48-747 Shape Grammars

SHAPE GRAMMARS
computation
Giacomo Barozzi da Vignola (1507-73)

An elementary treatise on architecture comprising the complete study of five orders, with indication on their shadows and the first principles of construction

Constructing the profile of a classical tapered column

There are five steps.

1. Determine the height and largest diameter of the column, \( d \). There are clear rules about preferred proportions between the height and diameter of various types of classical columns (doric, ionic etc.) These measures are normally related to each other as integral multiples of a common module, \( m \). Figure shows the shaft of a column with diameter 2m and height 12m. That is, the proportion between diameter and height is 2:12 or 1:6.

2. At of the shaft's height, draw a straight line, \( l \), across the shaft and draw a semi-circle, \( c \), about the center point of \( l \), \( C \), with radius \( d \) (1m in the figure). The shaft will have the uniform diameter \( d \) below line \( l \).

3. Determine the smallest diameter at the top of the shaft (1.5m in our case). Draw a perpendicular, \( l' \), through an end-point of the diameter. \( l' \) intersects \( c \) at a point \( P \). The line through \( P \) and \( C \) defines together with \( l \) a segment of \( c \).

4. Divide the segment into segments of equal size and divide the shaft above \( l \) into the same number of sections of equal height.

5. Each of these segments intersects \( c \) at a point. Draw a perpendicular line through each of these points and find the intersection point with the corresponding shaft division as shown in the Figure. Each intersection point is a point of the profile.

shape grammars have something to do with computation
**Constructing the profile of a classical column with enthasis**

Here, too, there are five steps.

1. Determine the height and its diameter (or radius) where it is widest and at the top. Following Vignola, the base is again assumed to be 2m wide, and the height is 16m; that is, the proportion of the diameter to height is 1:8. The widest radius occurs at of the total height and is 1+1/9m. The radius at the top is 5/6m.

2. Draw a line, \( l \), through the column where it is widest. Call the center point of the column on that line \( Q \) and the point at distance 1+1/9m from \( Q \) on \( l \), \( P \).

3. Call the point at distance 5/6m from the center at the top and on the same side as \( P \), \( M \). Draw a circle centered at \( M \) with radius 1+1/9m, that is, the circle with the widest radius of the column. This circle intersects the center line of the column at point \( R \).

4. Draw a line through \( M \) and \( R \) and find its intersection, \( O \), with \( l \).

5. Draw a series of horizontal lines that divide the shaft into equal sections. Any such line intersects the center line at a point \( T \). Draw a circle about each \( T \) with radius \( m \). The point of intersection, \( S \), between this circle and the line through \( O \) and \( T \) is a point on the profile.
creative
descriptive
‘mechanical’

computation is …
what is a shape grammar?
Illustration by Peter Murray, "the Architecture of the Italian Renaissance", Shocken Books Inc. 1963, Pp.96
derivation
analyses

original designs

as explanatory devices

how are they used?
mughal gardens

parti subdivision

dealing with canals

dealing with borders
another - **ice-ray designs**
apartment building in manhattan
cultural museum in LA
developing a shape grammar
shapes
spatial relations
rules
shape grammar
designs

**stages** in a shape grammar **development**
basic components of grammars and design

shapes
Shape is a finite arrangement of lines of non-zero length with respect to a coordinate system.

Shapes can be formed by *addition* of shapes which consists of lines in both shapes.

Shapes can be formed by *subtraction* of shapes which consists of the lines in the first shape that are not in the second.

Shapes can be formed by combinations of the two under an *Euclidean transformation*.

A central notion in this definition of shapes is *pictorial equivalence*.

for our purposes, *shape is ...*
euclidean transformations
reflection
glide reflection – example of a combination
euclidean transformations + scale
When two or more shapes combine they form a *spatial relation*. That is, a set of shapes specifies a spatial relation.
<table>
<thead>
<tr>
<th>Shape</th>
<th>$A, B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relation</td>
<td>$A + B$</td>
</tr>
<tr>
<td>Rule</td>
<td>$A \rightarrow A + B$</td>
</tr>
<tr>
<td></td>
<td>$B \rightarrow A + B$</td>
</tr>
</tbody>
</table>

**shape rules**
from **relation** to **rule**
The application of Euclidean transformation does not alter the spatial relation
to derivation
labeling rules helps control rule application
labeled rule based derivation
Labeled rule

\[
\begin{array}{ccc}
& \bullet & \\
\rightarrow & & \\
& & \bullet \\
\end{array}
\]

derivation

\[
\begin{array}{cccc}
& \bullet & \\
\rightarrow & & & \\
& & \bullet & \\
& & \bullet & \\
\rightarrow & & & \\
& & \bullet & \\
& & \bullet & \\
\rightarrow & & & \\
& & & \bullet & \\
\end{array}
\]

labeled rule based \textit{derivation}
Labeled rule

\[ \rightarrow \]

\[ \rightarrow \]

\[ \rightarrow \]

\[ \rightarrow \]

derivation

\[ \rightarrow \]

\[ \rightarrow \]

\[ \rightarrow \]

labeled rule based derivation
Labeled rule

derivation

labeled rule based derivation
shape grammar

rules

shape

grammars

designs
shape grammar
vocabulary

shapes made up from these vocabulary
In the general case, shapes are parameterized schemes

production rules
(or rules of change, encapsulate a spatial relation)

seed shape (we have to start somewhere)

+ a “notion” of rule application

shape grammar embodies change implies generation
shape grammar $G = (S, L, I, R)$

**initial (seed) shape**
belongs to the *universe* of labeled shapes made up of shapes in $S$ and labels in $L$

$R$ contain rules of the form $a \rightarrow b$ where $a$ and $b$ belong to the universe of labeled shapes made of shapes in $S$ and labels in $L$ except $a$ cannot be empty

formally: a shape grammar is
Vocabulary is a limited set of shapes no two of which are similar. The vocabulary provides the basic building blocks by means of which shapes can be generated through shape arithmetic and geometric (euclidean) transformations.
If we are given a set of shapes $S$, then we can create a set $U$ called the *universe* of $S$ in the following manner:

- The empty shape is in $U$
- Every shape in $S$ is in $U$
- If $s$ and $t$ are shapes in $U$, their sum $s+t$ is in $U$
- If $s$ and $t$ are shapes in $U$, their sum under transformations $f$ and $g$, $f(s)+g(t)$, is in $U$

$U$ is thus closed under shape addition and the Euclidean transformations.

**What can you say about the universe of the set of shapes consisting just one shape, a single line of unit length, $\{(0,0),(1,0)\}$?**

- $f$ and $g$, $f(s)+g(t)$, is in $U$

$U$ is thus closed under shape addition and the Euclidean transformations.

**universe**
A rule is *applicable* to the *current shape* which is either the initial shape or a shape produced from the initial shape whenever the left hand side of the rule ‘occurs’ in the object *in which case* it is *replaced* by the right hand side of the rule under rule application

**shape rule application**
A rule $a \rightarrow b$ is applies only if $a$ ‘occurs’ in the given shape $u$ under some ‘transformation’ $T$ in which case $T(a)$ is substituted by $T(b)$ in the current shape.

Rule application

$$v \leftarrow u - T(a) + T(b) \text{ provided } T(a) \leq u$$

We describe this as $u \Rightarrow v$

rule application
Implicit in this definition is the fact that 'parts' of shapes are recognizable in arbitrary ways.
addition and subtraction
A shape is a *subshape* of another if all the lines in the first shape are lines in the second.

A subshape identifies a *part* of a shape.

A shape has *indefinitely many* subshapes.

*subshapes*
decomposing a shape
reduction rules for add and subtraction
addition and subtraction
a metaphor for shape grammars
a pencil –
lead at one end to add marks
an eraser at the other to subtract marks

leaded side has a shape
eraser side has another shape, not necessarily the same

together they specify a rule
– that is, see something
take it away and its place do (make/add) something else

metaphor for shape grammars
deriving designs as a sequence of designs
based on the work of Terry Knight  http://www.mit.edu/~tknight/IJDC/
The language of a grammar $G$, is the set of all shapes (i.e., without non-terminals) that are produced from the initial shape through rule application

$\text{Language} = \{ \text{shape} \mid \text{initial-shape} \Rightarrow^* \text{shape} \}$

By labeling shapes we can “scaffold” the process of generating shapes.
a shape relation and a corresponding rule
consider the symmetry of a square

some possible labeling positions for a square
labeled rules
a labeled rule and sample derivations
another labeled rule and sample derivations
another shape relation and two corresponding rules
some possible labeling positions
and possible shape rules
example labeling

labeling 8,2

labeling 6,3

labeling 4,2

labeling 6,4
derivation using rule with 4,4 labeling
application in a real design context
inspiration
every house worth considering as a work of art
must have a grammar of its own

”grammar” in this sense, means the same thing in any
construction, whether it be of words or of stone or wood.
the worlds we study can be understood by
capturing underlying relationships

the “grammar” of the house is the manifest articulation of
its parts. this will be the “speech” it uses
to be achieved, construction must be grammatical

frank lloyd wright The Natural House