quad p_1 = p_2 \]
Perfect Complements

- Optimal choice: \( x_2 = x_1 \)

- Budget line:
  \[
  p_1 x_1 + p_2 x_2 = m
  \]

- Demand function for goods 1 and 2:
  \[
  x_1 = x_2 = \frac{m}{p_1 + p_2}
  \]
Example: Choosing a Tax

Q: Suppose that the government wants to raise a certain amount of revenue. Is it better to raise it via a **quantity** tax or an **income** tax?
Before the Tax

- **Budget line:**
  \[ p_1x_1 + p_2x_2 = m \]

- **Well-behaved preferences**

- **Consumer chooses:**
  \( (x_1^o, x_2^o) \)
Quantity Tax

- Budget line with quantity tax:

\[(p_1 + t)x_1 + p_2x_2 = m\]

- Tax revenues:

\[R^* = tx_1^*\]
Income Tax

New budget line:

\[ p_1 x_1 + p_2 x_2 = m - R^* \]

where

\[ R^* = t x_1^* \]
Questions

Q1: What is the slope of the budget line with the income tax?

Q2: Which kind of tax is this income tax?

Q3: Can the consumer afford $X^*$ when he is paying the income tax?
Answers

- A1: Slope of the budget line with income tax is:
  \[ \frac{-p_1}{p_2} \]

- A2: Lump-sum.
A3: Yes. $X^*$ Is still Affordable

- **Budget line with income tax:**

$$p_1 x_1 + p_2 x_2 = m - tx_1^*$$

- **Substitute $X^*$:**

$$p_1 x_1^* + p_2 x_2^* = m - tx_1^*$$

- **Rearrange:**

$$\left( p_1 + t \right) x_1^* + p_2 x_2^* = m$$
On the Graph

- Budget line with income tax:

- Budget line with quantity tax:

\[ \frac{p_1}{p_2} \]
Q: Is $X^*$ Optimal with the Income Tax?
A: No, it is not.

The budget line is \textbf{not} tangent to indifference curve at \( X^* \).

Optimal choice: \( \overline{X} \)
Q: Which Tax Does the Consumer Prefer?
A: Income Tax Gives Higher Utility to the Consumer

Caution:

- A **uniform** income tax for **all** consumers is not necessarily better than a **uniform** quantity tax.

- Assumption that income tax is lump-sum is key.
Another Question

Suppose that the government can use a uniform value tax on goods 1 and 2 to raise revenue (e.g. a sales tax). Does the consumer still prefer an income tax?
Answer

With a uniform value tax, the budget consumer line reads

\[(1 + t)p_1x_1 + (1 + t)p_2x_2 = m\]

Rearranging:

\[p_1x_1 + p_2x_2 = \frac{m}{1 + t}\]
Rearranging:

\[ p_1x_1 + p_2x_2 = \frac{m}{(1 + t)} \]

Budget line with income tax:

\[ p_1x_1 + p_2x_2 = m - R^* \]

\[ R^* = tp_1x_1^* + tp_2x_2^* = \frac{t}{1 + t} m \]
Answer Continued

- **Budget line with income tax:**

\[ p_1 x_1 + p_2 x_2 = \frac{m}{1 + t} \]

- **Same as budget line under value tax:**

\[ p_1 x_1 + p_2 x_2 = \frac{m}{1 + t} \]