Technology

- Production functions
- Short run and long run
- Examples of technology
- Marginal product
- Technical rate of substitution
- Returns to scale
### Analogies with Consumer Theory

<table>
<thead>
<tr>
<th>Consumers</th>
<th>Firms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximize utility</td>
<td>Maximize profits</td>
</tr>
<tr>
<td>Constraint: budget line</td>
<td>Constraint: production function</td>
</tr>
</tbody>
</table>
Factors of Production

- **Inputs:** labor, land, capital, raw materials
  
- **Physical** capital: inputs that are themselves produced goods, such as tractors, buildings, computers, etc.
  
- **Financial** capital: money used to start up or run a business (not an input to production)
Production Function

- Function that describes the maximum amount of output that can be produced for a given level of inputs.
- Inputs and outputs are measured in flow units.
- E.g., with labor, L, and capital, K, as inputs:

\[ Y = F(K, L) \]
Technology, Flexibility, and Returns to Scale

Input Flexibility
- How flexible is a firm’s technology?
- To obtain a particular output is it possible to substitute one input for another?
- At what rate?

Changing Scale of Operations
- If a firm doubles all inputs, what happens to output?
- Returns to scale.
Short Run and Long Run

**Short run**: some factors of production are fixed at predetermined levels.

**Example**: in short run, a firm cannot easily vary the size of a plant. It can use machines more intensively.

**E.g.**:

\[ Y = F(K, L) \]
Short Run and Long Run

- **Long run**: all factors of production can be varied. No fixed factors.

- How long is the long run? It depends on the specific type of production.

- For automobile manufacturer it can take years to change size of plant. For travel agency months.
Fixed Proportions: \( y = \min(x_1, x_2) \)
Perfect Substitutes: \( y = ax_1 + x_2 \)
Cobb-Douglas

- **Production function:**
  \[ f(x_1, x_2) = Ax_1^a x_2^b \]

- **Isoquants:**
  \[ x_2 = \frac{1}{y^b} \left( Ax_1^a \right)^{\frac{1}{b}} \]
The Marginal Product

- Consider a firm using inputs \((x_1, x_2)\)
- **Q:** by how much is output going to increase if the firm uses “a bit” more of input 1, while keeping input 2 fixed?
- **A:** Marginal product of factor 1:

\[
\frac{\partial f (x_1, x_2)}{\partial x_1}
\]
The Marginal Product

- Typical assumption in economics: the marginal product of a factor decreases as we use more and more of that factor.

- E.g.: nurses producing radiographies using given machine.
The Marginal Product: Cobb-Douglas

Consider Cobb-Douglas production function in K and L: \( Y = AK^a L^b \)

- Marginal product of capital: \( \frac{\partial Y}{\partial K} = aAK^{a-1}L^b \)
- Marginal product of labor: \( \frac{\partial Y}{\partial L} = bAK^aL^{b-1} \)
Technical Rate of Substitution

- **TRS** is the slope of an isoquant at a given point X.
- Measures the rate at which the firm has to substitute one input for another to keep output constant.
Computing the TRS

**Q:** What is the slope of an isoquant (TRS)?

**A:**

$$TRS(x_1, x_2) = -\frac{\partial f(x_1, x_2)}{\partial x_2}$$
Computing the TRS: Cobb-Douglas

Cobb-Douglas case:

\[ Y = AK^a L^b \]

\[ TRS(K, L) = -\frac{aK^{a-1}L^b}{bK^a L^{b-1}} = -\frac{a}{b} \frac{L}{K} \]
Returns to Scale

**Q:** What happens to output when you double the amount of all the inputs?

**A1:** If output doubles, the technology exhibits *constant returns to scale.*

**E.g.:** travel agency that uses office space and travel agents as inputs.
Increasing Returns to Scale

- A2: If output more than doubles, the technology exhibits increasing returns to scale.
- E.g.: oil pipeline. Double diameter and quadruple cross-section of the pipe.
Increasing Returns to Scale

- If there are increasing returns, it is economically advantageous for a large firm to produce, rather than many small firms.
- Issue of monopoly.
A3: If output less than doubles, the technology exhibits **decreasing returns to scale**.

Applies to firms with large-scale operations where it is more difficult to coordinate tasks and maintain communication between managers and workers.
Cobb-Douglas and Returns to Scale

**Cobb-Douglas:** \[ f(x_1, x_2) = Ax_1^a x_2^b \]

**Scale inputs by a factor** \( t > 1 \):

\[
 f(tx_1, tx_2) = A(tx_1)^a (tx_2)^b = t^{a+b} Ax_1^a x_2^b
\]
Cobb-Douglas and Returns to Scale

- Scale inputs by a factor \( t > 1 \):

\[
f(tx_1, tx_2) = A(tx_1)^a(tx_2)^b = t^{a+b}Ax_1^a x_2^b
\]

- If \( a + b > 1 \), then \( t^{a+b} > t \):

\[
f(tx_1, tx_2) > tAx_1^a x_2^b
\]
Cobb-Douglas and Returns to Scale

- Scale inputs by a factor $t > 1$:

$$f(tx_1, tx_2) = A(tx_1)^a(tx_2)^b = t^{a+b}Ax_1^a x_2^b$$

- If $a + b = 1$, then $t^{a+b} = t$:

$$f(tx_1, tx_2) = tAx_1^a x_2^b$$
Cobb-Douglas and Returns to Scale

- Scale inputs by a factor $t > 1$:

$$f(tx_1, tx_2) = A(tx_1)^a (tx_2)^b = t^{a+b} A x_1^a x_2^b$$

- If $a + b < 1$, then $t^{a+b} < t$:

$$f(tx_1, tx_2) < t A x_1^a x_2^b$$