Lecture 1

Large Scale Financial Systems

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Goals of the course

• Teach the students to \textit{systematically design} algorithms and systems for problems in the finance domain.

• Taking finance ideas from the \textit{academic} domain and making them real.

• \textbf{Maxim:} Learn by doing.
Nature of the course

- Largely independent in nature.

- The TAs and I are here to guide you, but you will work as independent teams.

- Think of the course as a \textit{structured independent study course}. 
What we will do in class

• The TAs and I will go through a systematic design of a system for pricing mortgage backed securities or MBSs.

• Use the lectures as a guide for your project.

• There will be five phases to the project. These phases will be defined later.
Logistics

- Students will form a team of 3-4.

- Each team will select a paper from a set of papers provided to you.

- Each team will design a system based on the paper they select.

- Each team will go through the five phases (to be described later).
Some useful tips

- Pick a *balanced team*.

- Stick to the schedule for each phase.
Grading

- Grading will be done depending on the outcome of each phase.

- No tests and homeworks.
Five phases

Description of these phases will be provided later, but here they are.

- Requirements phase (Phase 1).

- High-level design phase (Phase 2).

- Low-level design phase (Phase 3).
Phases (Contd)

- In class presentation (Phase 4).

- Prototype (Phase 5)
Prototype

- This will be a scaled back version of the design.

- Make as many limiting assumptions as possible, but state them carefully.

- You can use C, C++, or JAVA to implement the prototype. Make sure you inform the TAs about the environment you are using for developing your code.
Presentation

• There will be one presentation per team.

• The presentation will be a synopsis of phase 1, 2, and 3.

• Each presentation will be 20 minutes long.

• Last three lectures will be all presentations.
Paper 1

Paper 2

Paper 3

Paper 4

Paper 5

Paper 6

Paper 7

Paper selection

- Do not choose papers 5 or 6 unless you are comfortable with stochastic calculus.

- Focus on the techniques and algorithms in the paper. It is OK if you do not understand all the mathematical derivations.
Goals of reading the paper

- Decide what financial instruments you want to price after reading the paper. Pick 2-3 instruments. You will be required to understand these instruments completely.

- You should have a clear idea about the algorithm proposed in the paper.

- Make a note of advantages/disadvantages of this technique/algorithm.
Requirements document

• Describe the financial instrument in great detail.

• Describe the assets the instruments depend upon. State the assumptions on the prices of these assets. Describe the cash-flow characteristics.

• Describe the financial instruments and there cash-flow characteristics.
Requirements document

- In an abstract sense we are describing what is the semantics of each operation that the user can do.

- In this *very specific example* this amounts to defining the precise semantics of *mortgage backed securities (MBSs)*.
Describing Mortgages

• *Fixed Rate*: The annual interest rate of the mortgage stays fixed throughout the life of the mortgage.

• *Adjustable Rate Mortgages (ARMs)*: The annual interest rate can be adjusted by the loaning agency.
Fixed Rate Mortgages

• Let $MB_0$ be the original mortgage balance.

• Let $c$ be the simple monthly interest rate.

• Let $MP$ be the monthly mortgage payment.

• Let $n$ be the number of months.
Relationship between $MP$ and $MB_0$

- The following equation should hold between $MP$ and $MB_0$:

\[ MB_0 = MP \sum_{i=1}^{n} (1 + c)^{-i} \]

\[ = MP \frac{1 - (1 + c)^{-n}}{c} \]

- Hence the monthly mortgage payment $MP$ is given in terms of the mortgage amount $MB_0$ using the following formula:

\[ MP = MB_0 \frac{c(1+c)^n}{(1+c)^n-1} \]
Principal at time $t$

- Let $MB_t$ the remaining mortgage balance at time $t$.

- We have the following relationship between $MB_t$ and $MP$.

$$MB_t = MP \frac{1 - (1 + c)^{-(n-t)}}{c}$$

- So we have the following equation between $MB_0$ and $MB_t$:

$$MB_t = MB_0 \frac{(1 + c)^n - (1 + c)^t}{(1 + c)^n - 1}$$
Breaking the mortgage payments

- At time $t$ the mortgage balance is $MB_{t-1}$ ($t \geq 1$).

- The interest $I_t$ on this is mortgage balance is:
  \[ cMB_{t-1} \]

- The mortgage payment $MP$ at time $t$ is broken into two parts: interest payment $I_t$ and payment applied towards principal $P_t$. We have the following equation:
  \[ MP = I_t + P_t \]
Scenarios

- A requirements document for a large software system has a huge number of scenarios.

- Basically, *scenarios* describe what should happen in specific cases.

- For example, in the requirements document for an online brokerage system a scenario might describe what should happen when a user logs on and buys a stock.
Examples

• In this case, scenarios are simply examples of cash flows.

• Consider a mortgage of 100,000, annual mortgage rate (12c) of 9.5% and time period of 30 years (360 months).

• Check that the monthly mortgage rate $MP$ is 840.85.
Example continued

- Check that $I_0 = 791.67$ and $P_0 = 49.19$.

- Check that $I_{215} = 574.95$ and $P_{215} = 265.90$.

- $I_t$ is a decreasing function of $t$ and $P_t$ is an increasing function of $t$. (Why?).
ARMs

- ARM\(s\) start out with an initial interest rate.

- ARM\(s\) interest rate can be adjusted by a margin \(m\) at a frequency specified in the contract.

- *Lifetime cap* \(c_L\): This is an upper bound that the interest rate cannot exceed.

- *Lifetime floor* \(c_F\): This is a lower bound on the interest rate.
ARMs

- Let us the interest rate be $c(t - 1)$ at time $t - 1$ and suppose we are adjusting at time $t$,

- The new interest rate $c(t)$ is given by the following cases:
  - if $x(t) + m > c(t - 1)$
    $$\min [x(t) + m, c_L, c(t - 1) + c_P]$$
  - if $x(t) + m \leq c(t - 1)$
    $$\max [x(t) + m, c_F, c(t - 1) - c_P]$$
Explanation of terms

- $x(t)$: Underlying index specified in the contract. Two widely used indices are cost of funds index (COFI) and a constant maturity (one year or five year) Treasury index.

- $c_L$ and $c_F$ are the lifetime cap and floor respectively.

- $c_P$ denotes the ARMs periodic cap, i.e., cannot adjust by more than this amount in a single period.
New Mortgage Payment

• Assume that current time is $t$, the interest rate, mortgage balance at time $t - 1$ are $c(t - 1)$ and $MB_{t-1}$.

• The new *adjusted* interest rate is $c(t)$.

• The new mortgage payment at time $t$

  $MP(t)$ is given by the following expression:

  \[ MB_{t-1} \frac{1 - (1 + c(t))^{-(n-t)}}{c(t)} \]

• Everything else stays the same.
Action Items

- Pick your teams (3-4 students) and send e-mail to the TAs or me.

- Pick a paper from the bundle of papers provided.

- Papers should be in London and New York.

- Read the paper and decide on the financial instruments you are going to price.
- Start reading about the financial instruments you are going to price.