Cost Minimization

- An alternative approach to the decision of the firm
- Long run and short run costs
- Returns to scale and the cost function
- Different types of costs
Alternative Approach

- Thus far: firm chooses inputs in order to maximize profits
- Alternative approach:
  1. Firm chooses inputs in order to minimize the cost of producing given level of output
  2. Firm chooses level of output that maximizes profits
Cost Minimization

Given factors of production $K$ and $L$, with rental prices $w_K$ and $w_L$, find cheapest way to produce a given level of output $y$:

$$\min_{L,K} (w_L L + w_K K)$$

such that:

$$y = F(L, K)$$
Solution to minimization problem is cost function:

\[ c(w_L, w_K, y) = w_L L^*(w_L, w_K, y) + w_K K^*(w_L, w_K, y) \]
Finding the Cost Function

- Cost of using $K$ and $L$:

$$C = w_L L + w_K K$$

- Isocost lines:

$$K = \frac{C}{w_K} - \frac{w_L}{w_K} L$$
Isocost Lines: \[ K = \frac{C}{w_K} - \frac{w_L}{w_K} L \]

\[ C_1 > C_2 > C_3 \]
Cost Minimization

\[ y = F(L, K) \]
Cost Minimization:

Optimal choice:

\[ TRS(L^*, K^*) = -\frac{w_L}{w_K} \]

where

\[ TRS(L^*, K^*) = -\frac{MP_L(L^*, K^*)}{MP_K(L^*, K^*)} \]
Cost Minimization

To find solution use optimality condition plus production function (2 equations in 2 unknowns):

\[ TRS(L^*, K^*) = -\frac{w_L}{w_K} \]

\[ y = F(L^*, K^*) \]
Short-Run and Long-Run Cost Functions

- In the **short run** some factors of production are **fixed**: short-run cost function gives the minimum cost to produce a given level of output, **only** adjusting the variable factors of production.

- In the **long run** all factors are **variable**: long run cost function gives the minimum cost to produce a given level of output, adjusting **all** factors of production.
Example: Short Run

- Find the **short run** cost function in the example of consulting firm:

  $$w_L = 70 \quad y = (3000)^{0.2} (x_l)^{0.6}$$

- Quantity of labor used to produce $y$:

  $$x_l = \left( \frac{y}{(3000)^{0.2}} \right)^{\frac{1}{0.6}}$$
Example: Short Run

- Quantity of labor used to produce $y$:

$$x_l = \left( \frac{y}{(3000)^{0.2}} \right)^{\frac{1}{0.6}}$$

- Short run cost function:

$$c(y) = 70\left( \frac{y}{(3000)^{0.2}} \right)^{\frac{1}{0.6}}$$
Example: Inputs are Perfect Substitutes \[ y = L + 2K \]
Perfect Substitutes

If $\frac{w_L}{w_k} < \frac{1}{2}$ labor only input:

$$L^* = y$$

Cost function is:

$$c(w_L, w_K, y) = w_L y$$
Perfect Substitutes

If \( \frac{w_L}{w_K} > \frac{1}{2} \)  

capital only input:  

\[ K^* = \frac{y}{2} \]

Cost function is:  

\[ c(w_L, w_K, y) = \frac{w_K}{2} y \]
Perfect Substitutes

Summarizing, cost function is:

\[
c(w_L, w_K, y) = \min\left(w_L, \frac{w_K}{2}\right)y
\]
Fixed Proportions Production Function: \( y = \min(L, K) \)
Fixed Proportions

- No matter what input prices are:
  \[ K^* = L^* \]
  \[ y = \min(K^*, L^*) = L^* = K^* \]

- Cost function:
  \[ c(w_L, w_K, y) = w_L L^* + w_K K^* = (w_L + w_K)y \]
Cost Function and Returns to Scale

- **Constant returns:** to double output, need to double all inputs $\rightarrow$ double cost
- **Decreasing returns:** to double output, need to more than double inputs $\rightarrow$ more than double cost
- **Increasing returns:** to double output, need to less than double inputs $\rightarrow$ less than double cost
Cost Function and Returns to Scale

- **Constant returns:**
  \[ c(w_L, w_K, 2) = 2c(w_L, w_K, 1) \]

- **Decreasing returns:**
  \[ c(w_L, w_K, 2) > 2c(w_L, w_K, 1) \]

- **Increasing returns:**
  \[ c(w_L, w_K, 2) < 2c(w_L, w_K, 1) \]