Renting or buying capital
Profit maximization and returns to scale
Renting Capital

- If physical capital is one of the firm’s inputs, the firm can either rent capital or buy it
- E.g.: firm can lease computers

- Problem solved by firm:
  \[
  \max [pF(K, L) - w_L L - w_K K]
  \]
Buying Capital

What problem would the firm solve in the case it decides to **buy** rather than to **rent** capital?

Buying a machine has an impact on the firm’s revenue for several years.

**Q:** how do we compare revenue tomorrow to revenue today? How do we account for risk?
No Uncertainty

Firm can borrow and lend at interest rate \( r = 0.10 \)

Firm is considering how many computers to buy today. Each computer:

- Costs $10,000
- Will be used for two years and then discarded (zero resale value)
No Uncertainty

Objective of the firm is to maximize the present value of profits: present value of revenues minus the present value of costs

- **buy computers**
- **$R \& C$**
- **$R \& C$**

- $t = 0$
- $t = 1$
- $t = 2$
Computing the Present Value

What is the value today of having $1 one year from now, if the interest rate is \( r = 0.1 \)?

\[
\frac{\$1}{1 + 0.1} \approx \$0.9
\]

What is the value today of having $1 two years from now?

\[
\frac{\$1}{(1 + 0.1)^2} \approx \$0.82
\]
Present Value of the Firm at t=0

\[ PV = -p_k K + \frac{R_1 - C_1}{(1 + 0.1)} + \frac{R_2 - C_2}{(1 + 0.1)^2} \]

\[ R_1 - C_1 = p_1 F(K, L_1) - w_{L1} L_1 \]

\[ R_2 - C_2 = p_2 F(K, L_2) - w_{L2} L_2 \]
Maximizing the Present Value

The firm should decide how many computers to buy and how much labor to hire in order to maximize its present value.

\[ V^* = \max_{K,L_1,L_2} [PV] \]

Q: How much would you be willing to pay to buy this firm at time t=0?

\[ V^* \]
What are the Firms Profits?

- Cost of buying computers must be amortized across their lifetime
- To construct cost as a flow consider:
  1. Annual economic depreciation
  2. Opportunity cost due to foregone interest
User Cost of Capital

- **Year 1:**
  1. Annual economic depreciation: $5,000
  2. Opportunity cost of funds: ($10,000)0.10

- **Year 2:**
  1. Annual economic depreciation: $5,000
  2. Opportunity cost of funds: ($5,000)0.10
Profits

Profits = annual revenue - labor cost - user cost of capital
Buy or Rent?

- If the rental rate is larger than the user cost, then it is convenient to buy capital.
- If the rental rate is lower than the user cost, then it is convenient to rent capital.
- If the capital market is competitive, the rental rate should equal the user cost: firm indifferent between buying and renting.
Uncertainty

- Suppose there is uncertainty about price of the product firm is selling.
- Problem gets more complicated because:

1. Firm must take expectation of output price
2. Discount factor must be adjusted to take risk into account
Q: How much profit does a competitive firm with a constant returns to scale technology make in the long-run?
Profit Maximization and Returns to Scale

- A: Zero!
- Suppose it makes positive profits:

\[ \Pi^* = p y^* - w_1 x_1^* - w_2 x_2^* > 0 \]

- Double all inputs:

\[ 2 \Pi^* = p (2 y^*) - w_1 (2 x_1^*) - w_2 (2 x_2^*) > \Pi^* \]
Profit Maximization and Returns to Scale

- Double all inputs:

\[ 2 \Pi^* = p(2y^*) - w_1(2x_1^*) - w_2(2x_2^*) > \Pi^* \]

- This means that the firm was not choosing inputs optimally before! Contradiction!
- Thus, zero profits is the only possibility
Interpretation

Suppose you are the owner of a firm that produces software with a constant returns to scale technology:

\[ y = f(x_L, x_M) \]

where \( x_L \) represents workers and \( x_M \) managers (including yourself)
Interpretation

Then, this firm’s profits in the long run are zero:

- Pay wage to workers $x_L$
- Pay salary to managers $x_M$ (including yourself because of opportunity cost)

Nothing else is left