Objectives

- Options
  - basic strategies
  - introduction to some pricing restrictions
  - introduction to binomial model

Payoff Diagrams
Some strategies

- Naked
- Protective put
- Covered call
- Straddle

Protective put

- Purchase underlying security
- Purchase put option, exercise price X
Algebraically

<table>
<thead>
<tr>
<th>Position</th>
<th>$S_T &lt; X$</th>
<th>$S_T &gt; X$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Underlying</td>
<td>$S_T$</td>
<td>$S_T$</td>
</tr>
<tr>
<td>Put</td>
<td>$X - S_T$</td>
<td>$S_T$</td>
</tr>
<tr>
<td>Net</td>
<td>$X$</td>
<td>$S_T$</td>
</tr>
</tbody>
</table>

Why do this?
Covered call

- Purchase underlying
- Write call option against it

Straddle

- Buy put and call, both at the same strike
Other spreads

• combination of 2+ calls or puts, same asset with differing exercise prices or times to expiration
  – Vertical or money spread
    • Same maturity and different exercise price
  – Horizontal or time spread
    • different maturities

Main Points

• Lots of strategies possible
• Options allow you to customize cash flows across states in the future...
Put call parity

- Relationship between price of European call and put
- Independent of assumptions about randomness in underlying
  - stocks
  - indexes
  - bonds, currencies, etc.

Final Payoffs: Long call and short put
Arbitrage relationship

Example

- Stock price = $10
- maturity 1 year, interest rate 10%
- exercise price = $9.90
  - Put price = $1
  - Call price ?
Call price < stock price

- Suppose stock price = 10, call price = 11, exercise price = 4
- Strategy:
  - purchase stock
  - short option

Payoffs on strategy

<table>
<thead>
<tr>
<th>Item</th>
<th>Today</th>
<th>Maturity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$S_T &lt; 4$</td>
</tr>
<tr>
<td>Long stock</td>
<td>-$10</td>
<td>$S_T$</td>
</tr>
<tr>
<td>Short option</td>
<td>+$11</td>
<td>0</td>
</tr>
<tr>
<td>Net</td>
<td>+$1</td>
<td>$S_T$</td>
</tr>
</tbody>
</table>
American calls

• Don’t exercise early without dividends

Binomial Model

• Workhorse model for derivatives valuation
• Very flexible
• Basic Assumption about stock price moves
  – binomial process for stock price: either goes up or down next period
1 period example

- \( S_0 = 10 \), current stock price, \( r = 1.1 \) = riskfree rate
- probability stock increase = \( q = 0.5 \)
  - \( u \): multiplicative upward movement = 2
  - \( d \): downward movement = 0.5
  - \( u > r > d \)
- Why?

Stock price movements

\[
S_i = \begin{cases} 
  uS_0 & \text{probability} = 0.5 \\
  dS_0 & \text{probability} = 0.5 
\end{cases}
\]

\( S_0 = 10 \)

- \( uS_0 = 20 \), prob 0.5
- \( dS_0 = 5 \), prob 0.5
Call option, \( X=10, \) 1 period maturity

Valuation

- Basic idea: come up with strategy of stock and bonds with same payoff as option
  - arbitrage says: cost of strategy = cost of option
- \( \alpha \): number of shares, \( \beta \): number of bonds
  - ‘u’: \( \alpha 20 + \beta 1.05 = 10 \)
  - ‘d’: \( \alpha 5 + \beta 1.05 = 0 \)
  - Solving: \( \alpha = \frac{2}{3}, \ \beta = -\frac{2}{3}(\frac{5}{1.1}) \)
Call value

• Value of call=value of portfolio
  \[ \alpha(10)+\beta+=(2/3)(10)-(2/3)(5/1.1)=3.64 \]

Implications

• 2 states in future: needed 2 securities (stock and bond) to hedge/price option
• Call = long stock and short bond: levered position in portfolio
• What happened to probability of u and d?
Recap

- Pricing via replication
- probabilities of up and down doesn’t matter!
- Steps:
  - check if #securities $\geq$ # states
  - if so, match cash flows state by state

Summary

- Options
  - strategies
  - basic pricing restrictions
  - introduction to binomial model
Next Time

- Extend binomial model to deal with multiple periods
- Applying model to portfolio strategies
  – Sharpe and Perold in readings packet